

On the Local Value of a Vertex Degree Function Index of Graphs

Kinkar Chandra Das

*Department of Mathematics, Sungkyunkwan University, Suwon 16419,
Republic of Korea*

kinkardas2003@gmail.com, kinkar@skku.edu

(Received May 23, 2026)

Abstract

Let f be a real-valued function defined on the integers in the interval $[1, n - 1]$, and let G be a graph with n non-isolated vertices. The concept of the local value of a degree-function index was introduced by Rada in [Local value of a vertex-degree function index of a graph, *MATCH Commun. Math. Comput. Chem.* **95** (2026) 111–124]. The degree-function index $H_f(G)$ is defined by

$$H_f(G) = \sum_{u \in V(G)} f(d_G(u)),$$

where $d_G(u)$ denotes the degree of the vertex u in G . The local value of H_f at a vertex u , denoted by $f_G(u)$, measures the contribution of u to the total value of the index. In the same paper, Rada posed four questions concerning the local value of a degree-function index and the zeroth-order general Randić index.

In this paper, we provide answers to these questions. We establish an upper bound for the difference between the local values of a vertex-degree function index f at a vertex u in a graph G and in an induced subgraph H of G . Furthermore, we derive an upper bound for the difference between the local values of f at two vertices of a graph G . We also obtain some upper and lower bounds for the zeroth-order general Randić local value $(\mathcal{R}_\alpha)_G(u)$. As applications

of these results, we identify the vertices attaining the maximum local value of the zeroth-order general Randić index $\mathcal{R}_\alpha$ for $0 < \alpha < 1$, and the minimum local value for $\alpha < 0$. Finally, we present some concluding remarks and discuss several possible directions for future research.

1 Introduction

Let G be a simple graph with vertex set $V = V(G)$ and edge set $E = E(G)$, where $|V(G)| = n$. The degree of a vertex $u \in V(G)$ is denoted by $d_G(u)$, and when the graph is clear from the context, we simply write $d_u = d_G(u)$. The neighborhood of a vertex u is denoted by $N_G(u)$. Let S_1 and S_2 be two sets of vertices. We denote by $S_1 \setminus S_2$ the set of vertices belonging to S_1 but not to S_2 . Similarly, $S_1 \cap S_2$ denotes the set of vertices common to both S_1 and S_2 . Other undefined notations and terminology on the graph theory can be found in [5].

Let n be a positive integer, and let

$$f : [1, n-1] \cap \mathbb{N} \rightarrow \mathbb{R}$$

be a real-valued function. Following the notation introduced in the literature, we use the same symbol f to denote the corresponding vertex-degree function index defined by

$$f(G) = \sum_{v \in V(G)} f(d_v). \quad (1)$$

This concept was first introduced by Linial and Rozenman in 2002 [16]. Since then, vertex-degree function indices have attracted considerable attention and have been extensively studied in recent years; see, for example, [1–4, 6, 7, 11, 20–23].

One of the most important classes of vertex-degree function indices was introduced by Li and Zheng in 2005 [15], namely the zeroth-order general Randić index. For a graph G with no isolated vertices, the zeroth-order

general Randić index is defined by

$$\mathcal{R}_\alpha(G) = \sum_{u \in V(G)} \mathcal{R}_\alpha(d_u) = \sum_{u \in V(G)} (d_u)^\alpha.$$

Some mathematical properties on $\mathcal{R}_\alpha$ index have been established in [8, 12–14, 18, 19]. Two particularly important special cases are the following. For $\alpha = 2$, the index reduces to the first Zagreb index [10]:

$$R_2(G) = \sum_{u \in V(G)} d_u^2 = \sum_{vw \in E(G)} (d_v + d_w).$$

For $\alpha = 3$, the index is known as the forgotten index, a terminology introduced in [9, 10]:

$$R_3(G) = \sum_{u \in V(G)} d_u^3 = \sum_{vw \in E(G)} (d_v^2 + d_w^2).$$

Very recently, Rada [17] defined and investigated the local value of a vertex-degree function index f at a vertex u in a graph G , denoted by $f_G(u)$. Intuitively, $f_G(u)$ measures the contribution of the vertex u to the global index $f(G)$. The definition of the local value is constructed in such a way that

$$f(G) = \sum_{u \in V(G)} f_G(u).$$

The remainder of the paper is organized as follows. In Section 2, we present the necessary definitions and preliminary results related to local values of vertex-degree function indices. In Section 3, we establish an upper bound for the difference between the local values of a vertex-degree function index f at a vertex u in a graph G and in its induced subgraph H . We also derive an upper bound for the difference between the local values of f at two vertices of a graph G . In Section 4, we obtain some upper and lower bounds for the zeroth-order general Randić local value $(\mathcal{R}_\alpha)_G(u)$. Using these bounds, we characterize the vertices attaining the maximal local values of the zeroth-order general Randić index for $0 < \alpha < 1$, as well as the vertices attaining the minimal local values for $\alpha < 0$. Finally,

Section 5 concludes the paper and discusses several possible directions for future research.

2 Definitions and preliminaries

Throughout this paper, let n be a positive integer, and let $f : [1, n-1] \cap \mathbb{N} \rightarrow \mathbb{R}$ be a real-valued function. Given a graph G with n non-isolated vertices, it is clear that for every vertex $u \in V(G)$, the degree $d_G(u) \in [1, n-1] \cap \mathbb{N}$. Therefore, the vertex-degree function index

$$f(G) = \sum_{u \in V(G)} f(d_G(u))$$

is well defined, as given in Equation (1).

Definition 1. Let G be a graph with n non-isolated vertices. The local value of the degree-function index f at a vertex $u \in V(G)$, denoted by $f_G(u)$, is defined as:

$$f_G(u) = (1 - d_G(u)) f(d_G(u)) + \sum_{w \in N_G(u)} f(d_G(w)). \quad (2)$$

This quantity $f_G(u)$ can be interpreted as the contribution of the vertex u to the total value of the degree-function index $f(G)$.

The following results were established in [17].

Lemma 1. [17] *Let G be a graph with n non-isolated vertices. Then*

$$\sum_{u \in V(G)} \sum_{w \in N_G(u)} f(d_w) = \sum_{u \in V(G)} d_u f(d_u). \quad (3)$$

Lemma 2. [17] *Let G be a graph with n non-isolated vertices. Then the sum of the local values over all vertices equals the total degree-function index of the graph, that is,*

$$\sum_{u \in V(G)} f_G(u) = f(G).$$

3 Relations between two local values of a vertex-degree function index

Rada [17] mentioned that several natural questions arise when studying the local value of a vertex-degree function index f at a vertex u in a graph G :

Problem 1. Let H be an induced subgraph of a graph G , and let $u \in V(H)$. Is there a relation between $f_G(u)$ and $f_H(u)$?

Problem 2. Let $u \in V(G)$. If $v \in V(G)$ is such that $d_v > d_u$, is there a relation between $f_G(u)$ and $f_G(v)$?

We are looking to find an answer of the above questions. First we concentrate on the **Problem 1**.

Theorem 1. *Let H be an induced subgraph of a non-isolated graph G . If $f(x)$ is an non-negative increasing function on x , then*

$$f_H(u) - f_G(u) \leq \left(d_G(u) - 1\right) f(d_G(u)) - \left(d_H(u) - 1\right) f(d_H(u)).$$

Proof. Since H is an induced subgraph of a graph G , for all $u \in H$, we have

$$N_G(u) = \left(N_G(u) \setminus N_H(u)\right) \cup N_H(u) \quad \text{and} \quad \left(N_G(u) \setminus N_H(u)\right) \cap N_H(u) = \emptyset.$$

Thus we have

$$\sum_{w \in N_G(u)} f(d_G(w)) = \sum_{w \in N_H(u)} f(d_G(w)) + \sum_{w \in N_G(u) \setminus N_H(u)} f(d_G(w)). \quad (3)$$

Again since H is an induced subgraph of a graph G , we have $d_G(u) \geq d_H(u)$ for all $u \in H$. Since $f(x)$ is an increasing function, we have $f(d_G(u)) \geq f(d_H(u))$ for all $u \in H$. Since $f(x)$ is an non-negative function, we have $f(d_G(w)) \geq 0$ and hence

$$\sum_{w \in N_G(u) \setminus N_H(u)} f(d_G(w)) \geq 0.$$

Using the above results in (3), we obtain

$$\sum_{w \in N_G(u)} f(d_G(w)) \geq \sum_{w \in N_H(u)} f(d_G(w)) \geq \sum_{w \in N_H(u)} f(d_H(w)).$$

as $f(d_G(u)) \geq f(d_H(u))$ for all $u \in H$.

Using this, from (2), we obtain

$$\begin{aligned} f_G(u) &= \left(1 - d_G(u)\right) f(d_G(u)) + \sum_{w \in N_G(u)} f(d_G(w)) \\ &\geq \left(1 - d_G(u)\right) f(d_G(u)) + \sum_{w \in N_H(u)} f(d_H(w)). \end{aligned}$$

For $u \in V(H)$, we have

$$f_H(u) = \left(1 - d_H(u)\right) f(d_H(u)) + \sum_{w \in N_H(u)} f(d_H(w)).$$

From the above two results, we obtain

$$f_H(u) - f_G(u) \leq \left(d_G(u) - 1\right) f(d_G(u)) - \left(d_H(u) - 1\right) f(d_H(u)).$$

This completes the proof of the theorem. ■

Corollary 1. *Let H be an induced subgraph of a graph G with maximum degree Δ . If $f(x)$ be a non-negative increasing function on x , then*

$$f_H(u) - f_G(u) \leq (\Delta - 1) f(\Delta)$$

for any non-isolated vertex $u \in V(H)$.

Proof. Since $f(x)$ is a non-negative increasing function on x , and u is any non-isolated vertex in H , we have $d_H(u) \geq 1$ and $f(d_H(u)) \geq 0$, and hence $\left(d_H(u) - 1\right) f(d_H(u)) \geq 0$. Moreover, $d_G(u) \leq \Delta$ and $f(d_G(u)) \leq f(\Delta)$, and hence $\left(d_G(u) - 1\right) f(d_G(u)) \leq (\Delta - 1) f(\Delta)$. From Theorem 1, we get the required result. ■

Corollary 2. *Let H be an induced subgraph of a graph G of order n . If $f(x)$ is a non-negative increasing function on x , then*

$$f_H(u) - f_G(u) \leq (n-2)f(n-1)$$

for any non-isolated vertex $u \in V(H)$.

Proof. Since $\Delta \leq n-1$, from Corollary 1, we get the result. ■

We now discuss on the **Problem 2**.

Theorem 2. *Let G be a graph with maximum degree Δ and minimum degree δ . If $f(x)$ is an increasing function on x , then for $u, v \in V(G)$,*

$$f_G(u) - f_G(v) \leq \left(d_G(u) + d_G(v) - |N_G(u) \cap N_G(v)| - 1 \right) \left(f(\Delta) - f(\delta) \right)$$

with equality if and only if $d_G(w) = \Delta$ for $w \in N_G(u) \setminus N_G(v)$ or $d_G(u) = |N_G(u) \cap N_G(v)|$, $d_G(w) = \delta$ for $w \in N_G(v) \setminus N_G(u)$ or $d_G(v) = |N_G(u) \cap N_G(v)|$, and $d_G(v) = \Delta$, $d_G(u) = \delta$.

Proof. One can easily see that

$$N_G(u) = \left(N_G(u) \cap N_G(v) \right) \cup \left(N_G(u) \setminus N_G(v) \right).$$

For $u \in V(G)$, using the above, from (2), we obtain

$$\begin{aligned} f_G(u) &= \left(1 - d_G(u) \right) f(d_G(u)) + \sum_{w \in N_G(u)} f(d_G(w)) \\ &= \left(1 - d_G(u) \right) f(d_G(u)) + \sum_{w \in N_G(u) \cap N_G(v)} f(d_G(w)) \\ &\quad + \sum_{w \in N_G(u) \setminus N_G(v)} f(d_G(w)). \end{aligned}$$

Similarly, for $v \in V(G)$, from (2), we obtain

$$\begin{aligned} f_G(v) &= \left(1 - d_G(v)\right) f(d_G(v)) + \sum_{w \in N_G(u) \cap N_G(v)} f(d_G(w)) \\ &\quad + \sum_{w \in N_G(v) \setminus N_G(u)} f(d_G(w)). \end{aligned}$$

From the above two results, we obtain

$$\begin{aligned} f_G(u) - f_G(v) &= \left(d_G(v) - 1\right) f(d_G(v)) - \left(d_G(u) - 1\right) f(d_G(u)) \\ &\quad + \sum_{w \in N_G(u) \setminus N_G(v)} f(d_G(w)) - \sum_{w \in N_G(v) \setminus N_G(u)} f(d_G(w)). \end{aligned} \tag{4}$$

Since G has maximum degree Δ and minimum degree δ , we have $\delta \leq d_G(u) \leq \Delta$ for all $u \in V(G)$. Again since $f(x)$ is an increasing function, we have $f(\delta) \leq f(d_G(u)) \leq f(\Delta)$ for all $u \in V(G)$. Thus we obtain

$$\sum_{w \in N_G(u) \setminus N_G(v)} f(d_G(w)) \leq \left(d_G(u) - |N_G(u) \cap N_G(v)|\right) f(\Delta)$$

with equality if and only if $d_G(w) = \Delta$ for $w \in N_G(u) \setminus N_G(v)$ or $d_G(u) = |N_G(u) \cap N_G(v)|$, and

$$\sum_{w \in N_G(v) \setminus N_G(u)} f(d_G(w)) \geq \left(d_G(v) - |N_G(u) \cap N_G(v)|\right) f(\delta)$$

with equality if and only if $d_G(w) = \delta$ for $w \in N_G(v) \setminus N_G(u)$ or $d_G(v) = |N_G(u) \cap N_G(v)|$. Using the above results in (4), we obtain

$$\begin{aligned} f_G(u) - f_G(v) &\leq (d_G(v) - 1) f(\Delta) - (d_G(u) - 1) f(\delta) + \left(d_G(u) \right. \\ &\quad \left. - |N_G(u) \cap N_G(v)|\right) f(\Delta) - \left(d_G(v) - |N_G(u) \cap N_G(v)|\right) f(\delta) \\ &= \left(d_G(u) + d_G(v) - |N_G(u) \cap N_G(v)| - 1\right) \left(f(\Delta) - f(\delta)\right). \end{aligned}$$

This is the first part of the proof. Moreover, the above equality holds if and only if $d_G(w) = \Delta$ for $w \in N_G(u) \setminus N_G(v)$ or $d_G(u) = |N_G(u) \cap N_G(v)|$, $d_G(w) = \delta$ for $w \in N_G(v) \setminus N_G(u)$ or $d_G(v) = |N_G(u) \cap N_G(v)|$, and $d_G(v) = \Delta$, $d_G(u) = \delta$. ■

Corollary 3. *Let G be a graph with n non-isolated vertices. If $f(x)$ is an increasing function on x , then*

$$f_G(u) - f_G(v) \leq \left(d_G(u) + d_G(v) - |N_G(u) \cap N_G(v)| - 1 \right) \left(f(n-1) - f(1) \right)$$

with equality if and only if $d_G(w) = n - 1$ for $w \in N_G(u) \setminus N_G(v)$ or $d_G(u) = |N_G(u) \cap N_G(v)|$, $d_G(w) = 1$ for $w \in N_G(v) \setminus N_G(u)$ or $d_G(v) = |N_G(u) \cap N_G(v)|$, and $d_G(v) = n - 1$, $d_G(u) = 1$.

Proof. Since G has non-isolated vertices, the maximum degree $\Delta \leq n - 1$ and the minimum degree $\delta \geq 1$ in G . From Theorem 2, we get the result. Moreover, the equality holds if and only if $d_G(w) = n - 1$ for $w \in N_G(u) \setminus N_G(v)$ or $d_G(u) = |N_G(u) \cap N_G(v)|$, $d_G(w) = 1$ for $w \in N_G(v) \setminus N_G(u)$ or $d_G(v) = |N_G(u) \cap N_G(v)|$, and $d_G(v) = n - 1$, $d_G(u) = 1$. ■

Remark 1. The bound in Corollary 3 is tight. If G is a star graph with central vertex v and pendant vertex u , then $d_G(u) = 1$, $d_G(v) = n - 1$ and $|N_G(u) \cap N_G(v)| = 0$. Thus we have

$$f_G(u) = f(n-1) \text{ and } f_G(v) = -(n-2)f(n-1) + (n-1)f(1),$$

and hence

$$\begin{aligned} f_G(u) - f_G(v) &= (n-1) \left(f(n-1) - f(1) \right) \\ &= \left(d_G(u) + d_G(v) - |N_G(u) \cap N_G(v)| - 1 \right) \left(f(n-1) - f(1) \right). \end{aligned}$$

4 Maximum or minimum local value of the zeroth-order general Randić index $\mathcal{R}_\alpha$

Let n be a positive integer, and let $\mathcal{R}_\alpha : [1, n-1] \cap \mathbb{N} \rightarrow \mathbb{R}$ be the function defined by

$$\mathcal{R}_\alpha(x) = x^\alpha,$$

where $\alpha \in \mathbb{R}$. For a graph G with vertex set $V(G)$, the corresponding function index is given by

$$\mathcal{R}_\alpha(G) = \sum_{u \in V(G)} \mathcal{R}_\alpha(d_u) = \sum_{u \in V(G)} (d_u)^\alpha,$$

which is known as the zeroth-order Randić index of G .

In this setting, the local value of $\mathcal{R}_\alpha$ at a vertex $u \in V(G)$ is defined by

$$(\mathcal{R}_\alpha)_G(u) = (1 - d_u)d_u^\alpha + \sum_{w \in N_G(u)} d_w^\alpha,$$

where d_u denotes the degree of the vertex u , and $N_G(u)$ represents the neighborhood of u .

In [17], Rada mentioned that the following natural questions when studying on bounds for $(\mathcal{R}_\alpha)_G(u)$:

Problem 3. Find vertices with maximal local value of the zeroth-order general Randić index $\mathcal{R}_\alpha$ when $0 < \alpha < 1$.

Problem 4. Find vertices with minimal local value of the zeroth-order general Randić index $\mathcal{R}_\alpha$ when $\alpha < 0$.

Here we give an upper bound on the local value of the zeroth-order general Randić index $\mathcal{R}_\alpha$ when $0 < \alpha < 1$.

Theorem 3. *Let G be a graph with n non-isolated vertices, $u \in V(G)$, and $0 < \alpha < 1$. Then*

$$(\mathcal{R}_\alpha)_G(u) \leq \max \left\{ (1 - y_1) y_1^\alpha + y_1 (n - 1)^\alpha, (1 - y_2) y_2^\alpha + y_2 (n - 1)^\alpha \right\}$$

with $y_1 = \lfloor y \rfloor$ and $y_2 = \lceil y \rceil$, where y is the unique solution of

$$(n-1)^\alpha = y^{\alpha-1} \left((\alpha+1)y - \alpha \right).$$

Proof. For $u \in V(G)$,

$$\begin{aligned} (R_\alpha)_G(u) &= \left(1 - d_G(u)\right) d_G(u)^\alpha + \sum_{w \in N_G(u)} d_G(w)^\alpha \\ &\leq \left(1 - d_G(u)\right) d_G(u)^\alpha + \sum_{w \in N_G(u)} (n-1)^\alpha \\ &= \left(1 - d_G(u)\right) d_G(u)^\alpha + d_G(u) (n-1)^\alpha. \end{aligned}$$

Let us consider a function

$$h(x) = (1-x)x^\alpha + x(n-1)^\alpha, \quad 1 \leq x \leq n-1 \quad \text{and} \quad 0 < \alpha < 1.$$

Then we have

$$h'(x) = \alpha x^{\alpha-1} - (\alpha+1)x^\alpha + (n-1)^\alpha, \quad h''(x) = -\alpha(1-\alpha)x^{\alpha-2} - (\alpha+1)\alpha x^{\alpha-1}.$$

Since $0 < \alpha < 1$ and $1 \leq x \leq n-1$, from the above, we have $h''(x) < 0$ and hence $h(x)$ is strictly concave on $[1, n-1]$. So it has a unique maximizer determined by $h'(x) = 0$. Thus the maximum value is attained at the unique solution $x = y \in (1, n-1)$ satisfying

$$\alpha y^{\alpha-1} - (\alpha+1)y^\alpha + (n-1)^\alpha = 0.$$

Since $d_G(u)$ is an integer, we have

$$\begin{aligned} (R_\alpha)_G(u) &\leq h(d_G(u)) \leq \max \left\{ h(y_1), h(y_2) \right\} \\ &= \max \left\{ (1-y_1)y_1^\alpha + y_1(n-1)^\alpha, (1-y_2)y_2^\alpha + y_2(n-1)^\alpha \right\} \end{aligned}$$

with $y_1 = \lfloor y \rfloor$ and $y_2 = \lceil y \rceil$, where y is the unique solution of

$$(n-1)^\alpha = y^{\alpha-1} \left((\alpha+1)y - \alpha \right). \quad \blacksquare$$

Remark 2. From Theorem 3, it follows that the vertices y_1 or y_2 attain

the maximal local value of the zeroth-order general Randić index R_α when $0 < \alpha < 1$, where $y_1 = \lfloor y \rfloor$ and $y_2 = \lceil y \rceil$, with y being the unique solution of

$$(n-1)^\alpha = y^{\alpha-1}((\alpha+1)y - \alpha).$$

Proceeding similarly to Theorem 3, we next present a lower bound on the local value of the zeroth-order general Randić index R_α when $-1 < \alpha < 0$, and therefore omit the proof.

Theorem 4. *Let G be a graph with n non-isolated vertices, $u \in V(G)$, and $-1 < \alpha < 0$. Then*

$$(R_\alpha)_G(u) \geq \min \left\{ (1-y_1)y_1^\alpha + y_1(n-1)^\alpha, (1-y_2)y_2^\alpha + y_2(n-1)^\alpha \right\}$$

with $y_1 = \lfloor y \rfloor$ and $y_2 = \lceil y \rceil$, where y is the unique solution of

$$(n-1)^\alpha = y^{\alpha-1}((\alpha+1)y - \alpha).$$

Remark 3. From Theorem 4, it follows that the vertices y_1 or y_2 attain the minimal local value of the zeroth-order general Randić index R_α when $-1 < \alpha < 0$, where $y_1 = \lfloor y \rfloor$ and $y_2 = \lceil y \rceil$, with y being the unique solution of

$$(n-1)^\alpha = y^{\alpha-1}((\alpha+1)y - \alpha).$$

5 Concluding remarks

In this paper, we established an upper bound for the difference between the local values of a vertex-degree function index f at a vertex u in a graph G and in an induced subgraph H of G . Furthermore, we derived an upper bound for the difference between the local values of f at two vertices of a graph G . We also obtained some upper and lower bounds for the zeroth-order general Randić local value $(\mathcal{R}_\alpha)_G(u)$. As applications of these results, we identified the vertices attaining maximal (respectively, minimal) local values of the zeroth-order general Randić index R_α for $0 < \alpha < 1$ (respectively, $\alpha < 0$).

The results presented in this paper suggest several interesting directions

for future research. In particular, the following problems merit further investigation:

Problem 5. Determine upper and lower bounds for $f_H(u) - f_G(u)$, where H is an induced subgraph of a graph G , in terms of various graph parameters, and characterize the extremal graphs attaining these bounds.

Problem 6. Determine upper and lower bounds for $f_G(u) - f_G(v)$, for vertices $u, v \in V(G)$, in terms of different graph parameters, and characterize the corresponding extremal graphs.

Problem 7. Determine upper and lower bounds for the zeroth-order general Randić local value $(R_\alpha)_G(u)$, in terms of graph parameters, and characterize the extremal graphs for which equality holds.

Acknowledgment: The author is grateful to the anonymous referee for their careful reading of the manuscript and for their valuable comments and suggestions, which have significantly improved the presentation of this paper. K. C. Das is supported by National Research Foundation funded by the Korean government (Grant No. RS-2026-25475577).

References

- [1] A. Ali, K. C. Das, A. M. Alanazi, I. Tomescu, On diameter-constrained unicyclic graphs and BID indices, *Mediterranean Jour. Math.* **23** (41) (2026) 1–22.
- [2] A. Ali, K. C. Das, A. M. Alotaibi, T. S. Hassan, On diameter-constrained trees and BID indices, *J. Appl. Math. Comput.* **72** (2026) #71.
- [3] A. Ali, K. C. Das, A. A. Attiya, Bond incident degree indices of fixed-order unicyclic graphs, *Discr. Math. Lett.* **16** (2025) 135–142.
- [4] S. Bermudo, R. Cruz, J. Rada, Vertex-degree function index on tournaments, *Commun. Comb. Optim.* **10** (2025) 443–452.
- [5] J. A. Bondy, U.S R. Murty, *Graph Theory*, Springer, Berlin, 2008, pp. 1–676.

-
- [6] X. Cheng, X. Li, Some bounds for the vertex-degree function index of connected graphs with given minimum and maximum degrees, *MATCH Commun. Math. Comput. Chem.* **90** (2023) 175–186.
 - [7] K. C. Das, On the vertex degree function of graphs, *Comput. Appl. Math.* **44** (2025) #183.
 - [8] K. C. Das, M. Dehmer, Comparison between the zeroth-order Randić index and the sum-connectivity index, *Appl. Math. Comput.* **274** (2016) 585–589.
 - [9] B. Furtula, I. Gutman, A forgotten topological index, *J. Math. Chem.* **53** (2015) 1184–1190.
 - [10] I. Gutman, N. Trinajstić, Graph theory and molecular orbitals. Total π -electron energy of alternant hydrocarbons, *Chem. Phys. Lett.* **17** (1972) 535–538.
 - [11] D. He, Z. Ji, C. Yang, K. C. Das, Extremal graphs to vertex degree function index for convex functions, *Axioms* **12** (2023) #31.
 - [12] Y. Hu, X. Li, Y. Shi, T. Xu, Connected (n, m) -graphs with minimum and maximum zeroth-order general Randić index, *Discr. Appl. Math.* **155** (2007) 1044–1054.
 - [13] Y. Hu, X. Li, Y. Shi, T. Xu, I. Gutman, On molecular graphs with smallest and greatest zeroth-order general Randić index, *MATCH Commun. Math. Comput. Chem.* **54** (2005) 425–434.
 - [14] X. Li, Y. Shi, (n, m) -graphs with maximum zeroth-order general Randić index for $\alpha \in (-1, 0)$, *MATCH Commun. Math. Comput. Chem.* **62** (2009) 163–170.
 - [15] X. Li, J. Zheng, An unified approach to the extremal trees for different indices, *MATCH Commun. Math. Comput. Chem.* **51** (2005) 195–208.
 - [16] N. Linial, E. Rozenman, An extremal problem on degree sequences of graphs, *Graphs Comb.* **18** (2002) 573–582.
 - [17] J. Rada, Local value of a vertex-degree function index of a graph, *MATCH Commun. Math. Comput. Chem.* **95** (2026) 111–124.
 - [18] G. Su, J. Tu, K. C. Das, Graphs with fixed number of pendent vertices and minimal zeroth-order general Randić index, *Appl. Math. Comput.* **270** (2015) 705–710.

-
- [19] G. Su, L. Xiong, X. Su, G. Li, Maximally edge-connected graphs and zeroth-order general Randić index for $\alpha < -1$, *J. Comb. Optim.* **31** (2016) 182–195.
 - [20] I. Tomescu, Extremal vertex-degree function index for trees and unicyclic graphs with given independence number, *Discr. Appl. Math.* **306** (2022) 83–88.
 - [21] T. Vetrík, Degree-based function index of trees and unicyclic graphs, *J. Appl. Math. Comput.* **71** (2024) 2115–2133.
 - [22] S. Xu, B. Wu, (n, m) -Graphs with maximum vertex-degree function-index for convex functions, *MATCH Commun. Math. Comput. Chem.* **91** (2024) 197–234.
 - [23] J. Yang, H. Liu, Y. Wang, Vertex-degree function index for concave functions of graphs with a given clique number, *J. Appl. Math. Comput.* **70** (2024) 2197–2208.