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Topological Properties and Computation of Silicate Networks Using Graph-Invariants

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Abstract

Silicon, oxygen and aluminium are found in large quantities on the earth's surface. Silicates are the minerals which contain silicon and oxygen in tetrahedral $(SiO)_4^{4-}$ units, which are linked together in several patterns. In chemical graph theory, atoms are represented as vertices and chemical bonds as edges. In a silicate network, a tetrahedron comprises of one central silicon atom and four surrounding oxygen atoms. Cement, ceramic and glass industries use silicates for manufacturing purposes. The computation of silicate networks using graph-invariant has been introduced in this article. Using graph-invariant parameters has applications in studying the topological properties of silicate networks. We consider chain silicate (CS_m), cyclic silicate (CYS_m), double chain silicate (DCS_m), sheet silicate (SS_m), honeycomb network (HC_m) and m $\times$ m grid network ($G_{m\times m}$). To compute the topological properties of silicate networks, we investigate the graph-invariants such as maximum cliques(ω), minimum chromatic number(χ), maximum independence number(α), matching ratio(m_r) and minimum domination number(γ). Here, we observe the clique number of chain silicate, cyclic silicate, double chain silicate, and sheet silicate is 3, whereas honeycomb network and m $\times$ m grid network is 2. Similarly, the chromatic number of chain silicate, cyclic silicate and sheet silicate is 4. Moreover, the chromatic number of the honeycomb

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network, m × m grid network is 2 and the double chain silicate is 4. Then, the independence number of chain silicate, cyclic silicate, double chain silicate and sheet silicate is m. These characteristics show the structural behaviour of silicate networks. For instance, the clique number gives the molecular structure of the silicate network whereas the chromatic number gives the least number of atom types necessary to represent a molecule.

1 Introduction

Graph theory is a branch of mathematics which deals with the way the graphs are connected. A graph consists of nodes/vertices and edges. Graph theory has a wide range of applications in chemistry, computer science, economics, combinatorics, probability, numerical analysis etc. Chemical graph theory is a combination of chemistry and graph theory which uses graph theory to model and analyze molecular structures and chemical reactions [4]. Here, atoms are represented as vertices and chemical bonds are represented as edges in a graph. Chemical graph theory aims to study the graph properties to gain insights into the characteristics and behaviour of molecules. In [23] K. Zu et al. studied the survey on graphs extremal to distance-based topological indices.

A silicate network is a 3-dimensional structure composed of silicon atoms and oxygen atoms which leads to the formation of tetrahedral $(SiO)_4^{4-}$. Silicate networks are used in construction materials, glass manufacturing, ceramics, soil improvement, electronics, water treatment etc. In chemistry, the corner nodes have 4 oxygen nodes and the central node has one-silicon node [1]. The minerals are formed by combining oxygen nodes of two tetrahedra of unique silicates. We obtain the different types of silicates from how these tetrahedra are framed [1].

Here, we consider chain silicate (CS_m), cyclic silicate (CYS_m), double chain silicate (DCS_m), honeycomb network (HC_m), sheet silicate (SS_m) and $m \times m$ grid network. When tetrahedra are formed linearly, chain silicates are obtained [1]. The physical properties of Chain silicates are: they have strong covalent bonds between silicon and oxygen in the $(SiO)_4^{4-}$ tetrahedra and can react with strong acids or bases. The chemical properties of chain silicates are: they are hard, transparent, translucent, and opaque depending on the mineral. They are the most prevalent group of silicate minerals, constituting the major portion of Earth's crust and mantle. They are strong and fibrous making them ideal for pyroxene minerals. Due to its high melting point, some chain silicates like wollastonite are useful in ceramics and paints. J. Li et al. [21] studied the highly stable pure silica zeolite, ZEO-3 and proved it has a high performance for volatile organic compounds compared with other zeolites. J. A. Bowey et al. [3] studied the infrared spectra of crystalline chain silicates at room temperature.

When tetrahedra are arranged cyclically, cyclic silicate is formed [1]. These silicates are found in tourmaline used in gemstones and beryl, including emeralds. Certain cyclic silicates like zeolites act as catalysts in chemical reactions by providing a space for reactants to interact. Cyclic silicate can also be used in water purification based on size or polarity. The physical properties of cyclic silicates are: they are resistant towards weathering conditions due to their strong covalent bond between oxygen and silicon. They can undergo reactions with very strong acids or bases. The chemical properties of cyclic silicates are: They are hard and dense in nature. They execute wider range of colors. N. Sahai et al. [20] proposed that the cyclic silicate trimer is the universal active site for heterogeneous, stereochemically promoted nucleation on silicate-based bioactive ceramics.

Double-chain silicate is formed by linking two single chains of $(SiO)_4^{4-}$ tetrahedral units together through oxygen atoms. Each $(SiO)_4^{4-}$ tetrahedron shares three oxygen atoms with neighbouring tetrahedra and thus forming two-dimensional sheets. Unlike chain silicate and cyclic silicate, even double chain silicate exhibits a strong covalent between silicon and oxygen along with it they also undergo reactions with very strong acids or bases. They are hard, dense and transparent. Certain type of double-chain silicate like spodumene is a source of lithium in batteries. Due to their strong covalent bonds some of the double chain silicates can withstand high temperatures. G. Giuli et al. [22] studied the nickel site distribution and clustering in synthetic double-chain silicates on the basis of experimental and theoretical XANES spectroscopy.

A honeycomb network is formed from a hexagon in various ways. The honeycomb network HC_1 is of the hexagon shape. The honeycomb network HC_2 is formed by attaching six hexagons to the outer edges of HC_1. Similarly, honeycomb network HC_m is formed from HC_{m-1} by attaching a layer of hexagons around the outer layer of $HC_{(m-1)}$ [1]. The strong connections between the elements in the honeycomb network provide good resistance and deformation. Different materials in honeycomb networks have different chemical properties like reactivity, corrosion resistance and thermal stability with strong covalent bonds. The hexagonal arrangement of carbon atoms represents aromatic hydrocarbons like the structure of benzene. By the arrangement of bonds and electrons within the honeycomb network, chemists can predict the stability and reactivity of aromatic compounds. This is required in designing new drugs, and catalysts with specific properties.

$m \times m$ grid is a square matrix arrangement of nodes of the form $m \times m$, where $m \times m$ represents the total number of rows and the total number of columns respectively [2]. $m \times m$ grid network helps understand molecule arrangement in a crystal lattice. Chemists can analyse crystal structure with the arrangement of molecules on the network. The spacing between squares indicates the porosity of the network. The material's thermal conductivity will influence how heat passes through the network. A square grid material will be highly soluble in water and very resistant.

Here, we compute the topological properties of chain silicate (CS_m), cyclic silicate (CYS_m), double chain silicate (DCS_m), sheet silicate (SS_m), honeycomb network (HC_m) and m $\times m$ grid network ($G_{m \times m}$) by calculating the maximum clique number(ω), minimum chromatic number(χ), maximum independence number(α), perfect matching(σ), matching ratio(m_r), minimum domination number(γ).

2 Related work

Implementing chemical graph theory to investigate the topological properties has been a trending area in recent times. Khan et al. [2] computed the topological properties of neural networks using graph-theoretic parameters. Manuel et al. [1] computed the topological properties of silicate networks. Babar et al. [5] computed the multiplicative topological property of the honeycomb graph. Jia et al. [6] calculated the topological indices such as atom bond connectivity, Randic, geometric arithmetic, and Zagreb index for the m^{th} silicate network. Akhtar et al. [7] determined the topological indices such as atom-bond connectivity, Randic, geometric arithmetic, and Zagreb index for chain silicates and hexagonal networks. Bharati et al. [8] determined the Zagreb, ABC, Randic index of silicate, honeycomb and hexagonal networks.

Zhang et al. [9] determined the degree-based topological properties of chain silicates, honeycomb networks and hexagonal networks. Muthukumar et al. [10] computed the total domination number of silicate networks and oxide networks. Keerthi et al. [11] computed the secure domination number of silicate chains, silicate networks and line graphs. Anita et al. [12] computed the equal domination and independent domination numbers for chain silicate and cyclic silicate networks. Chithra et al. [13] computed the total chromatic number of the honeycomb network. Stephen et al. [18] computed the power domination in polyphenylene dendrimers, Rhenium Trioxide.

2.1 Motivation

Based on the literature survey, in this section, we discuss the drawbacks observed in the articles which motivate our article.

- In [1] Manuel et al. limited to topological structure of silicate networks.
- In [10] Muthukumar et al. limited to minimum domination number of even number of tetrahedral in chain silicate.
- Liu et al. [6] studied the topological indices of silicate networks.

2.2 Scientific contributions

Here, we made the following investigations:

- We discuss the graph-invariant such as maximum cliques(ω), minimum chromatic number (χ), maximum independence number (α), perfect matching (σ), minimum domination number (γ) of (CS_m), (CYS_m), (DC_m), (SS_m), (HC_m) and ($G_{m\times m}$).
- These general results provide detailed information about the structural behaviour related to the networks.
- We prove that the clique number(ω), chromatic number(χ) of chain silicate, cyclic silicate and sheet silicate is the same.
- We have proven that the clique number(ω), chromatic number(χ) of honeycomb network, m $\times$ m grid network is same.
- We have proven that the minimum domination number of chain silicate, cyclic silicate, and double chain silicate is $\lceil \frac{m}{2} \rceil$.
- We show that the independence number of chain silicate, cyclic silicate, double chain silicate and sheet silicate is m whereas independence number of honeycomb network is $3m^2$ and independence number of m $\times m$ is $\lceil \frac{m^2}{2} \rceil$.

3 Mathematical preliminaries

Here we consider finite, simple and connected.

λ is a graph with ordered pair (V_λ, E_λ), such that V_λ is the set of nodes and a set of joints called edges E_λ. The cardinality of vertices is called order. A graph ψ is known as a sub-graph of λ, if $V_\psi \subset V_\lambda$. A graph λ is called an induced graph if it is formed by removing some nodes. The degree of a vertex v is defined as the number of edges incident to the vertex v and it is denoted as deg(v).

A complete graph is a directed graph in which every pair of distinct vertices is connected by a pair of different edges. A graph λ is called bipartite if the vertices can be separated into two different sets. A graph is planar if there are no edge crossings.

Theorem 1. *[14] A graph λ with (V_λ, E_λ) is called bipartite if it consists of no cycle of odd length.*

The maximum complete graph from the induced subgraph C where C $\subset V_\lambda$ is known as a clique. The cardinality of the maximum clique in a graph λ is called a clique number and is denoted by $\omega(\lambda)$. A set with the mutually disjoint set of nodes in λ forms an independent set and it is denoted by I. The independence number of a graph λ is the maximum cardinality of an independent set of vertices. The independence number is denoted by $\alpha(\lambda)$. A k-coloring of a graph G is a mapping f: V(λ) $\rightarrow$ S, where | S | = k. Let P be the set of colours, the nodes of one colour form a colour class. A k-colouring is proper if neighbouring nodes have different labels. The least number of colours which obeys the proper colouring condition is called a chromatic number. It is denoted as $\chi(\lambda)$. A matching in a graph λ is a set of non-loop edges with no common endpoints. The nodes adjacent to the edges of a matching M are saturated by M; the others are unsaturated. A maximal matching in a graph λ is a matching that cannot be extended by attaching an edge. A matching of maximum size compared to all other matchings in the graph λ is a matching. A perfect matching in λ is a matching that covers every vertex. The matching ratio is defined as $m_r(\lambda) = \frac{|M_m|}{|E_\lambda|}$, where M_m is the maximum matching.

Theorem 2. *[17] Every planar graph is 4-colorable.*

Proposition 3. *[14] For a v-node graph λ, we have $\chi(\lambda) \geq \omega(\lambda)$.*

Lemma 1. *[15] If there is a existence of a perfect matching in a graph λ, then, $m_r(\lambda) = \frac{|V_\lambda|}{2\times|E_\lambda|}$.*

In graph theory, a dominating set S is a subset of its nodes such that each vertex in the graph λ is either in the dominating set S or adjacent to a vertex in the dominating set. A dominating set with the least number of vertices necessary to ensure that every node in the graph λ is either in the dominating set or neighbouring to a vertex in the dominating set is known as the minimum dominating set. It is denoted as $\gamma(\lambda)$. The order of the minimum dominating set is known as the minimum domination number.

We compute the clique number $\omega(\lambda)$, chromatic number $\chi(\lambda)$ and the independence number $\alpha(\lambda)$ of chain silicate.

Theorem 4. *Let λ be the chain silicate (CS_m). Then, $\omega(\lambda) = 3$, $\chi(\lambda) = 4$ and $\alpha(\lambda) = m$ where m is the number of tetrahedral.*

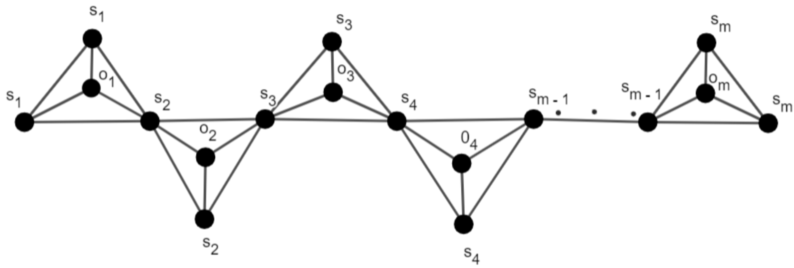

Figure 1. Graph of the chain silicate (CS_m)

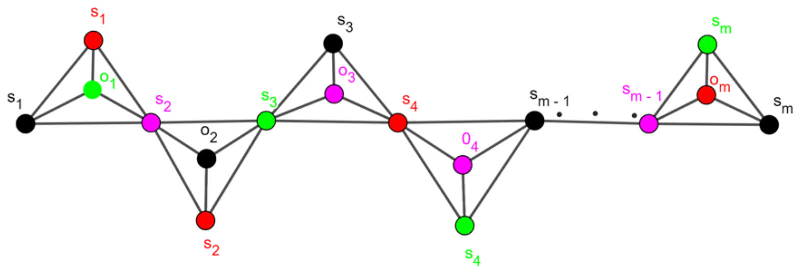

Figure 2. 4-coloring of Chain silicate (CS_m)

Proof. Consider V to be the set of nodes of the chain silicate (CS_m) such that $\mid V(\lambda) \mid$ = 3m + 1. Firstly we show that λ is a triangle. From Fig. 1, in each tetrahedron, $(SiO)_4^{4-}$ one vertex is connected to every other vertex and also has an odd cycle of length 3. This shows that λ contains a triangle. This implies that $\omega(\lambda) \geq 3$ and since λ consists of a tetrahedron which has 4 vertices in each tetrahedron, this implies that $\chi(\lambda) \geq 4$. Let $\{s_1, s_1, s_2\}$ be the nodes which forms triangle in λ. Because there is no shared neighbour of s_i's in λ, it does not consist of a clique number of cardinality 4. Using Proposition 3, we obtain $\omega(\lambda) \leq 3$, which shows that $\omega(\lambda) = 3$. Fig. 2, represents the 4-coloring of λ which implies that $\chi(\lambda) \leq 4$. Also, from Theorem 2., we know that $\chi(\lambda) = 4$ as λ is a planar graph.

Therefore, $\chi(\lambda) = 4$.
Next, we compute the independence number of λ. On observing that a central silicon vertex forms an independent set in a tetrahedron. In general, for m tetrahedrons the independence number is m. Therefore, $\alpha(\lambda) =$ m. Hence the proof. ■

Now, we compute the clique number $\omega(\lambda)$, chromatic number $\chi(\lambda)$ and the independence number $\alpha(\lambda)$ of cyclic silicate.

Theorem 5. *Let λ be the cyclic silicate (CYS_m). Then, $\omega(\lambda) = 3$, $\chi(\lambda) = 4$ and $\alpha(\lambda) = m$.*

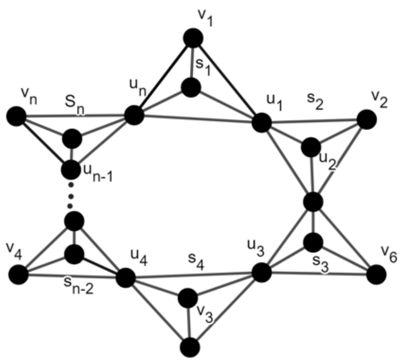

Figure 3. Graph of the Cyclic silicate (CYS_m)

Proof. Let V be the set of vertices of the cyclic silicate (CYS_m) such that $\mid V(\lambda) \mid = 3$m. Firstly, we show that λ is a triangle. From Fig. 3, in each tetrahedron $(SiO)_4^{4-}$, one vertex is connected to every other vertex and also has an odd cycle of length 3. This shows that λ contains a triangle. This implies that $\omega(\lambda) \geq 3$ and since λ consists of a tetrahedron which has 4 vertices in each tetrahedron, this implies that $\chi(\lambda) \geq 4$. Let Q $= \{v_1, u_1, s_1\}$ be the nodes which forms a triangle in λ. Because there is no shared neighbour of the set Q in λ, it does not consist of a clique of cardinality 4. Using Proposition 3, we obtain $\omega(\lambda) \leq 3$, which shows that $\omega(\lambda) = 3$. Fig. 4, represents the proper 4-colouring of λ which implies that $\chi(\lambda) \leq 4$. Also, from Theorem 2., we know that $\chi(\lambda) = 4$ as λ is a planar graph. Therefore, $\chi(\lambda) = 4$.

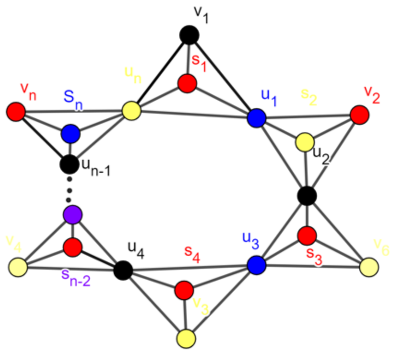

Figure 4. 4-coloring of cyclic silicate (CYS_m)

Next, we compute the independence number of λ. On observing that a central silicon vertex forms an independent set in a tetrahedron. In general, for m tetrahedrons the independence number is m. Therefore, $\alpha(\lambda) = $ m. Hence the proof. ■

Next, we compute the clique number $\omega(\lambda)$, chromatic number $\chi(\lambda)$ and the independence number $\alpha(\lambda)$ of double chain silicate.

Theorem 6. *Let λ be the double chain silicate (DCS_m). Then, $\omega(\lambda) = 3$, $\chi(\lambda) = 4$ and $\alpha(\lambda) = m$, where m is the number of tetrahedral.*

Proof. Let V be the set of vertices of the double chain silicate (DCS_m) such that $|\ V(\lambda)\ | = $ 3m - n where m is the number of tetrahedral and n is the dimension. Firstly, we show that λ is a triangle. From Fig. 5, in each tetrahedron $(SiO)_4^{4-}$, one vertex is connected to every other vertex and also has an odd cycle of length 3. This shows that λ contains a triangle. This implies that $\omega(\lambda) \geq 3$ and since λ consists of a tetrahedron which has 4 vertices in each tetrahedron, this implies that $\chi(\lambda) \geq 4$. Let T $= \{v_1, s_1, e_{12}\}$ be the nodes which induce triangle in λ. Because there is no shared neighbour of the set T in λ, it does not consist of a clique of cardinality 4. Using Proposition 3, we obtain $\omega(\lambda) \leq 3$, which shows that

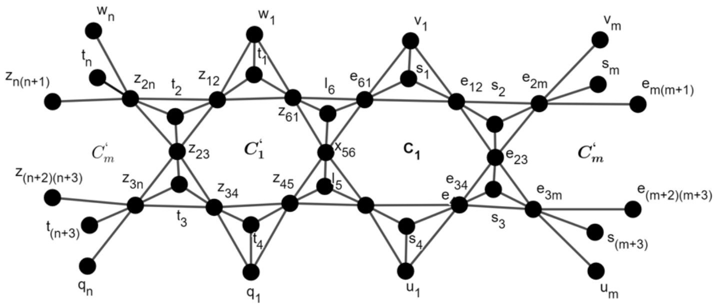

Figure 5. Graph of double chain silicate (DCS_m)

$\omega(\lambda) = 3$. Fig. 6, represents the proper 4-coloring of λ which implies that $\chi(\lambda) \leq 4$, which shows that $\chi(\lambda) = 4$.

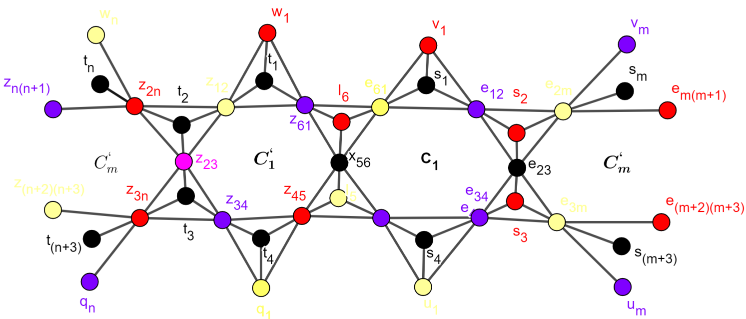

Figure 6. 5-coloring of double chain silicate (DCS_m)

Next, we compute the independence number of λ, by observing that a central silicon vertex forms an independent set in a tetrahedron. In general, for m tetrahedrons the independence number is m. Therefore, $\alpha(\lambda) = $ m. Hence the proof. ∎

Next we compute the clique number $\omega(\lambda)$, chromatic number $\chi(\lambda)$ and the independence number $\alpha(\lambda)$ of sheet silicate.

Theorem 7. *Let λ be the sheet silicate (SS_m). Then, $\omega(\lambda) = 3$, $\chi(\lambda) = 4$ and $\alpha(\lambda) = m$, where m is the number of tetrahedral.*

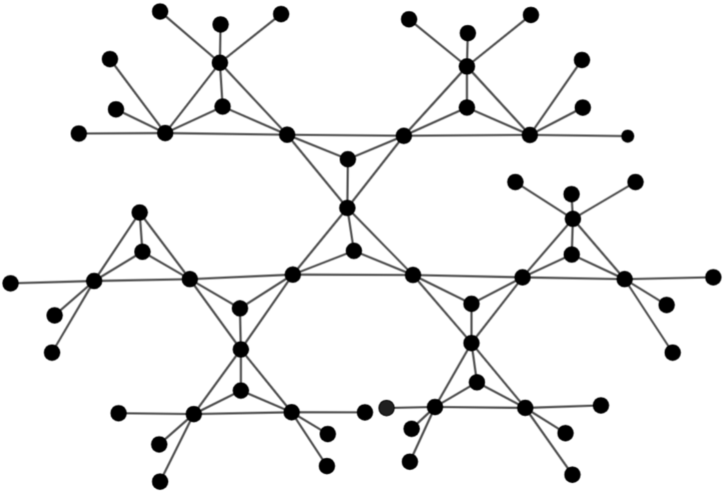

Figure 7. Graph of the sheet silicate (SS_m)

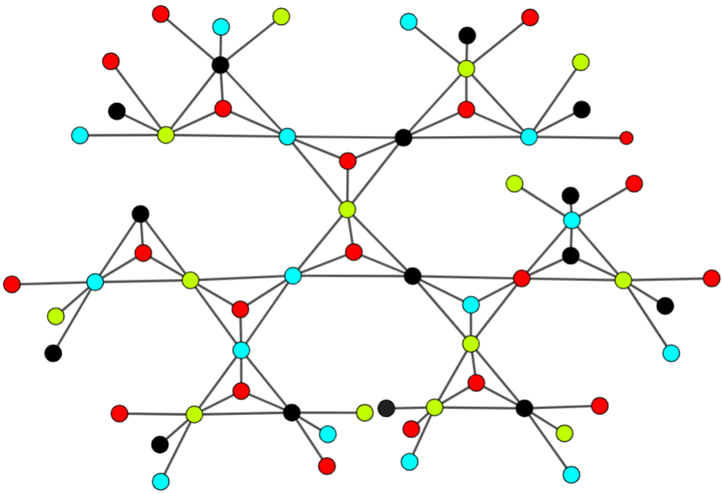

Figure 8. 4-coloring of sheet silicate (SS_m)

Proof. Consider V be the set of nodes of the sheet silicate (SS_m) in which $\mid V(\lambda) \mid$ = 3m + 1. Firstly, we show that λ is a triangle. From Fig. 7, in each tetrahedral $(SiO)_4^{4-}$, one vertex is connected to every other vertex and also has an odd cycle of length 3. This shows that λ contains a

triangle. This implies that $\omega(\lambda) \geq 3$ and since λ consists of a tetrahedron which has 4 vertices in each tetrahedron, this implies that $\chi(\lambda) \geq 4$. Let $\{y_1, y_2, y_3\}$ be the nodes which form a triangle in λ. Because there is no shared neighbour of y_i's in λ, it does not consist of a clique of cardinality 4. Using Proposition 3, we obtain $\omega(\lambda) \leq 3$, which shows that $\omega(\lambda) = 3$. Fig. 8, represents the proper 4-coloring of λ, which implies that $\chi(\lambda) \leq 4$, which shows that $\chi(\lambda) = 4$.
Next, we compute the independence number of λ. On observing that a central silicon vertex forms an independent set in a tetrahedron. In general, for m tetrahedrons the independence number is m. Therefore, $\alpha(\lambda) = $ m. Hence the proof. ■

Now we compute the clique number $\omega(\lambda)$, chromatic number $\chi(\lambda)$ and the independence number $\alpha(\lambda)$ of honeycomb network.

Theorem 8. *Let λ be the honeycomb network (HC_m). Then, $\omega(\lambda) = 2$, $\chi(\lambda) = 2$ and $\alpha(\lambda) = 3m^2$ where m is the dimension of (HC_m).*

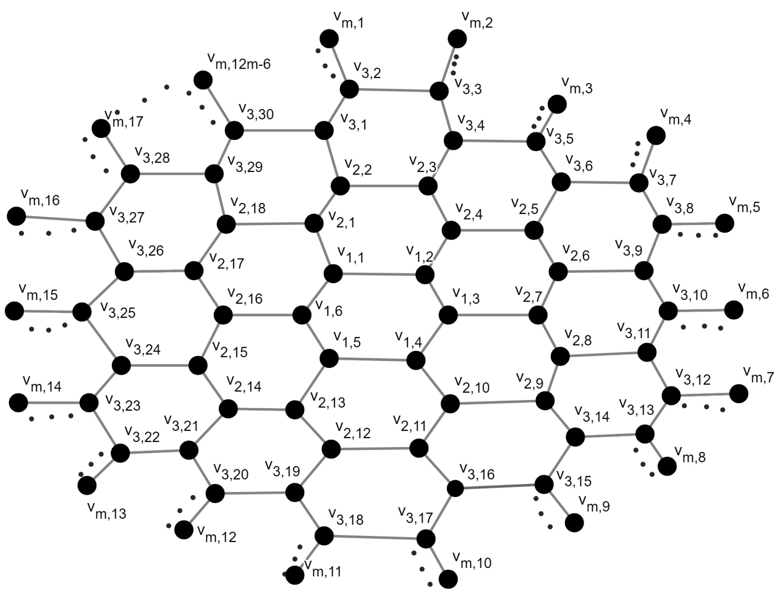

Figure 9. Graph of the honeycomb network (HC_m)

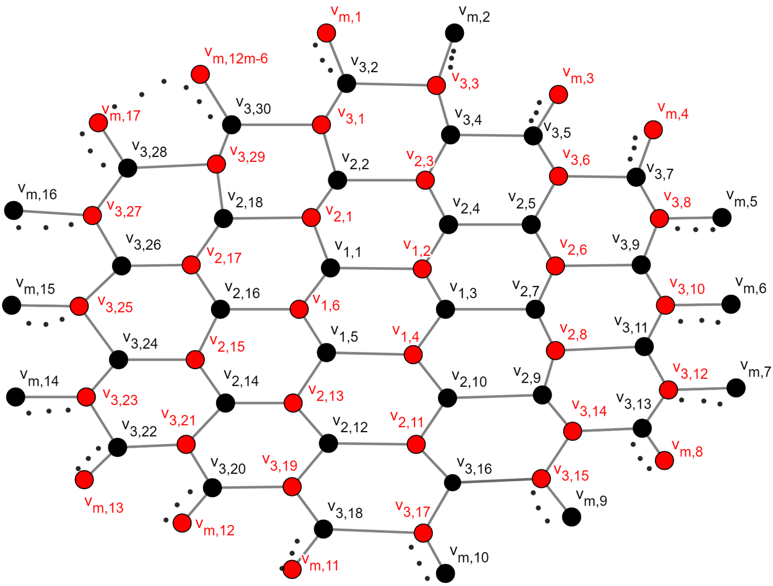

Figure 10. 2-coloring of honeycomb network (HC_m)

Proof. Consider V be the set of nodes of the honeycomb network (HC_m) in which $\mid V(\lambda) \mid = 6m^2$. Firstly, we show that λ is bipartite. Since λ has no odd cycle and represents the hexagonal pattern from Theorem 1., it forms a bipartite graph. From Fig. 9, we can easily partition the vertices into two sets, one set for the hexagon vertices at the corners of the honeycomb cells and another set for the vertices at the center of the cells. This partitioning ensures that every edge connects a vertex in one set to a vertex in the other set, satisfying the definition of a bipartite graph. Since λ consists of no triangles, λ does not contain K_s with s ≥ 3 as an induced sub-graph. Because K_2 is the maximum clique in λ, this proves that $\omega(\lambda)$ $= 2$. Since, λ is bipartite, hence it is 2-colorable. Fig. 10, depicts a proper coloring of λ. Hence, $\chi(\lambda) = 2$.

Next, we compute the independence number of λ. By using the induction hypothesis, we have.

Base case: When m = 1, i.e., when there is one hexagon that has 3 independent vertices which are non-adjacent. Since there is only one hexagon, the independence number required is 3. This satisfies the base case condition.

Induction hypothesis: Assume for m-dimensional honeycomb network, the independence number is $3m^2$.

Induction step: For (m + 1) dimensional honeycomb network. Attaching hexagons to the outer layer of the m-dimensional honeycomb network leads to constructing a (m + 1) dimensional honeycomb network. From the Base case, we know that it requires 3 independent vertices for each hexagon. For (m + 1)-dimensional honeycomb network the independence number is $3(m+1)^2 = 3(m^2+2m+1)$ which shows that the independence number of λ is $3m^2$. Therefore, $\alpha(\lambda) = 3m^2$. Hence the proof. ■

Next, we compute the clique number $\omega(\lambda)$, chromatic number $\chi(\lambda)$ and the independence number $\alpha(\lambda)$ of $m \times m$ grid network.

Theorem 9. *Let λ be the $m \times m$ grid $(G_{m\times m})$. Then, $\omega(\lambda) = 2$, $\chi(\lambda) = 2$ and $\alpha(\lambda) = \lceil \frac{m^2}{2} \rceil$.*

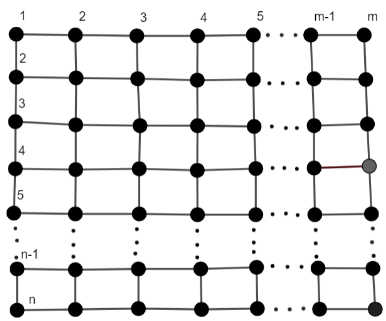

Figure 11. Graph of the Grid network $(G_{m\times m})$

Proof. Let V be the set of nodes such that $\mid V(\lambda) \mid = m^2$. From Theorem 1., λ is a bipartite graph consisting of no cycle of odd length. Because we can divide λ into 2 disjoint sets, one for the rows and the other for the columns. From Fig. 11, each edge then connects a row to a column, which satisfies the condition of a bipartite graph. This implies λ is bipartite and has no cycle of odd distance. Since λ does not contain a triangle, λ does not contain K_s with s $\geq$ 3 as an induced sub-graph. Because K_2 is the

maximum clique in λ, this proves that $\omega(\lambda) = 2$.
Moreover, λ is bipartite, hence it is 2-colorable. Fig. 12, depicts a proper coloring of λ. This proves that $\chi(\lambda) = 2$. Next, we compute the inde-

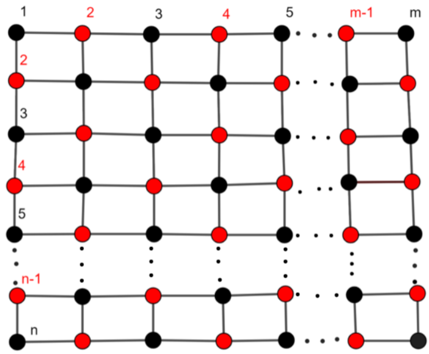

Figure 12. 2-coloring of $m \times m$ grid network ($G_{m\times m}$)

pendence number of λ. Because the colouring is proper, the nodes which belong to the similar colour class form an independent set. The alternating nodes in every row and column receive a similar colour, the highest order of a colour class in λ is $\lceil \frac{m^2}{2} \rceil$. Thus, we get $\alpha(\lambda) \geq \lceil \frac{m^2}{2} \rceil$. Now we prove that the size of any independent sets of nodes in λ of measurement m^2 do not cross $\lceil \frac{m^2}{2} \rceil$. On observing there are m columns in λ of dimension m^2 and each column is a path. Hence, λ of measurement m $\times$ m is divided into m paths. The order of any independent set of cardinality m does not cross $\lceil \frac{m^2}{2} \rceil$. Therefore, the size of any independent set of λ with measurement m $\times$ m does not cross $\lceil \frac{m^2}{2} \rceil$. Hence, $\alpha(\lambda) \leq \lceil \frac{m^2}{2} \rceil$. This proves that the set of the highest size in a colour class is the highest independent set of λ. Hence, $\alpha(\lambda) = \lceil \frac{m^2}{2} \rceil$. Hence the proof. ■

4 Perfect matching and matching ratio in silicate networks

The below-mentioned lemma gives the total number of nodes and the total number of edges in the chain silicate (CS_m).

Lemma 2. *Let* $\lambda = CS_m$. *Then,* $\mid V(\lambda) \mid$ = *3m + 1,* $\mid E(\lambda) \mid$ = *6m. The following theorem provides the perfect matching condition. For such values, we calculate the matching ratio.*

Theorem 10. *Let* λ *be the chain silicate* (CS_m). *Then,* λ *contains a perfect matching if* λ *has an odd number of tetrahedral. Further, the matching ratio of* λ *is* $m_r = \frac{(3m+1)}{12m}$.

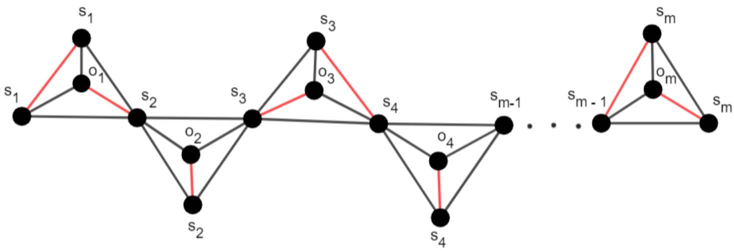

Figure 13. Perfect matching in chain silicate (CS_m)

Proof. Observe that there is no perfect matching in a graph λ if the tetrahedral is even. Since, $\mid V(\lambda) \mid$ = 3m + 1 is even if m is odd. Fig. 13, shows the perfect matching in λ. Thus, λ consists of a perfect matching if several tetrahedral is odd.
Then, λ consists of perfect matching and by Lemma 1, the matching ratio (m_r) is a ratio of the cardinality of the vertex set and cardinality of the edge set of λ. By Lemma 2, we obtain $m_r = \frac{(3m+1)}{12m}$. Hence the proof. ∎

The below-mentioned lemma computes the result for the total number of nodes and the total number of edges in the cyclic silicate (CYS_m).

Lemma 3. *Let* $\lambda = CYS_m$. *Then,* $\mid V(\lambda) \mid$ = *3m,* $\mid E(\lambda) \mid$ = *6m - 1.*

The following theorem provides the perfect matching condition. For such values, we calculate the matching ratio.
The next theorem gives conditions on cyclic silicate such that the cyclic silicate consists of perfect matching. For such λ we compute the matching ratio.

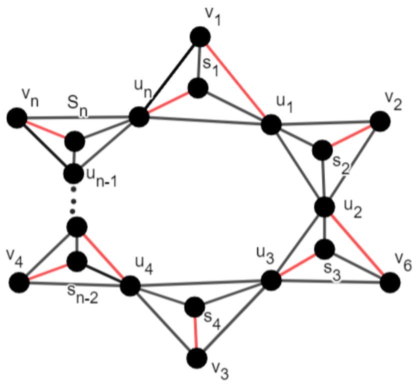

Figure 14. Perfect matching of cyclic silicate (CYS_m)

Theorem 11. *Let λ be the cyclic silicate (CYS_m). Then, λ consists of a perfect matching if λ has an even number of tetrahedral. Further, the matching ratio of λ is $m_r = \frac{(3m)}{12m-2}$.*

Proof. On observing that there exists a perfect matching if the number of tetrahedral is even whereas if the number of tetrahedral is odd, the perfect matching is not satisfied. Hence, $\mid V(\lambda) \mid = 3$m which is even when m is even. Fig. 14, shows the perfect matching in λ. Thus, λ consists of a perfect matching if m is even. Then, λ consists of perfect matching and by Lemma 1., the matching ratio (m_r) is a ratio of the vertex set's and edge set's cardinality. By Lemma 3, we have $m_r = \frac{(3m)}{12m-2}$. Hence the proof. ■

Theorem 12. *Let λ be the double chain silicate (DCS_m). Then, perfect matching is not satisfied in λ.*

Proof. Each tetrahedron in a double chain silicate connects to three other tetrahedra such as two within the chain and one connected with a different chain. This violates the perfect matching condition, as each tetrahedron needs to be paired with exactly one other tetrahedron in a perfect matching, which is impossible with the structure of double chain silicate. Hence the proof. ■

Theorem 13. *Let λ be the sheet silicate (SS_m). The perfect matching is not satisfied in λ for m >1.*

Proof. Each tetrahedron in a sheet silicate connects to three other tetrahedra forming a hexagonal structure. Similar to double chain silicate, the structure of sheet silicate does not allow each tetrahedron to be paired with just one other, not fulfilling the perfect matching condition. Hence the proof. ■

Lemma 4. *Let λ = (HC_m). Then, $\mid V(\lambda) \mid = 6m^2$, $\mid E(\lambda) \mid = 9m^2 - 3m$.*

The following theorem provides the perfect matching condition. For such values, we calculate the matching ratio.
The next theorem gives conditions on the honeycomb network such that the honeycomb network consists of perfect matching. For such λ we compute the matching ratio.

Theorem 14. *Let λ be the honeycomb network (HC_m) where m >1. Then λ consists of a perfect matching if m is even in λ for m >1. Then, λ consists of a perfect matching. Hence, $m_r(\lambda) = \frac{6m^2}{18m^2-6m}$.*

Proof. On observing that there is no perfect matching if m is odd for m >1. We have $\mid V(\lambda) \mid = 6m^2$ is even when m is both even and odd. This implies that the perfect matching is satisfied for even dimension. Fig. 15, shows the perfect matching with even dimension.
Hence, λ consists of a perfect matching, by Lemma 1, the matching ratio (m_r) is a ratio of cardinality of vertices and cardinality of edges of λ. By Lemma 4, we get $m_r(\lambda) = \frac{6m^2}{18m^2-6m}$. Hence the proof. ■

The below-mentioned Lemma computes the results for the total number of nodes and the total number of edges in the $m \times m$ grid network $(G_{m\times m})$.

Lemma 5. *Let $\lambda = G_{m\times m}$, where m is the dimension of $G_{m\times m}$. Then, $\mid V(\lambda) \mid = m^2$, $\mid E(\lambda) \mid = 2m^2$ - 2m.*

The below-mentioned theorem provides the perfect matching condition. For such values, we calculate the matching ratio.
The next theorem gives conditions on the $m \times m$ grid network such that

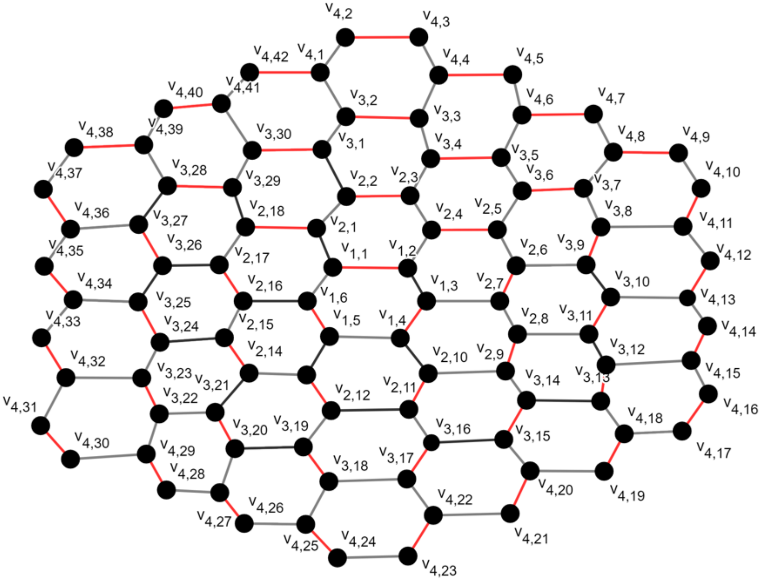

Figure 15. Perfect matching of honeycomb network (HC_m)

the grid network consists of perfect matching. For such λ we compute the matching ratio.

Theorem 15. *Let λ be the $m \times m$ grid network $(G_{m\times m})$. Then, λ consists of a perfect matching if λ is even product. Further, the matching ratio of λ is $m_r = \frac{m^2}{4m^2-4}$.*

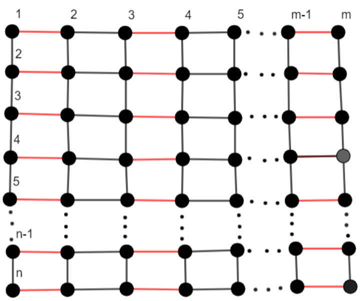

Figure 16. Perfect matching of $m \times m$ grid network ($G_{m\times m}$)

Proof. We observe that there is no existence of perfect matching in λ for the odd product of $m \times m$. The $\mid V(\lambda) \mid = m^2$ is even if $m \times m$ grid network is even. Fig. 16, shows the perfect matching in even product of λ. Hence, λ consists of perfect matching, by Lemma 1, the matching ratio(m_r) is a ratio of the vertex set and edge set cardinality. By Lemma 5, we obtain $m_r = \frac{m^2}{4m^2-4}$. Hence the proof. ∎

5 Minimum dominating sets in silicate networks

Here, we discuss the minimum dominating sets in the silicate networks. The below-mentioned theorem computes the results of the minimum dominating sets in silicate networks.

Theorem 16. *Let λ be the chain silicate (CS_m). Then, $\gamma(\lambda) = \lceil \frac{m}{2} \rceil$.*

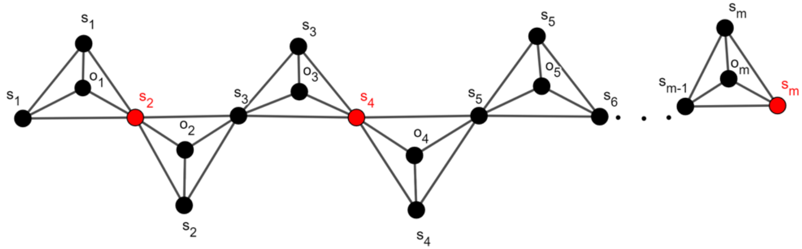

Figure 17. Minimum dominating set in even number of tetrahedral in chain silicate (CS_m)

Proof. We will prove this using the contradiction method: Suppose the minimum domination number in (CS_m) is $\lceil \frac{m}{2} \rceil - 1$. Consider it to be k.
Case 1: When m is even which implies that we can equally divide the chain silicate into 2 tetrahedra. Each group is dominated by a vertex with a maximum degree. By continuing this process at last one tetrahedron will be undominated which contradicts our assumption.
Case 2: When m is odd it implies that we can divide the chain silicate

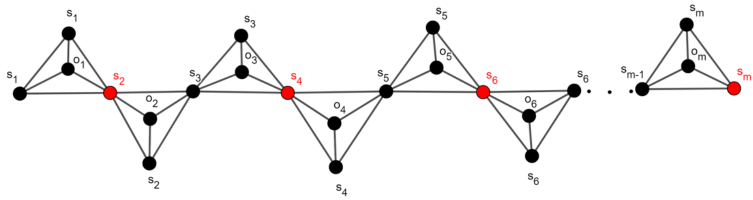

Figure 18. Minimum dominating set in odd number of tetrahedral in chain silicate (CS_m)

into groups of 3 tetrahedrons. Each group will be dominated by a vertex with a maximum degree and one tetrahedron will be undominated in all the groups which again contradicts our assumption. We conclude that $\gamma(\lambda) =$ k does not exist in both cases. Therefore, $\gamma(\lambda) = \lceil \frac{m}{2} \rceil$. Hence the proof. ∎

The next theorem shows the minimum domination number of cyclic silicate.

Theorem 17. *Let λ be the cyclic silicate (CYS_m). Then, $\gamma(\lambda) = \lceil \frac{m}{2} \rceil$.*

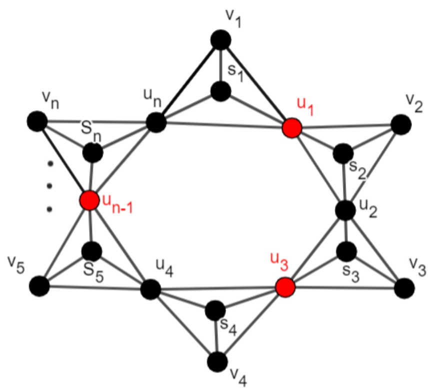

Figure 19. Minimum dominating set in even number of tetrahedral in cyclic silicate (CYS_m)

Proof. We will prove this using the contradiction method:
Suppose the minimum domination number in (CYS_m) is $\lceil \frac{m}{2} \rceil - 1$. Consider

it to be s.

Case 1: When m is even which implies that we can equally divide the cyclic silicate into 2 tetrahedra. Each group is dominated by a vertex with a maximum degree. By continuing this process at last one tetrahedron will be undominated which contradicts our assumption.

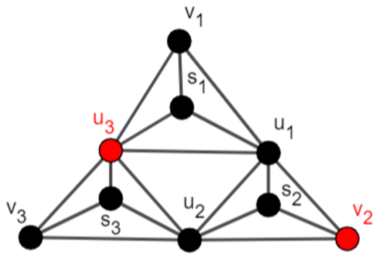

Figure 20. Minimum dominating set in odd number of tetrahedral in cyclic silicate (CYS_m)

Case 2: When m is odd it implies that we can divide the chain silicate into groups of 3 tetrahedrons. Each group will be dominated by a vertex with a maximum degree and one tetrahedron will be undominated in all the groups which again contradicts our assumption. In both cases, we conclude that $\gamma(\lambda)$ = s does not exist. Therefore, $\gamma(\lambda) = \lceil \frac{m}{2} \rceil$. Hence the proof. The next theorem shows the minimum domination number of double chain silicate. ∎

Theorem 18. *Let λ be the double chain silicate (DCS_m). Then,* $\gamma(\lambda) = \frac{m}{2}$.

Proof. In a double chain silicate, for each pair of tetrahedrons, the minimum domination number required is one, the minimum dominating set in a pair of tetrahedrons will be the vertex with the maximum degree. In general, the minimum domination number required is $\frac{m}{2}$. Fig. 21, shows the minimum domination number of double-chain silicate. Therefore, $\gamma(\lambda) = \frac{m}{2}$. Hence the proof. ∎

The following theorem computes the minimum domination number of sheet silicate (SS_m).

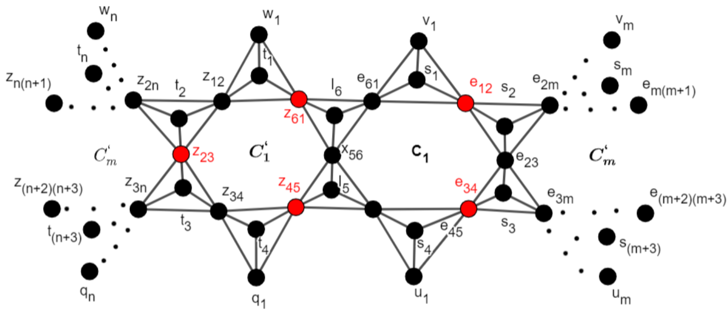

Figure 21. Minimum dominating set in double chain silicate (DCS_m)

Theorem 19. *Let λ be the sheet silicate (SS_m). Then, $\gamma(\lambda) = \lceil \frac{2m}{3} \rceil$ where m is the number of tetrahedra.*

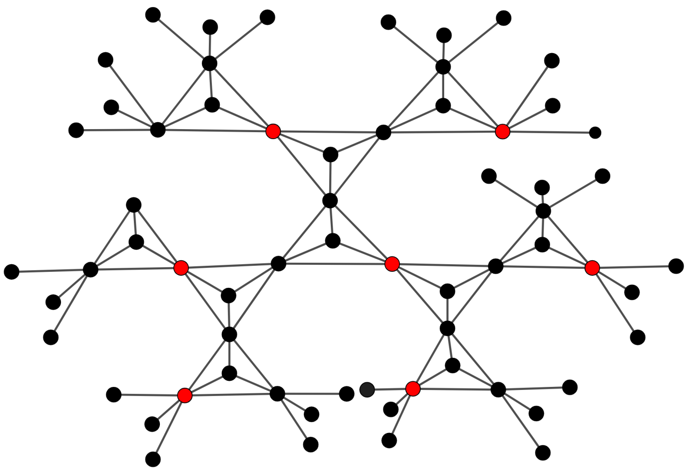

Figure 22. Minimum dominating set in sheet silicate (SS_m)

.

Proof. Assume the minimum dominating set for a sheet silicate has cardinality $\lceil \frac{2m}{3} \rceil$ where m represents the number of tetrahedra in the sheet silicate. We consider $\lceil \frac{2m}{3} \rceil$ - 1 = k. In λ the vertices with degree 3 dominate more vertices than the highest degree. There are (3m + 1) vertices in λ in which k dominates just (3m - 2) vertices which contradicts the fact that (k + 1) is the minimum dominating set in λ. Fig. 22, depicts the minimum dominating set in λ. Therefore, $\gamma(\lambda) = \lceil \frac{2m}{3} \rceil$. Hence the proof. ■

The following theorem gives the minimum domination number of the honeycomb network (HC_m).

Theorem 20. *Let λ be the honeycomb network* (HC_m)*. Then,* $\gamma(\lambda) = \frac{3m^2}{2}$ *when m is even dimension.*

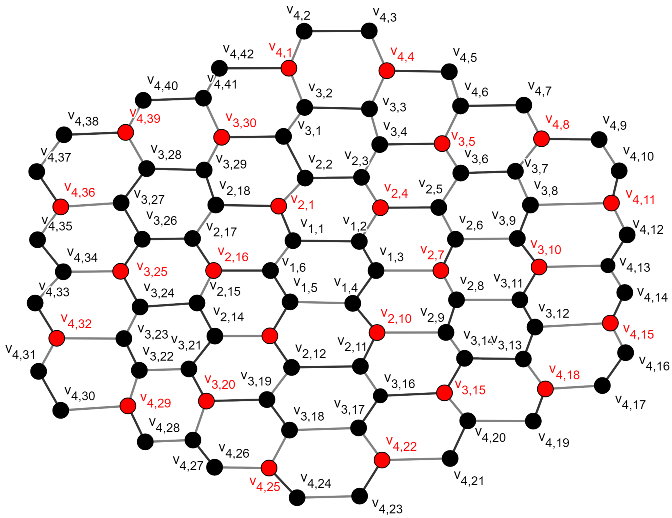

Figure 23. Minimum dominating set in even dimension honeycomb network (HC_m)

Proof. Assume a vertex t ∈ $V(\lambda)$, we know that there are deg(t) + 1 vertices dominated by t which are also the vertices adjacent to the vertex t. Therefore, in the even dimension in the honeycomb network, the vertices with degree 3 dominate more vertices than the vertices of the least degree. Since $\Delta(\lambda) = 3$, let s ∈ $V(\lambda)$ such that deg(s) = 3. If u = $6m^2$ is the order of λ, then s dominates 4 vertices excluding the u - 4 vertices of λ.

The remaining u - 4 undominated vertices dominate themselves. Fig. 23 provides the minimum dominating set in the honeycomb network. Next consider that $\gamma(\lambda)$ = q. Consider that the minimum dominating set is D = $\{t_1, \cdots, t_q\} \subset$ V(λ). We know that every t_l dominates 1 + $deg(t_l)$ vertices in λ, where $1 \leq l \leq$ q. We obtain, $\sum_{l=1}^{q}(1+deg(t_l)) \geq u$. Further, $1 + deg(t_l) \leq 1 + \Delta(\lambda) = 4$ for $1 \leq l \leq$ q. Thus, we have u $\leq \sum_{l=1}^{q}(1 + deg(t_l)) \leq$ 4q and 4q $\geq$ u. Since q = $\gamma(\lambda)$. We get, $\gamma(\lambda) \geq \frac{3m^2}{2}$. Also, we have u $\geq 1 + deg(t_l)$. Further, $1 + deg(t_l) \geq 1 + \Delta(\lambda) = 4$ for $1 \leq l \leq$ q. We get $u \geq 1 + deg(t_l) \geq$ 4q then u $\geq$ 4q. Since u = $6m^2$ and q = $\gamma(\lambda)$ then $6m^2 \geq 4\gamma(\lambda)$. So we get, $4\gamma(\lambda) \leq 6m^2$. Then, $\gamma(\lambda) \leq \frac{3m^2}{2}$. Hence the proof. ■

Theorem 21. *Let λ be the honeycomb network (HC_m). Then, $\gamma(\lambda) = \lceil \frac{3m^2}{2} \rceil + \lfloor \frac{m}{2} \rfloor$ when m is odd dimension.*

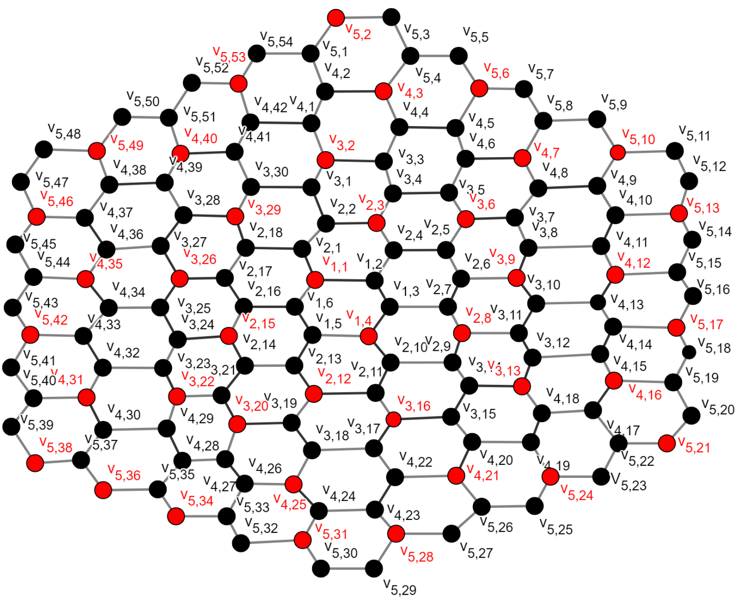

Figure 24. Minimum dominating set in odd dimension honeycomb network (HC_m)

.

Proof. Assume a vertex t $\in V(\lambda)$, we know that there are deg(t) + 1 vertices dominated by t which are also the vertices adjacent to the vertex t. Therefore, in the odd dimension of the honeycomb network, the vertices with degree 3 dominate more vertices than the vertices of the least degree. Since $\Delta(\lambda) = 3$, let s $\in V(\lambda)$ such that deg(s) = 3. If u = $6m^2$ is the order of λ, then s dominates 4 vertices excluding the u - 4 vertices of λ. The remaining u - 4 undominated vertices dominate themselves. Fig. 24 provides the minimum dominating set in the honeycomb network. Next consider that $\gamma(\lambda)$ = q. Consider that the minimum dominating set is D = $\{t_1, \cdots, t_q\}$ $\subset$ V(λ). We know that every t_l dominates 1 + $deg(t_l)$ vertices in λ, where $1 \leq l \leq$ q. We obtain, $\sum_{l=1}^{q}(1 + deg(t_l)) \geq u$. Further, $1 + deg(t_l) \leq 1$ + $\Delta(\lambda)$ = 4 for $1 \leq l \leq$ q. Thus, we have $m^2 \leq \sum_{l=1}^{q}(1 + deg(t_l)) \leq$ 4q and 4q $\geq 6m^2$. Since q = $\gamma(\lambda)$. We get, $\gamma(\lambda) \geq \lceil \frac{3m^2}{2} \rceil$. Also, we have u $\geq 1 + deg(t_l)$. Further, 1 + $deg(t_l) \leq 1 + \Delta(\lambda) = 4$ for $1 \leq l \leq$ q. We get u $\geq 1 + deg(t_l) \geq$ 4q. Then, u $\geq$ 4q and $6m^2 \geq 4\gamma(\lambda)$. Now, $4\gamma(\lambda) \leq 6m^2$ $\gamma(\lambda) \leq \lceil \frac{3m^2}{2} \rceil$.

Also, to dominate the remaining vertices we are assuming a vertex k $\in V(\lambda)$, we know that there are d(k) + 1 vertices dominated by k which are also the vertices adjacent to the vertex k. Therefore, in the odd dimension of the honeycomb network, the vertices with degree 3 dominate more vertices than the vertices of the least degree. Since $\Delta(\lambda) = 3$, let v $\in V(\lambda)$ such that deg(v) = 3. If u = 2m is the order of the undominated vertices, then v dominates 4 vertices excluding the u - 4 vertices of λ. The remaining u - 4 undominated vertices dominate themselves. Fig. 24 provides the minimum dominating set in the honeycomb network.

Next consider that $\gamma(\lambda)$ = q. Consider that the minimum dominating set is D = $\{k_1, \cdots, k_q\}$ $\subset$ V(λ). We know that every k_l dominates 1 + $deg(k_l)$ vertices in λ, where $1 \leq l \leq$ q. We obtain, $\sum_{l=1}^{q}(1 + deg(k_l)) \geq u$. Further, 1 + $deg(k_l) \leq 1 + \Delta(\lambda) = 4$ for $1 \leq l \leq$ q. Thus, we have $2m \leq \sum_{l=1}^{q}(1 + deg(k_l)) \leq$ 4q and 4q $\geq 2m$. Since q = $\gamma(\lambda)$. We get, $\gamma(\lambda) \geq \lfloor \frac{m}{2} \rfloor$. Also, we have u $\geq 1 + deg(k_l)$. Further, 1 + $deg(k_l) \leq 1 + \Delta(\lambda)$ = 4 for $1 \leq l \leq$ q. We get $u \geq 1 + deg(k_l) \geq$ 4q. Then, u $\geq$ 4q. Since u = 2m and q = $\gamma(\lambda)$. We get, $2m \geq 4\gamma(\lambda)$. Then, $4\gamma(\lambda) \leq 2m$. Also, $\gamma(\lambda) \leq \lfloor \frac{m}{2} \rfloor$. By adding we get, $\gamma(\lambda) = \lceil \frac{3m^2}{2} \rceil + \lfloor \frac{m}{2} \rfloor$. Hence the proof. ■

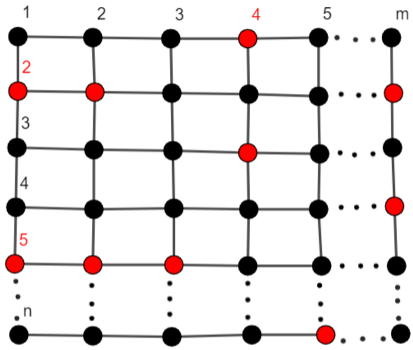

Figure 25. Minimum dominating set for m ≡ 2(mod 4) in grid network ($G_{m\times m}$)

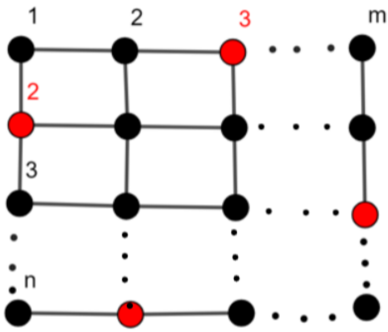

Figure 26. Minimum dominating set for m ≡ 0(mod 4) in grid network ($G_{m\times m}$)

The following theorem computes the minimum domination number of $m \times m$ grid network.

Theorem 22. *Let λ be the $m \times m$ even product grid network $(G_{m\times m})$. Then,* $\gamma(\lambda)$ =

$$\begin{cases} \frac{m^2}{4} + \lfloor \frac{m}{4} \rfloor, & \textit{when } m \equiv 2 (\textit{mod } 4), \textit{ for } m > 2 \\ \frac{m^2}{4}, & \textit{when } m \equiv 0 (\textit{mod } 4) \end{cases}$$

.

Proof. Assume a vertex t $\in V(\lambda)$, we know that there are deg(t) + 1 vertices which are dominated by t which are also the vertices adjacent to the vertex t. Therefore, in the even product of m $\times$ m grid network, the vertices with degree 3 dominate more vertices than the vertices of the highest degree. Since $\delta(\lambda) = 3$, let s $\in V(\lambda)$ such that deg(s) = 3. If u = m^2 is the order of λ, then s dominates 4 vertices excluding the u - 4 vertices of λ. The remaining u - 4 undominated vertices dominate themselves. Fig. 26 provide the minimum dominating set in $m^2 \equiv 0$(mod 3) and $m^2 \equiv 2$(mod 4).

Next consider that $\gamma(\lambda)$ = q. Consider that the minimum dominating set is D = $\{t_1, \cdots, t_q\} \subset$ V(λ). We know that every t_l dominates 1 + $deg(t_l)$ vertices in λ, where $1 \leq l \leq$ q. We obtain, $\sum_{l=1}^{q}(1 + deg(t_l)) \geq u$. Further, 1 + $deg(t_l) \leq$ 1 + $\delta(\lambda)$ = 4 for $1 \leq l \leq$ q. Thus, we have $m^2 \leq \sum_{l=1}^{q}(1 + deg(t_l)) \leq$ 4q and 4q $\geq m^2$. Since q = $\gamma(\lambda)$. We get, $\gamma(\lambda) \geq \frac{m^2}{4}$. Also, we have u $\geq 1 + deg(t_l)$. Further, 1 + $deg(t_l) \leq$ 1 + $\delta(\lambda)$ = 4 for $1 \leq l \leq$ q. We get, $u \geq 1 + deg(t_l) \geq$ 4q. Then, u $\geq$ 4q. Since u = m^2 and q = $\gamma(\lambda)$. We get, $m^2 \geq 4\gamma(\lambda)$. Further, $4\gamma(\lambda) \leq m^2$. Then, $\gamma(\lambda) \leq \frac{m^2}{4}$.

Also, to dominate the remaining vertices we are assuming a vertex k $\in V(\lambda)$, we know that there are deg(k) + 1 vertices dominated by k which are also the vertices adjacent to the vertex k. Therefore, in the even product grid network, the vertices with degree 3 dominate more vertices than the vertices of the least degree. Since $\Delta(\lambda) = 3$, let v $\in V(\lambda)$ such that deg(v) = 3. If u = m is the order of the undominated vertices, then v dominates 4 vertices excluding the u - 4 vertices of λ. The remaining u - 4 undominated vertices dominate themselves.

Next consider that $\gamma(\lambda)$ = q. Consider that the minimum dominating set is D = $\{k_1, \cdots, k_q\} \subset$ V(λ). We know that every k_l dominates 1 + $deg(k_l)$ vertices in λ, where $1 \leq l \leq$ q. We obtain, $\sum_{l=1}^{q}(1 + deg(k_l)) \geq u$. Further, $1 + deg(k_l) \leq$ 1 + $\Delta(\lambda)$ = 4 for $1 \leq l \leq$ q. Thus, we have m $\leq \sum_{l=1}^{q}(1 + deg(k_l)) \leq$ 4q and 4q $\geq$ m. Since q = $\gamma(\lambda)$. We get, $\gamma(\lambda) \geq \lfloor \frac{m}{4} \rfloor$. Also, we have u $\geq 1 + deg(k_l)$. Further, 1 + $deg(k_l) \geq$ 1 + $\Delta(\lambda)$ = 4 for $1 \leq l \leq$ q. We get $u \geq 1 + deg(k_l) \geq$ 4q. We get, u $\geq$ 4q. Since u = m and q = $\gamma(\lambda)$. Then, $2m \geq 4\gamma(\lambda)$. Further, $4\gamma(\lambda) \leq m$. Also, $\gamma(\lambda) \leq$

$\lfloor \frac{m}{4} \rfloor$. By adding we get, $\gamma(\lambda) = \frac{m^2}{4} + \lfloor \frac{m}{4} \rfloor$ when m $\equiv$ 2(mod 4).
Assume a vertex k $\in V(\lambda)$, we know that there are deg(k) + 1 vertices dominated by k which are also the vertices adjacent to the vertex k. Therefore, in the even product of m $\times$ m grid network, the vertices with degree 3 dominate more vertices than those of the highest degree. Since $\delta(\lambda) =$ 3, let v $\in V(\lambda)$ such that deg(v) = 3. If u = m^2 is the order of λ, then v dominates 4 vertices excluding the u - 4 vertices of λ. The remaining u - 4 undominated vertices dominate themselves. Fig. 27 provides the minimum dominating set in $m \equiv 0(mod 4)$.
Next consider that $\gamma(\lambda)$ = q. Consider that the minimum dominating set is D = $\{k_1, \cdots, k_q\} \subset$ V(λ). We know that every k_l dominates 1 + $deg(k_l)$ vertices in λ, where 1 $\leq l \leq$ q. We obtain, $\sum_{l=1}^{q}(1 + deg(k_l)) \geq u$. Further, 1 + $deg(k_l) \leq$ 1 + $\delta(\lambda)$ = 4 for 1 $\leq l \leq$ q. Thus, we have $m^2 \leq \sum_{l=1}^{q}(1 + deg(k_l)) \leq$ 4q and 4q $\geq m^2$. Since q = $\gamma(\lambda)$. We get, $\gamma(\lambda) \geq \frac{m^2}{4}$. Also, we have u $\geq 1 + deg(k_l)$. Further, 1 + $deg(k_l) \geq$ 1 + $\delta(\lambda)$ = 4 for 1 $\leq l \leq$ q. We get $u \geq 1 + deg(k_l) \geq$ 4q. Then, u $\geq$ 4q. Since u = m^2 and q = $\gamma(\lambda)$. Further, $m^2 \geq 4\gamma(\lambda)$. Then, $4\gamma(\lambda) \leq m^2$. Also, $\gamma(\lambda) \leq \frac{m^2}{4}$ when m $\equiv$ 0(mod 4). Hence the proof. ■

Theorem 23. *Let λ be the m $\times$ m odd product grid network ($G_{m\times m}$). Then,* $\chi(\lambda) = \lceil \frac{m^2}{4} \rceil$.

Proof. Assume a vertex t $\in V(\lambda)$, we know that there are deg(t) + 1 vertices dominated by t which are also the vertices adjacent to the vertex t. Therefore, in the odd product of m $\times$ m grid network, the vertices with degree 3 dominate more vertices than the vertices of the highest degree. Since $\delta(\lambda) = 3$, let s $\in V(\lambda)$ such that deg(s) = 3. If u = m^2 is the order of λ, then s dominates 4 vertices excluding the u - 4 vertices of λ. The remaining u - 4 undominated vertices dominate themselves. Fig. 27 and Fig. 28 provide the minimum dominating set in $m^2 \equiv$ 0(mod 3) and $m^2 \equiv$ 1(mod 3).

Next consider that $\gamma(\lambda)$ = q. Consider that the minimum dominating set is D = $\{t_1, \cdots, t_q\} \subset$ V(λ). We know that every t_l dominates 1 + $deg(t_l)$ vertices in λ, where 1 $\leq l \leq$ q. We obtain, $\sum_{l=1}^{q}(1 + deg(t_l)) \geq u$. Further, 1 + $deg(t_l) \leq$ 1 + $\delta(\lambda)$ = 4 for 1 $\leq l \leq$ q. Thus, we have $m^2 \leq$

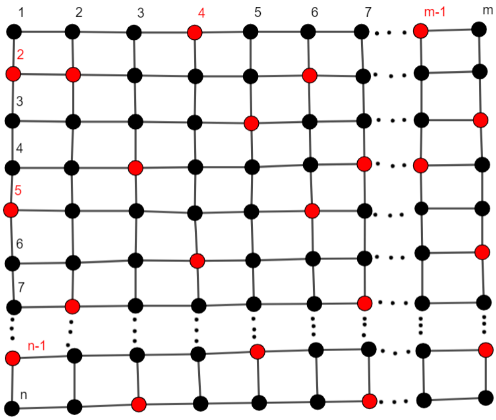

Figure 27. Minimum dominating set for $m^2 \equiv 0 \pmod 3$ in grid network $(G_{m\times m})$

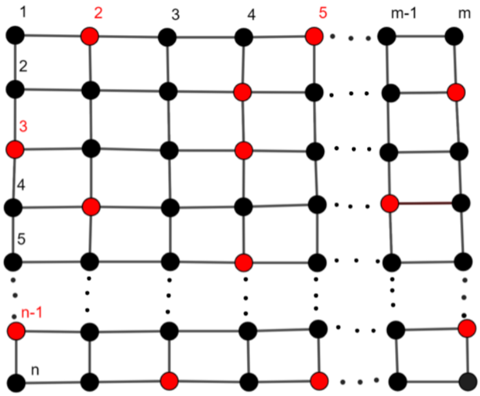

Figure 28. Minimum dominating set for $m^2 \equiv 1 \pmod 3$ in grid network $(G_{m\times m})$

$\sum_{l=1}^{q}(1 + deg(t_l)) \leq$ 4q and 4q $\geq m^2$. Since q $= \gamma(\lambda)$. We get, $\gamma(\lambda) \geq \lceil \frac{m^2}{4} \rceil$. Also, we have u $\geq 1 + deg(t_l)$. Further, $1 + deg(t_l) \geq 1 + \delta(\lambda) =$ 4 for $1 \leq l \leq$ q. We get $u \geq 1 + deg(t_l) \geq$ 4q. Then, u $\geq$ 4q. Since u $=$ m^2 and q $= \gamma(\lambda)$. Also, $m^2 \geq 4\gamma(\lambda)$. Further, $4\gamma(\lambda) \leq m^2$. Also, we have $\gamma(\lambda) \leq \lceil \frac{m^2}{4} \rceil$. Hence the proof. ∎

Table 1. Comparative study using graph-invariants

Literature Methodology	**Our method**
Manuel(2009) [1] Limited to topological structures of honeycomb network, silicate networks	Computed the topological properties for CS_m, CYS_m, DCS_m, SS_m, HC_m and $G_{m\times m}$.
Muthukumar(2020) [10] Limited to minimum domination number of even number of tetrahedral in chain silicate	Computed for both even and odd numbers of tetrahedral in CS_m.
Liu(2019) [6] Limited to topological indices of silicate networks	Computed the graph-invariant which depicts the topological properties of silicate networks.

6 Conclusion

Here, we compute the structural properties of silicate networks using the graph-invariant such as clique number, chromatic number, independence number, perfect matching, matching ratio and minimum domination number. These invariants have been selected on observing restrictions of methodology in the literature. The general results mentioned in this article provide some interesting facts about the structural properties of silicate networks.

- In this article we observe that the clique number of chain silicate, cyclic silicate, double chain silicate, and sheet silicate is 3 whereas the honeycomb network and $m \times m$ grid is 2.
- The chromatic number of chain silicate, cyclic silicate and sheet silicate is 4 whereas double chain silicate is 4, honeycomb network and $m \times m$ grid is 2.
- The independence number of chain silicate, cyclic silicate, double chain silicate, sheet silicate and $m \times m$ grid network is m.
- The minimum domination number of chain silicate, cyclic silicate, and sheet silicate is $\lceil \frac{m}{2} \rceil$.

7 Applications

The general results in this article have some interesting applications for silicate networks. These are as follows:

- Clique number in silicate networks provides the molecular structures.
- Chromatic number is useful in determining the least number of different atom types necessary to represent a molecule. In chemical graph theory, atoms are denoted as vertices and chemical bonds are denoted as edges.
- Independence number provides the ability to fill non-interacting elements or divisions within the network.
- Perfect matching represents minimal pairings between some elements.
- Minimum dominating set represents an efficient use of elements in controlling other elements.

8 Future scope

In our study, the structural properties of silicate networks have been discussed using graph-invariant. However, their practical application needs to be studied. The below-mentioned problems need to be studied.

- **Problem 8.1.** Discuss the practical applications of general results obtained in this problem.
- **Problem 8.2.** Discuss the least metric dimension for all the silicate networks mentioned in this article.
- **Problem 8.3.** Discuss the problem statement considered in this article for other families of networks.
- **Problem 8.4.** We can implement monophonic pebbling number concept mentioned in [19] on the considered silicate networks.

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