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On Some Measures of Weak Associativity in Hypercompositional Structure and Implications in Chain Reactions

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Abstract

This study is based on the analysis of non-associativity in the hypercompositional algebraic structure of chain reaction and to discover the 'algebraic-behavior' of the elements using the probability of some non-associative properties. It was discovered that the hypercompositional algebraic structure that is represented in the chain reaction is non-associative. In addition, the 'algebraic-behavior' of each element based on the non-associative properties was analyzed, and elements with high, higher and highest probabilities were identified in the chain reaction. Some of the elements were found to

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be right nuclear, flexible, left alternative and right alternative. It was shown that the chain reaction structure has equal likelihood of being left nuclear or middle nuclear or right nuclear. Also, chain reaction structure was established to have equal likelihood of being flexible or left alternative. It was discovered that the chain reaction structure is more likely to be flexible or left alternative than being left nuclear or middle nuclear or right nuclear or right alternative. It was also discovered that the chain reaction structure is more likely to be right alternative than being left nuclear or middle nuclear or right nuclear. The differences in probabilities form the increasing sequence $0.0 < 0.008 < 0.08 < 0.088$.

1 Introduction

1.1 Hypercompositional algebraic structures

The concept of hypercompositional structure theory was formulated in 1934 by Marty [25] where the study was viewed in a theoretical point and its applications to different fields of mathematics that are pure and applied mathematics, as illustrated in [5, 12]. In a classical algebraic structure it was shown that the composition of two elements is an element while the composition of two elements in an hypercompositional algebraic structure is a set. Consequently, researchers are exploring its applications across various fields such as physical [15, 28], chemical [1, 2, 7–9, 11, 13, 20, 22, 27] and biological science [3, 4, 16, 29]. One of the motivations for studying hypercompositional structures is based on chemical reactions.

Some of the most popular types of hypercompositional algebraic structures are semihypergroup, quasihypergroup, hypergroup and H_v-group. Some new ones which are weak associative were introduced and studied by Ilori et al. [23] and Jaiyéọlá et al. [24].

Definition 1 (Semihypergroup, Quasihypergroup, Hypergroup, H_v-group)**.** An hypergroupoid or polygroupoid $(H, \circ)$ is the pair of a non-empty set H with an hyperoperation $\circ : H \times H \to P(H)\backslash\{\emptyset\}$ defined on it.

An hypergroupoid $(H, \circ)$ is called a semihypergroup if

(i) it obeys the associativity law $a \circ (b \circ c) = (a \circ b) \circ c$ for all $a, b, c \in H$, which means that

$$\bigcup_{u\in\ a\circ b} u\circ c = \bigcup_{v\in\ b\circ c} a\circ v$$

An hypergroupoid $(H,\circ)$ is called a quasihypergroup if

(ii) it obeys the reproduction axiom $x\circ H = H = H\circ x$ for all $x\in H$.

An hypergroupoid $(H,\circ)$ is called an hypergroup if it is a semihypergroup and a quasihypergroup.

A hypergroupoid $(H,\circ)$ is called an H_v-semigroup it obeys the weak associativity (WASS) condition

(iii) $x\circ(y\circ z)\cap(x\circ y)\circ z\neq\emptyset$ for all $x,y,z\in H$.

A hypergroupoid $(H,\circ)$ is called an H_v-group if it is a quasihypergroup and a H_v-semigroup.

2 Chemical hypercompositional structures in chain reaction

2.1 Radical reactions

A radical is an atom or a group of atoms with an unpaired electron. Such elements can either be electrically neutral or charged, called free radicals. Homolytic bond breakage, induced by heating in non-polar solvents or in the vapour phase, yields electrically neutral free radical pairs. High temperatures of exposure to ultraviolet light at room temperature cause molecular species to dissociate into radicals. Thus, homolysis or homolytic bonding takes place when a two-electron covalent bond and one electron are transferred to each of the resulting species. For instance, chlorine (Cl_2) forms chlorine radicals ($Cl^\bullet$) that is $X-X\to 2X^\bullet$ which can be expressed as $Cl-Cl\to 2Cl^\bullet$

Radical reactions tend to proceed as chain reaction processes which involve identical propagation steps, these steps are clearly stated in [26] as chain-initiating steps, chain-propagating steps, and chain-terminating steps.

2.2 The halogens in chain reactions

Recall that halogens are classified as non-metals. Though the physical forms of the halogens differ such as fluorine and chlorine are gases while bromine is a liquid and iodine exhibits solid at room temperature, each consists of diatomic molecules: F_2, Cl_2, Br_2 and I_2. Halogens react with hydrogen (H) to form gaseous compounds such as HF, HCl, HBr and HI which are all soluble in water. The reaction of these halogens with metals gives halides.

According to Davvaz *et al.* [6], they considered the chain reaction;

$$A_2 + B_2 \overset{\text{Heat or Light}}{\leftrightarrow} 2AB$$

then there exist molecules A_2, B_2, and AB and their fragment parts in the experiment such that elements of this collection can be combined with each other. Thus, all combinations for the set $S_H = \{A^\bullet, B^\bullet, A_2, B_2, AB\}$ are formed without energy as shown in Table 1.

Table 1. Multiplication table of $S_H = \{A^\bullet, B^\bullet, A_2, B_2, AB\}$

$+$	$A^\bullet$	$B^\bullet$	A_2	B_2	AB
$A^\bullet$	$A^\bullet, A_2$	$A^\bullet, B^\bullet, AB$	$A^\bullet, A_2$	$A^\bullet, B_2,$ $B^\bullet, AB$	$A^\bullet, AB,$ $A_2, B^\bullet$
$B^\bullet$	$A^\bullet, B^\bullet, AB$	$B^\bullet, B_2$	$A^\bullet, B^\bullet,$ AB, A_2	$B^\bullet, B_2$	$A^\bullet, B^\bullet,$ AB, B_2
A_2	$A^\bullet, A_2$	$A^\bullet, B^\bullet, AB, A_2$	$A^\bullet, A_2$	$A^\bullet, B^\bullet,$ A_2, B_2, AB	$A^\bullet, B^\bullet,$ A_2, AB
B_2	$A^\bullet, B^\bullet,$ B_2, AB	$B^\bullet, B_2$	$A^\bullet, B^\bullet,$ A_2, B_2, AB	$B^\bullet, B_2$	$A^\bullet, B^\bullet,$ B_2, AB
AB	$A^\bullet, AB,$ $A_2, B^\bullet$	$A^\bullet, B^\bullet,$ AB, B_2	$A^\bullet, B^\bullet,$ A_2, AB	$A^\bullet, B^\bullet,$ B_2, AB	$A^\bullet, B^\bullet, A_2,$ B_2, AB

The authors in [6], discovered a result that was stated as follows.

Theorem 1. *[6] $(S_H, +)$ is an H_v-group.*

Lemma 1. *The only H_v-subgroups of $(S_H, +)$ are $X = \{A^\bullet, A_2\}$ and $Y = \{B^\bullet, B_2\}$.*

Considering $A = H$ and $B \in \{F, Cl, Br, I\}$; for instance, when $B = F$, Table 2 shows the complete reaction.

Table 2. Multiplication table of $S_H = \{H^\bullet, F^\bullet, H_2, F_2, HF\}$

$+$	$H^\bullet$	$F^\bullet$	H_2	F_2	HF
$H^\bullet$	$H^\bullet, H_2$	$H^\bullet, F^\bullet, HF$	$H^\bullet, H_2$	$H^\bullet, F_2, F^\bullet, HF$	$H^\bullet, HF, H_2, F^\bullet$
$F^\bullet$	$H^\bullet, F^\bullet, HF$	$F^\bullet, F_2$	$H^\bullet, F^\bullet, HF, F_2$	$F^\bullet, F_2$	$H^\bullet, F^\bullet, HF, F_2$
H_2	$H^\bullet, H_2$	$H^\bullet, F^\bullet, HF, H_2$	$H^\bullet, H_2$	$H^\bullet, F^\bullet, H_2, F_2, HF$	$H^\bullet, F^\bullet, H_2, HF$
F_2	$H^\bullet, F^\bullet, F_2, HF$	$F^\bullet, F_2$	$H^\bullet, F^\bullet, H_2, F_2, HF$	$F^\bullet, F_2$	$H^\bullet, F^\bullet, F_2, HF$
HF	$H^\bullet, HF, H_2, F^\bullet$	$H^\bullet, F^\bullet, HF, F_2$	$H^\bullet, F^\bullet, H_2, HF$	$H^\bullet, F^\bullet, F_2, HF$	$H^\bullet, F^\bullet, H_2, F_2, HF$

2.3 Analysis of non-associativity in H_v-structures

2.3.1 Left nuclear element and probability of left nuclear element

Definition 2. (Left nuclear element) [22]

Let $(P, \cdot)$ be a polygroupoid. The left nucleus pair of $x \in P$ is denoted by $N_\lambda(x)$ and defined as $N_\lambda(x) = \{(y, z) \in P \times P \mid x \cdot (yz) = (xy) \cdot z\}$. $x \in P$ is said to be left nuclear if $N_\lambda(x) = P \times P$.

Definition 3. (Probability of left nuclear element/polygroupoid) [22]

Let $(P, \cdot)$ be a polygroupoid.

1. The probability of an element $x \in P$ being left nuclear is denoted by $Pr_{N_\lambda(P,\cdot)}(x)$ and defined as

$$Pr_{N_\lambda(P,\cdot)}(x) = \frac{|N_\lambda(x)|}{|P|^2}.$$

2. The probability of $(P, \cdot)$ being left nuclear is denoted by $Pr_{N_\lambda}(P, \cdot)$ and defined as

$$Pr_{N_\lambda}(P, \cdot) = \frac{\sum\limits_{x \in P} Pr_{N_\lambda(P,\cdot)}(x)}{|P|}.$$

Lemma 2. *[22] Let $(P,\cdot)$ be a polygroupoid. Let the left nucleus of $(P,\cdot)$ be defined as $N_\lambda(P,\cdot) = \{x \in P \mid x\cdot(yz) = (xy)\cdot z \ \forall\ (y,z) \in P\times P\}$. Then:*

1. $N_\lambda(P,\cdot) = \{x \in P \mid N_\lambda(x) = P\times P\} = \{x \in P \mid x$ *is left nuclear*$\}$.

2. $$Pr_{N_\lambda}(P,\cdot) = \frac{\sum_{x\in P} |N_\lambda(x)|}{|P|^3}.$$

2.3.2 Middle nuclear element and probability of middle nuclear element

Definition 4. (Middle nuclear element) [22]

Let $(P,\cdot)$ be a polygroupoid. The middle nucleus pair of $x \in P$ is denoted by $N_\mu(x)$ and defined as $N_\mu(x) = \{(y,z) \in P\times P \mid y\cdot(xz) = (yx)\cdot z\}$. $x \in P$ is said to be middle nuclear if $N_\mu(x) = P\times P$.

Definition 5. (Probability of middle nuclear element of polygroupoid) [22]

Let $(P,\cdot)$ be a polygroupoid.

1. The probability of an element $x \in P$ being middle nuclear is denoted by $Pr_{N_\mu(P,\cdot)}(x)$ and defined as
$$Pr_{N_\mu(P,\cdot)}(x) = \frac{|N_\mu(x)|}{|P|^2}.$$

2. The probability of $(P,\cdot)$ being middle nuclear is denoted by $Pr_{N_\mu}(P,\cdot)$ and defined as
$$Pr_{N_\mu}(P,\cdot) = \frac{\sum_{x\in P} Pr_{N_\mu(P,\cdot)}(x)}{|P|}.$$

Lemma 3. *[22] Let $(P,\cdot)$ be a polygroupoid. Let the middle nucleus of $(P,\cdot)$ be defined as $N_\mu(P,\cdot) = \{x \in P \mid y\cdot(xz) = (yx)\cdot z \ \forall\ (y,z) \in P\times P\}$. Then:*

1. $N_\mu(P,\cdot) = \{x \in P \mid N_\mu(x) = P\times P\} = \{x \in P \mid x$ *is middle nuclear*$\}$.

$$\textit{2. } Pr_{N_\mu}(P,\cdot) = \frac{\sum_{x\in P} |N_\mu(x)|}{|P|^3}.$$

2.3.3 Right nuclear element and probability of right nuclear element

Definition 6. (Right nuclear element) [22]

Let $(P,\cdot)$ be a polygroupoid. The right nucleus pair of $x \in P$ is denoted by $N_\rho(x)$ and defined as $N_\rho(x) = \{(y,z) \in P \times P \mid y\cdot(zx) = (yz)\cdot x\}$. $x \in P$ is said to be right nuclear if $N_\rho(x) = P \times P$.

Definition 7. (Probability of right nuclear element/polygroupoid) [22]

Let $(P,\cdot)$ be a polygroupoid.

1. The probability of an element $x \in P$ being right nuclear is denoted by $Pr_{N_\rho(P,\cdot)}(x)$ and defined as

$$Pr_{N_\rho(P,\cdot)}(x) = \frac{|N_\rho(x)|}{|P|^2}.$$

2. The probability of $(P,\cdot)$ being right nuclear is denoted by $Pr_{N_\rho}(P,\cdot)$ and defined as

$$Pr_{N_\rho}(P,\cdot) = \frac{\sum_{x\in P} Pr_{N_\rho(P,\cdot)}(x)}{|P|}.$$

Lemma 4. *[22] Let $(P,\cdot)$ be a polygroupoid. Let the right nucleus of $(P,\cdot)$ be defined as $N_\rho(P,\cdot) = \{x \in P \mid y\cdot(zx) = (yz)\cdot x \ \forall\ (y,z) \in P \times P\}$. Then:*

1. $N_\rho(P,\cdot) = \{x \in P \mid N_\rho(x) = P \times P\} = \{x \in P \mid x \text{ is right nuclear}\}$.

$$\textit{2. } Pr_{N_\rho}(P,\cdot) = \frac{\sum_{x\in P} |N_\rho(x)|}{|P|^3}.$$

2.3.4 Flexibility and alternativity

Definition 8. (Flexibility set of an element) [22]

Let $(P, \cdot)$ be a polygroupoid. The flexibility set of an element $x \in P$ is denoted by $FLEX(x)$ and defined as $FLEX(x) = \{y \in P \mid (xy)x = x(yx)\}$.

The set of flexible elements in P is denoted by $FLEX(P, \cdot)$ and defined as $FLEX(P, \cdot) = \{x \in P \mid FLEX(x) = P\}$.

Definition 9. (Probability of flexible element /polygroupoid) [22]

Let $(P, \cdot)$ be a polygroupoid. The probability of an element $x \in P$ being flexible is denoted by $Pr_{FLEX}(x)$ and defined as

$$Pr_{FLEX}(x) = \frac{|FLEX(x)|}{|P|}.$$

The probability of $(P, \cdot)$ being flexible is denoted by $Pr_{FLEX}(P, \cdot)$ and defined as

$$Pr_{FLEX}(P, \cdot) = \frac{\sum_{x \in P} Pr_{FLEX}(x)}{|P|}.$$

Lemma 5. *[22]*

Let $(P, \cdot)$ be a polygroupoid.

$$\textit{Then, } Pr_{FLEX}(P, \cdot) = \frac{\sum_{x \in P} |FLEX(x)|}{|P|^2}.$$

Definition 10. (Left alternative element) [22]

Let $(P, \cdot)$ be a polygroupoid. The left alternative set of an element $x \in P$ is denoted by $LAP(x)$ and defined as $LAP(x) = \{y \in P \mid (xx)y = x(xy)\}$. The set of left alternative element of $(P, \cdot)$ is denoted by $LAP(P, \cdot)$ and defined as $LAP(P, \cdot) = \{x \in P \mid LAP(x) = P\}$.

Definition 11. (Probability of left alternative element/polygroupoid) [22]

Let $(P, \cdot)$ be a polygroupoid. The probability of an element $x \in P$ being left alternative is denoted by $Pr_{LAP}(x)$ and defined as

$$Pr_{LAP}(x) = \frac{|LAP(x)|}{|P|}.$$

The probability of $(P,\cdot)$ being left-alternative is denoted by $Pr_{LAP}(P,\cdot)$ and defined as

$$Pr_{LAP}(P,\cdot)=\frac{\sum\limits_{x\in P} Pr_{LAP}(x)}{|P|}.$$

Lemma 6. *[22]*

Let $(P,\cdot)$ be a polygroupoid.

$$\textit{Then, } Pr_{LAP}(P,\cdot)=\frac{\sum\limits_{x\in P}|LAP(x)|}{|P|^2}.$$

Definition 12. (Right alternative element) [22]

Let $(P,\cdot)$ be a polygroupoid. Right alternative set of an element $x\in P$ is denoted by $RAP(x)$ and defined as $RAP(x)=\{y\in P \mid y(xx)=y(xx)\}$. The set of right alternative elements is denoted by $RAP(P,\cdot)$ and defined as $RAP(P,\cdot)=\{x\in P \mid RAP(x)=P\}$.

Definition 13. (Probability of right alternative element/polygroupoid) [22]

Let $(P,\cdot)$ be a polygroupoid. The probability of an element $x\in P$ being right alternative is denoted by $Pr_{RAP}(x)$ and defined as

$$Pr_{RAP}(x)=\frac{|RAP(x)|}{|P|}.$$

The probability of $(P,\cdot)$ being a right alternative is denoted by $Pr_{RAP}(P,\cdot)$ and defined as

$$Pr_{RAP}(P,\cdot)=\frac{\sum\limits_{x\in P} Pr_{RAP}(x)}{|P|}.$$

Lemma 7. *[22]*

Let $(P,\cdot)$ be a polygroupoid.

$$\textit{Then, } Pr_{RAP}(P,\cdot)=\frac{\sum\limits_{x\in P}|RAP(x)|}{|P|^2}.$$

In this work, our main objective is to analyze some non-associative properties in chain reaction which were identified in [6], [11] and [13]. Also, since the structure obtained is an H_v-group, this motivates us to quantify non-associative properties such as flexibility, left alternative property, right alternative property, left (middle, right) nuclear property. and their implications to chain reaction. A computer application program was used to identify and analyze weak associativity in chain reactions.

3 Main results

The following definition of terms are introduced and will be needed in the analysis.

Definition 14. Let $(P, \cdot)$ be a polygroupoid.

1. $x \in P$ will be said to be flexible if $FLEX(x) = P$.
2. $(P, \cdot)$ will be said to be flexible if $FLEX(P, \cdot) = P$.
3. $x \in P$ will be said to be left alternative if $LAP(x) = P$.
4. $(P, \cdot)$ will be said to be left alternative if $LAP(P, \cdot) = P$.
5. $x \in P$ will be said to be right alternative if $RAP(x) = P$.
6. $(P, \cdot)$ will be said to be right alternative if $RAP(P, \cdot) = P$.

We now present the analysis of non-associative properties for chain reactions carried out based on Definition 2 to Definition 13, Definition 14 and using Lemma 2 to Lemma 7.

3.1 Algebraic analysis for the triples found in chain reaction

3.1.1 Algebraic properties and probability of elements in chain reactions

Discussions:

Table 3. Algebraic analysis of chain reaction

Properties	Number of Triples					
		$A^\bullet$	$B^\bullet$	A_2	B_2	AB
Left Nucleus $\|N_\lambda(\cdot)\|$	True	14	24	24	24	23
	False	11	1	1	1	2
Middle Nucleus $\|N_\mu(\cdot)\|$	True	23	21	24	21	20
	False	2	4	1	4	5
Right Nucleus $\|N_\rho(\cdot)\|$	True	25	22	25	15	22
	False	0	3	0	10	3
Flexibility $\|FLEX(\cdot)\|$	True	5	5	5	4	5
	False	0	0	0	1	0
Left Alternative Property $\|LAP(\cdot)\|$	True	4	5	5	5	5
	False	1	0	0	0	0
Right Alternative Property $\|RAP(\cdot)\|$	True	5	4	5	4	4
	False	0	1	0	1	1

Table 4. Probability of elements in chain reaction S_H

Probability of Properties	$A^\bullet$	$B^\bullet$	A_2	B_2	AB
Left Nucleus $P_{N_\lambda}(\cdot)$	0.56	0.96	0.96	0.96	0.92
Middle Nucleus $P_{N_\mu}(\cdot)$	0.92	0.84	0.96	0.84	0.80
Right Nucleus $P_{N_\rho}(\cdot)$	1.00	0.88	1.00	0.60	0.88
Flexibility $P_{FLEX}(\cdot)$	1.00	1.00	1.00	0.80	1.00
Left Alternative Property $P_{LAP}(\cdot)$	0.80	1.00	1.00	1.00	1.00
Right Alternative Property $P_{RAP}(\cdot)$	1.00	0.80	1.00	0.80	0.80

1. Left Nuclearity:

- Observations: Since the left nuclear property reveals how an element algebraically acts in the left nucleus in chain reaction then, from the Table 4, it can be observed that the probability of $A^\bullet$ being a left nuclear element is the least compared to other elements of $(S_H, +)$. Next is AB. The elements with the highest probability of 0.96 are $B^\bullet$, A_2, B_2.
- Implications: Despite that $B^\bullet$, A_2, B_2 have the highest left nuclear probability, the trio does not form an H_v-subgroup of $(S_H, +)$. Whereas, by Lemma 1, the only H_v-subgroups of $(S_H, +)$ are $X = \{A^\bullet, A_2\}$ and $Y = \{B^\bullet, B_2\}$. This is be-

cause X and Y are both hypergroupoids and quasihypergroups. But they are not hypergroups because they are not semihypergroups; their elements have left nuclear-probabilities that are less than 1.

2. Middle Nuclearity:

- Observations: The middle nuclear property tells how an element algebraically acts in the middle nucleus in chain reaction. From the Table 4, it can be observed that the probability of AB being a middle nuclear element is the least compared to other elements of $(S_H, +)$. Next are $B^\bullet$ and B_2 with higher probabilities of middle nuclear property, while the elements with the highest probabilities of 0.92 and 0.96 are $A^\bullet$ and A_2 respectively.
- Implications: Interestingly, $A^\bullet$ and A_2 have the highest middle nuclear probabilities and the duo forms an H_v-subgroup X of $(S_H, +)$ according to Lemma 1. Furthermore, $B^\bullet$ and B_2 which have the higher middle nuclear probability of 0.84 form an H_v-subgroup Y of $(S_H, +)$ according to Lemma 1. The least middle nuclear probability in $(S_H, +)$ is possessed by AB; and this element is not contained in any of the two non-trivial H_v-subgroups of $(S_H, +)$. Note that even though X and Y are both hypergroupoids and quasihypergroups, they are not hypergroups because they are not semihypergroups; their elements have middle nuclear-probabilities that are less than 1.

3. Right Nuclearity:

- Observations: Right nuclear property tells how an element algebraically acts in the right nucleus in chain reaction. Based on Table 4, it can be deduced that the probability of B_2 being a right nuclear element is the least among other elements of $(S_H, +)$. Next are $B^\bullet$ and AB with higher probability of right nuclear property, while the elements with the highest probability of 1.0 are $A^\bullet$ and A_2 respectively.

- Implications: $A^{\bullet}$ and A_2 have the highest right nuclear probability and the duo forms an H_v-subgroup X of $(S_H, +)$ according to Lemma 1. Further, $B^{\bullet}$ and B_2 which have the higher and least right nuclear probabilities of 0.88 and 0.60 respectively, form an H_v-subgroup Y of $(S_H, +)$ according to Lemma 1. Even though an high right nuclear probability in $(S_H, +)$ is possessed by AB; and this element is not contained in any of the two non-trivial H_v-subgroups of $(S_H, +)$. Although the elements of X have right nuclear probability of 1.0, X is a H_v-subgroup of $(S_H, +)$ but not an hypergroup. This is because the elements of X do not have left and middle nuclear probabilities of 1.0. Nevertheless, $A^{\bullet}$ and A_2 are right nuclear elements of $(S_H, +)$.

4. Flexibility:

- Observations: Flexibility property tells how an element algebraically acts to be elastic in a symmetric manner in chain reaction. Based on Table 4, it can be observed that $A^{\bullet}, B^{\bullet}, A_2, AB$ have flexible probability of 1.0. But B_2 has a flexible probability of 0.80.

- Implications: $A^{\bullet}, B^{\bullet}, A_2, AB$ are flexible elements. Thus, $FLEX(S_H, +) = \{A^{\bullet}, B^{\bullet}, A_2, AB\}$. Among the flexible elements are $A^{\bullet}$ and A_2 which form an H_v-subgroup X of $(S_H, +)$ according to Lemma 1. Recall that these elements were earlier on found to be right nuclear. Further, $B^{\bullet}$ and B_2 which have the highest and least flexible probabilities of 1.0 and 0.80 respectively, form an H_v-subgroup Y of $(S_H, +)$ according to Lemma 1. Even though the highest flexible probability in $(S_H, +)$ is possessed by AB; and this element is not contained in any of the two non-trivial H_v-subgroups of $(S_H, +)$. Although the elements of X have flexible probability of 1.0, X is a H_v-subgroup of $(S_H, +)$ but not an hypergroup. This is because the elements of X do not have left and middle nuclear probabilities of 1.0. Nevertheless, elements of X are right nuclear elements of $(S_H, +)$.

5. Left Alternative Property:

- Observations: Left alternative property reveals how an element algebraically acts to be elastic in a left symmetric manner in chain reaction. Based on Table 4, it can be observed that $B^\bullet, A_2, B_2, AB$ have left alternative probability of 1.0. But $A^\bullet$ has a left alternative probability of 0.80.
- Implications: $B^\bullet, A_2, B_2, AB$ are left alternative elements. Thus, $LAP(S_H, +) = \{B^\bullet, A_2, B_2, AB\}$. Among the left alternative elements are $B^\bullet$ and B_2 which form an H_v-subgroup Y of $(S_H, +)$ according to Lemma 1. Recall that only of them was earlier on found to be right nuclear. Further, $A^\bullet$ and A_2 which have the least and highest left alternative probabilities of 0.80 and 1.0 respectively, form an H_v-subgroup X of $(S_H, +)$ according to Lemma 1. Though the highest left alternative probability in $(S_H, +)$ is possessed by AB; and this element is not contained in any of the two non-trivial H_v-subgroups of $(S_H, +)$. Although the elements of Y have left alternative probability of 1.0, X is a H_v-subgroup of $(S_H, +)$ but not an hypergroup. This is because the elements of X do not have left and middle nuclear probabilities of 1.0.

6. Right Alternative Property:

- Observations: Right alternative property reveals how an element algebraically acts to be elastic in a right symmetric manner in chain reaction. Based on Table 4, it can be observed that $A^\bullet, A_2$ have right alternative probability of 1.0. But $B^\bullet, B_2, AB$ have left alternative probability of 0.80.
- Implications: $A^\bullet, A_2$ are right alternative elements. Thus, $RAP(S_H, +) = \{A^\bullet, A_2\}$. The only right alternative elements form an H_v-subgroup X of $(S_H, +)$ according to Lemma 1. Recall that only these two elements were found to be right nuclear and flexible. Further, $B^\bullet$ and B_2 which have the least right alternative probability of 0.80, form an H_v-subgroup Y of $(S_H, +)$ according to Lemma 1.

3.2 Algebraic analysis for the structure $(S_H, +)$ in chain reaction

Table 5. Probability of algebraic properties in chain reaction $(S_H, +)$

Probability of Properties	$(S_H, +)$
Left Nucleus $P_{N_\lambda}(\cdot)$	0.872
Middle Nucleus $P_{N_\mu}(\cdot)$	0.872
Right Nucleus $P_{N_\rho}(\cdot)$	0.872
Flexibility $P_{FLEX}(\cdot)$	0.96
Left Alternative Property $P_{LAP}(\cdot)$	0.96
Right Alternative Property $P_{RAP}(\cdot)$	0.88

Discussions: From the Table 5, it can be observed that the probabilities of $(S_H, +)$ being left nuclear, middle nuclear and right nuclear are equal, which is 0.872 and it is the least among the probabilities. The higher probability is 0.88 for right alternativity. The chain reaction structure $(S_H, +)$ has the highest probability of 0.96 of being left alternative or being flexible. The differences in probabilities are: 0.00, $|0.96 - 0.88| = 0.08$, $|0.88 - 0.872| = 0.008$, $|0.96 - 0.872| = 0.088$. Which means that the chain reaction structure $(S_H, +)$ has equal likelihood of being left nuclear or middle nuclear or right nuclear. Also, chain reaction structure $(S_H, +)$ has equal likelihood of being flexible or left alternative. The chain reaction structure $(S_H, +)$ is more likely to be flexible or left alternative than being left nuclear or middle nuclear or right nuclear or right alternative The chain reaction structure $(S_H, +)$ is more likely to be right alternative than being left nuclear or middle nuclear or right nuclear. The differences in probabilities form the increasing sequence $0.0 < 0.008 < 0.08 < 0.088$.

4 Conclusion

This study discovered that hypercompositional algebraic structures representing chain reaction is non-associative hypercompositional algebraic structures. The algebraic behavior of each element based on the non-associative properties were analyzed. It can be concluded that the structure is more likely to be flexible or left alternative than being left nuclear or middle nuclear or right nuclear or right alternative Furthermore, it can be concluded that structure is more likely to be right alternative than being left nuclear or middle nuclear or right nuclear. The differences in probabilities form the increasing sequence $0.0 < 0.008 < 0.08 < 0.088$.

References

[1] S. Abdizadeh, S. Ostadhadi, O. Dehkordi and H. Aghabozorgi, Mathematical model for some chemical Redox reactions, *J. Algebraic Hyperstruct. Log. Algebras* **4** (2023) 97–105.

[2] F. Agusfrianto, S. Andromeda, M. Hariri, Bi-hyperstructures in chemical hyperstructures of redox reactions with three and four oxidation states, *J. Teor. Apl. Mat.* **8** (2024) 50–57.

[3] F. Agusfrianto, M. Al-Tahan, M. Hariri, Y. Mahatma, Examples of neutrohyperstructures on biological inheritance, *Neutrosophic Sets Sys.* **60** (2023) 583–592.

[4] M. Al Tahan, B. Davvaz, Algebraic hyperstructures associated to biological inheritance, *Math. Biosci.* **285** (2017) 112–118.

[5] P. Corsini, V. Leoreanu, *Applications of Hyperstructure Theory*, Kluwer Acad. Pub., Dordrecht, 2003.

[6] B. Davvaz, A. Dehghan-Nezhad, A. Benvidi, Chain reactions as experimental examples of ternary algebraic hyperstructures, *MATCH Commun. Math. Comput. Chem.* **65** (2011) 491–499.

[7] B. Davvaz, A. Dehghan-Nezhad, Dismutation reactions as experimental verifications of ternary algebraic hyperstructures, *MATCH Commun. Math. Comput. Chem.* **68** (2012) 551–559.

[8] B. Davvaz, A. Dehghan-Nezad, M. Mazloum-Ardakani, Chemical hyperalgebra: Redox reactions, *MATCH Commun. Math. Comput. Chem.* **71** (2014) 323–331.

[9] B. Davvaz, A. Dehghan-Nezad, M. Mazloum-Ardakani, M. A. Sheikh-Mohseneib, Describing the algebraic hyperstructure of all elements in radiolytic processes in cement medium, *MATCH Commun. Math. Comput. Chem.*, **72** (2014) 375–388.

[10] B. Davvaz, *Polygroup Theory and Related System*, World Sci. Pub., Singapore, 2013.

[11] B. Davvaz, A. Dehghan-Nezhad, A. Benvidi, Chemical hyperalgebra: dismutation reactions, *MATCH Commun. Math. Comput. Chem.* **67** (2012) 55–63.

[12] B. Davvaz, *Semihypergroup Theory*, Elsevier, London, 2016.

[13] B. Davvaz, Weak algebraic hyperstructures as a model for interpretation of chemical reactions, *Iran. J. Math. Chem.* **7** (2016) 267–283.

[14] B. Davvaz, A brief survey on algebraic hyperstructures: Theory and applications, *J. Algebraic Hyperstruct. Log. Algebras* **1** (2020) 15–29.

[15] B. Davvaz, A. Dehghan Nezhad, S. M. Moosavi Nejad, Algebraic hyperstructure of observable elementary particles including the Higgs boson, *Proc. Natl. Acad. Sci.* **90A** (2020) 169–176.

[16] B. Davvaz, A. Nezhad Dehghan, M. M. Heidari, Inheritance examples of algebraic hyperstructures, *Inform. Sci.* **224** (2013) 180–187.

[17] B. Davvaz, T. Vougiouklis (2019), *A Walk Through Weak Hyperstructures Hv-Structures*, World Sci. Pub., Singapore, 2019.

[18] J. Gadolin, K. Sv. Vet. Acad. Handl. **1788** 186–197.

[19] J. Gadolin, Crells Chem. Annalen 1790 (I), 260–273.

[20] D. Heidari, D. Mazaheri, B. Davvaz, Chemical salt reaction as algebraic hyperstructures, *Iran. J. Math. Chem.* **10** (2019) 93–102.

[21] A. F. Holleman, E. Wiberg, *Inorganic Chemistry*, Academic Press, San Diego, 2001.

[22] K. G. Ilori, T. G. Jaiyéọlá, O. O. Oyebola, Analysis of weak associativity in some hyper-algebraic structures that represent dismutation reactions, *MATCH Commun. Math. Comput. Chem.* **95** (2025) 385–486.

[23] K. G. Ilori, T. G. Jaiyéọlá, O. O. Oyebola, O. B. Ogunfolu, E. A. Alhassan, Polygroupoid, polyquasigroup, polyloop and their nuclei, *J. Nigerian Math. Soc.* **44** (2025) 115–137.

[24] T. G. Jaiyéọlá , K. G. Ilori, O. O. Oyebola, *On some non-associative hyper-algebraic structure*, pre-print.

[25] F. Marty, Sur une généralization de la notion de groupe, *8th Congress Math.* Scandenaves, Stockholm, 1934, pp. 45–49.

[26] R. T. Morrison, R. N. Boyd, *Organic Chemistry*, Prentice-Hall, Englewood Cliffs, 1992.

[27] S. Nawaz, M. Gulistan, S. Khan, Weak LA-hypergroups: Neutrosophy, enumeration and redox reaction, *Neutrosophic Sets Syst.* **36** (2020) 352–368.

[28] A. D. Nezhad, S. M. M. Nejad, M. Nadjafikhah, B. Davvaz, A physical example of algebraic hyperstructures: Leptons, *Indian J. Phys.* **86** (2012) 1027—1032.

[29] O. O. Oyebola, T. G. Jaiyéọlá, Non-associative algebraic hyperstructures and their applications to biological inheritance, *Monografías Matemáticas García de Galdeano* **42** (2019) 229—241.

[30] D. F. Shriver, P. W. Atkins, T. L. Overton, J. P. Rourke, M. T. Weller, F. A. Armstrong, *Inorganic Chemistry*, W. H. Freeman, New York, 2006.

[31] J. Ušan, R. Galić, On Hyperquasigroups, *Math. Moravica* **8** (2004) 63–71.

[32] T. Vougiouklis, *Hyperstructures and Their Representations*, Hadronic Press, Palm Harbor, 1994.