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Fuzzy Degree and Intuitionistic Fuzzy Degree for Chemical Hyperstructures

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Abstract

The notion of hyperstructures first introduced by Marty in 1934 as a generalization of algebraic structures. Next, by associating fuzzy sets, Corsini introduced new notion in hyperstructures, that is fuzzy hyperstructures. After fuzzy hyperstructures introduced by Corsini, Cristea introduced the notion of intuitionistic fuzzy hyperstructures as a generalization of fuzzy hyperstructures. In 2023, Violeta-Fotea related the notion of fuzzy hyperstructures with genetic inheritance. Inspired by this research, this paper aims to find relation between fuzzy and intuitionistic fuzzy hyperstructures with chemical hyperstructures, that is to finding the fuzzy degree and intuitionistic fuzzy degree on chemical hyperstructures such as redox reactions, electrochemical cells, equilibrium reactions, and acid rain reactions.

1 Introduction

Hyperstructures are a generalization of algebraic structures. This concept was introduced by Marty in 1934 [1]. If in algebraic structures the composition of two elements is an element, then in hyperstructures the composition of two elements is a set. Furthermore, by involving weak concept in

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hyperstructures, Vougiouklis defined a new type of hyperstructures called H_v-structures in 1990 [2]. Some research related to hyperstructures is the involvement of the concept of partial ordered (p.o) on hyperstructures such as semihypergroup, polygroup, and hyperring so that the concept called ordered hyperstructures is born [3 - 5], transposition hypergroups [6], determination of hypergroups and join spaces with relations [7], and the differential calculus on hyperrings [8]. Furthermore, in Corsini and Leoreanu-Fotea's book titled "Application of Hyperstructures Theory", there are various applications of hyperstructures, namely in automata, cryptography, geometry, coding theory, and artificial intelligence [9]. Additionally, hyperstructures also have applications in natural sciences, such as physics, biology, and chemistry. In physics, hyperstructures are needed to analyze the type of hyperstructures in particle physics, specifically leptons [10]. Furthermore, in biology, hyperstructures are needed to analyze the types of hyperstructures in the inheritance [11-12] and the results obtained in [11] and [12] are associated with the concept of ordered hyperstructures so that the application of ordered hyperstructures in biology is obtained [13]. Furthermore, in chemistry, hyperstructures are needed to analyze the type of hyperstructure in chemical reactions. This idea was first introduced by Davvaz and Dehghan-Nezhad in 2003 [14]. Inspired by this idea, hyperstructures were eventually used to analyze the types of hyperstructures in other chemical reactions, such as dismutation reactions [15], redox reactions [16], redox reactions with three oxidation states [17], redox reactions with four oxidation states [18-19], reactions in electrolysis cells [20-21], salt formation reactions [22], ozone depletion reactions [23-24], chemical equilibrium reactions [25], and biochemical reactions [26].

In contrast, the notion of fuzzy set is an extension of the set. The concept of fuzzy sets was introduced by Azerbaijani mathematician, L. A. Zadeh in 1965 [27]. Furthermore, in 1986, Bulgarian mathematician, K. T. Attanasov generalized the concept of fuzzy sets into a set concept that has a membership degree and a non-membership degree called intuitionistic fuzzy sets [28]. Both fuzzy sets and intuitionistic fuzzy sets have a relationship with hyperstructures. Corsini introduced the connection of fuzzy sets with hyperstructures [29-30], Stefanescu introduced the concept of fuzzy grade on hypergroups [31] and Cristea introduced the concept of intuitionistic fuzzy grade on hypergroups [32]. Furthermore, the concepts of fuzzy grade and intuitionistic fuzzy grade are used to determine fuzzy grade and intuitionistic fuzzy grade on hyperstructures related to genetics [33-36].

Motivated by previous research on determining the fuzzy grade and intuitionistic fuzzy grade on hyperstructures related to genetics, this research goal is to analyze the fuzzy grade and intuitionistic fuzzy grade on chemical hyperstructures. Chemical hyperstructures that are taken include chemical hyperstructures in redox reactions with three and four oxidation states, reactions in electrochemical cells, chemical equilibrium reactions, and acid rain reactions.

2 Preliminaries

In this section, recall the concept that needed in this paper. First, recall the definition of hypergroups. The following definitions are retrieved from [37].

Let H be a nonempty set and define a mapping $\circ : H \times H \to \mathcal{P}^*(H)$ with $\mathcal{P}^*(H)$ is the collection of all nonempty subsets of H. Then, a mapping "$\circ$" is called a hyperoperation on H and the mathematical system $(H, \circ)$ is called a hypergroupoid. Here, the meaning of the hyperoperation "$\circ$" is if $A, B \subseteq H$ and both are nonempty, for every $x \in H$, define:

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b,\ x \circ A = \{x\} \circ A,\ A \circ x = A \circ \{x\}.$$

Next, $(H, \circ)$ is called a semihypergroup if for every $x, y, z \in H$, $x \circ (y \circ z) = (x \circ y) \circ z$. A hypergroupoid $(H, \circ)$ is called a quasihypergroup if for every $x \in H$, $x \circ H = H \circ x = H$. The mathematical system $(H, \circ)$ is called a hypergroup if it is a semihypergroup and quasihypegroup. A hypergroupoid is called commutative if for every $x, y \in H$, $x \circ y = y \circ x$. Furthermore, the hypergroup $(H, \circ)$ is called a total hypergroup if for every $x, y \in H$, $x \circ y = H$.

Next, recall the definition of join spaces. The notion of join spaces first introduced by Prenowitz in 1950 [38 - 39]. Let $(H, \circ)$ is a commutative hypergroup, then $(H, \circ)$ is called a join space if for every $p, q, r, s, x \in H$,

$$p \in q \circ x,\, r \in s \circ x \Rightarrow p \circ s \cap q \circ r \neq \emptyset.$$

Now, recall the notion of fuzzy degree. This notion introduced by Corsini in 2003 [30]. Let $\mu : H \to [0,1]$ be a fuzzy subset of a nonempty set H. For any $a, b \in H$, define a hyperoperation $\circ_1$ on H as follows.

$$(z) : \, a \circ_1 b = b \circ_1 a = \{c \in H : (\mu(a) \wedge \mu(b)) \leq \mu(c) \leq (\mu(a) \vee \mu(b))\}$$

With $\wedge = min\{\mu(a), \mu(b)\}$ and $\vee = max\{\mu(a), \mu(b)\}$ hypergroupoid ${}^1H$ towards a hyperoperation $\circ_1$ is a join space [30]. in [30] and [31], Corsini defined fuzzy subsets from hypergroups with the following rules: For any hypergroup $(H, \circ)$, define a fuzzy subset $\mu : H \to [0,1]$ of H as follows:

$$\forall u \in H, Q(u) = \{(x, y) \in H^2 : u \in x \circ y\}$$
$$q(u) = |Q(u)|$$
$$\gamma(u) = \Sigma_{(x,y) \in Q(u)} \tfrac{1}{|x \circ y|}$$
$$(z'') : \, \mu(u) = \tfrac{\gamma(u)}{q(u)}$$

Next, recall the notion of fuzzy degree and strong fuzzy degree. Let $(({}^iH, \circ_i),\ \mu_i)$ is a sequence of fuzzy sets and join spaces that obtained with the following condition. If two consecutive hypergroups of the obtained sequence are isomorphic, then the sequence stops. Fuzzy degree of H is the length of the sequence of join spaces corresponding to H and denoted by $f.d(H)$. A hypergroupoid H has a fuzzy degree $m \in \mathbb{N}$ and written as $f.d(H) = m$ if for every i with $0 \leq i \leq m$, the join space ${}^iH$ is not isomorphic with the join space ${}^{i+1}H$ and for every $r > m$, ${}^rH$ and ${}^mH$ are isomorphic. If $f.d(H) = m$ and for every $r > m$, ${}^rH = {}^mH$, then H has a strong fuzzy degree, denoted by $s.f.d(H) = m$.

Now, the concept related to Attanasov's intuitionistic fuzzy degree is reminded. The notion of intuitionistic fuzzy sets is a generalization of fuzzy sets. This notion first introduced by Attanasov in 1986 [28]. Let A be a nonempty set, the notion of intuitionistic fuzzy sets in A have a form $B = \{(a, \mu_B(a), \lambda_B(a))\}$ where $a \in A$. $\mu_B(a)$ is a degree of membership of a and $\lambda_B(a)$ is a degree of non-membership of A and this condition is satisfied.

$$0 \leq \mu_B(a) + \lambda_B(a) \leq 1$$

Furthermore, Cristea and Davvaz [32] define an Attanasov's intuitionistic fuzzy degree. Let $(H, \circ)$ be a finite hypergroupoid with cardinality n and $n \in \mathbb{N}$. Define on H an Attanasov's intuitionistic fuzzy set B with the following rule:

For any $u \in H$, as $Q(u)$ denotes all $(a, b) \in H^2$ satisfying $u \in a \circ b$ and $\bar{Q}(u)$ denotes all $(a, b) \in H^2$ satisfying $u \notin a \circ b$ and consider

$$\mu(u) = \frac{\Sigma_{(x,y)\in Q(u)} \frac{1}{|x \circ y|}}{n^2}$$
$$\lambda(u) = \frac{\Sigma_{(x,y)\in \bar{Q}(u)} \frac{1}{|x \circ y|}}{n^2}$$

If $Q(u) = \emptyset$ (or $\bar{Q}(u) = \emptyset$), then $\mu(u) = 0$ (or $\lambda(u) = 0$).
Next, let $B = (\mu, \lambda)$ be a Attanasov's intuitionistic fuzzy set on H. Let we have two join spaces $(H, \circ_{\mu\wedge\lambda})$ and $(H, \circ_{mu\vee\lambda})$, where for any fuzzy set γ on H, the hyperoperation $\circ_\gamma$ is defined as

$$a \circ_\gamma b = \{u \in H | \gamma(a) \wedge \gamma(b) \leq \gamma(u) \leq \gamma(a) \vee \gamma(b)\}$$

Corsini show that $(H, \circ_\gamma)$ is a join space [29]. Now, let we have an arbitrary Attanasov's intuitionistic fuzzy set $B = (\mu, \lambda)$ on the set H. We associate the join space $({}_0H, \circ_{\mu\wedge\lambda})$ and we construct the Attanassov's intuitionistic fuzzy set $B_1 = (\mu_1, \lambda_1)$. Next, we associate again the join space $({}_1H, \circ_{\mu\wedge\lambda})$, then we determine its Attanasov's intuitionistic fuzzy set $A_2 = (\mu_2, \lambda_2)$ and we construct the join space $({}_2H, \circ_{\mu\wedge\lambda})$ and so on. We have the sequence $(({}_iH, \circ_{\mu_i\wedge\lambda_i}), A_i)_{i\geq 0}$ of the join spaces and Attanasov's intuitionistic fuzzy sets associated with H. In the same way, we can construct the second sequence $(({}^iH, \circ_{\mu_i\vee\lambda_i}), A_i)_{i\geq 0}$.

Next, we recall the notion of lower Attanasov's intuitionistic fuzzy degree and upper Attanasov's intuitionistic fuzzy degree of H. A set H with an Attanasov's intuitionistic fuzzy set has the lower (upper) Attanasov's intuitionistic fuzzy degree $m \in \mathbb{N}$, denoted by $l.i.f.d(H) = m$ $(u.i.f.d(H) = m)$ if for any $0 \leq i \leq m$, the join spaces $({}_iH, \circ_{\mu_i\wedge\lambda_i})$ (or $({}^iH, \circ_{\mu_i\vee\lambda_i})$) associated with H are not isomorphic and for any $r > m$, ${}_rH$ is isomorphic with ${}_{m-1}H$ (${}^rH$ is isomorphic with ${}^{m-1}H$).

3 Chemical redox reactions with three and four oxidation states

Redox reactions are the reactions in which a decrease in oxidation state (reduction reaction) and an increase in oxidation state (oxidation reaction) occur simultaneously [40]. In this section, we focused on redox reactions whose elements have three and four oxidation states.

3.1 Chemical hyperstructures for element with three oxidation states

This notion first generalize by Chung in 2014 [17]. In general, we have the Latimer diagram as follows.

$$A \xrightarrow{p} B \xrightarrow{q} C$$

with A, B, and C are elements, p and q are potential (in volts). There are two cases for hyperstructures in this case, that is $p > q$ or $p < q$. Let $X = \{A, B, C\}$ and define a hyperoperation $\odot$ as a product of all spontaneuos reactions between two elements in X. Based on [17], for case $p > q$, we have the following results.

Table 1. $(X, \odot)$

$\odot$	A	B	C
A	$\{A\}$	$\{A, B\}$	$\{A, B, C\}$
B	$\{A, B\}$	$\{A, C\}$	$\{A, C\}$
C	$\{A, B, C\}$	$\{A, C\}$	$\{C\}$

$(X, \odot)$ is a hypergroup [17].

Proposition 1. *Let $X = \{A, B, C\}$, then $s.f.d(X) = 2$.*

Proof. Simple calculation gives that

$$\mu(A) = \tfrac{25}{42} \quad \mu(B) = \tfrac{5}{12} \quad \mu(C) = \tfrac{19}{36}$$

We represent $({}_1X, \odot)$ by the following table:

Table 2. $({}_1X, \odot)$

$\odot$	A	B	C
A	$\{A\}$	$\{A,B,C\}$	$\{A,C\}$
B	$\{A,B,C\}$	$\{B\}$	$\{B,C\}$
C	$\{A,C\}$	$\{B,C\}$	$\{C\}$

Simple calculation gives that:

$$\mu(A) = \tfrac{8}{15} \quad \mu(B) = \tfrac{5}{12} \quad \mu(C) = \tfrac{19}{36}$$

We represent $({}_2X, \odot)$ by the following table:

Table 3. $({}_2X, \odot)$

$\odot$	A	B	C
A	$\{A\}$	$\{A,B,C\}$	$\{A,C\}$
B	$\{A,B,C\}$	$\{B\}$	$\{B,C\}$
C	$\{A,C\}$	$\{B,C\}$	$\{C\}$

Then, $s.f.d(X) = 2$ because ${}_1H = {}_2H$. ■

Remark. For case $p < q$, X not have a fuzzy degree.

now, the intuitionistic fuzzy degree is determined.

Proposition 2. *For case $p < q$, X have $l.i.f.d(X) = 2$ and $u.i.f.d(X) = 2$.*

Proof. Simple calculation gives that

$$\mu(A) = \tfrac{1}{2} \quad \mu(B) = 0 \quad \mu(C) = \tfrac{1}{2}$$
$$\lambda(A) = \tfrac{1}{9} \quad \lambda(B) = 0 \quad \lambda(C) = \tfrac{1}{9}$$

Based on the above results, we get

$$(\mu \wedge \lambda)(A) = \tfrac{1}{9} \quad (\mu \wedge \lambda)(B) = 0 \quad (\mu \wedge \lambda)(C) = \tfrac{1}{9}$$

Then, we have $({}_0H, \odot_{\mu\wedge\lambda})$ as follows.

Table 4. $({}_0H, \odot_{\mu\wedge\lambda})$

$\odot_{\mu\wedge\lambda}$	A	B	C
A	$\{A,C\}$	$\{A,B,C\}$	$\{A,C\}$
B	$\{A,B,C\}$	$\{B\}$	$\{A,B,C\}$
C	$\{A,C\}$	$\{A,B,C\}$	$\{A,C\}$

In the similar way as finding $({}_1H, \odot_{\mu\wedge\lambda})$, we get ${}_1H = {}_2H$. Thus, we get $l.i.f.g(X) = 2$ and in the similar way as finding $l.i.f.g(X)$, we get $u.i.f.g(X) = 2$. ■

Proposition 3. *For case $p > q$, X have $l.i.f.d(X) = 3$ and $u.i.f.d(X) = 3$*

Proof. Simple calculation gives that

$$\mu(A) = \tfrac{25}{54} \quad \mu(B) = \tfrac{5}{27} \quad \mu(C) = \tfrac{19}{54}$$
$$\lambda(A) = \tfrac{1}{9} \quad \lambda(B) = \tfrac{5}{18} \quad \lambda(C) = \tfrac{2}{9}$$

Based on the above results, we get

$$(\mu \wedge \lambda)(A) = \tfrac{1}{9} \quad (\mu \wedge \lambda)(B) = \tfrac{10}{54} \quad (\mu \wedge \lambda)(C) = \tfrac{2}{9}$$

We represent $({}_1H, \odot_{\mu_1\wedge\lambda_1})$ by the following table:

Table 5. $({}_1H, \odot_{\mu_1\wedge\lambda_1})$

$\odot_{\mu_1\wedge\lambda_1}$	A	B	C
A	$\{A\}$	$\{A, B\}$	$\{A, B, C\}$
B	$\{A, B\}$	$\{B\}$	$\{B, C\}$
C	$\{A, B, C\}$	$\{B, C\}$	$\{C\}$

Simple calculation gives that:

$$\mu_2(A) = \tfrac{10}{27} \quad \mu_2(B) = \tfrac{11}{27} \quad \mu_2(C) = \tfrac{10}{27}$$
$$\lambda_2(A) = \tfrac{3}{9} \quad \lambda_2(B) = \tfrac{1}{9} \quad \lambda_2(C) = \tfrac{3}{9}$$

Based on the above results, we get

$$(\mu_2 \wedge \lambda_2)(A) = \tfrac{3}{9} \quad (\mu_2 \wedge \lambda_2)(B) = \tfrac{1}{9} \quad (\mu_2 \wedge \lambda_2)(C) = \tfrac{3}{9}$$

We represent $({}_2H, \odot_{\mu_2\wedge\lambda_2})$ by the following table:

Table 6. $({}_2H, \odot_{\mu_2 \wedge \lambda_2})$

$\odot_{\mu_2 \wedge \lambda_2}$	A	B	C
A	$\{A, C\}$	$\{A, B, C\}$	$\{A, C\}$
B	$\{A, B, C\}$	$\{B\}$	$\{A, B, C\}$
C	$\{A, C\}$	$\{A, B, C\}$	$\{A, C\}$

Next, we have

$$(\mu_3 \wedge \lambda_3)(A) = \tfrac{1}{9} \quad (\mu_3 \wedge \lambda_3)(B) = \tfrac{2}{9} \quad (\mu_3 \wedge \lambda_3)(C) = \tfrac{1}{9}$$

We represent $({}_3H, \odot_{\mu_3 \wedge \lambda_3})$ by the following table:

Table 7. $({}_3H, \odot_{\mu_3 \wedge \lambda_3})$

$\odot_{\mu_3 \wedge \lambda_3}$	A	B	C
A	$\{A, C\}$	$\{A, B, C\}$	$\{A, C\}$
B	$\{A, B, C\}$	$\{B\}$	$\{A, B, C\}$
C	$\{A, C\}$	$\{A, B, C\}$	$\{A, C\}$

Since $({}_2H, \odot_{\mu_3 \wedge \lambda_3})$ and $({}_3H, \odot_{\mu_3 \wedge \lambda_3})$ are isomorphic, then $l.i.f.d(H) = 3$ and in the similar way as finding $l.i.f.g(H)$, we get $u.i.f.g(X) = 3$. ■

Example 1. Let $X = \{Fe, Fe^{2+}, Fe^{3+}\}$ and $Y = \{In, In^{+}, In^{3+}\}$ with Fe is an Iron and In is an Indium. Define a hyperoperation $\odot$ as a product of all spontaneous reaction between two elements in X and Y. Based on Chun [41], Fe has a Latimer diagram with condition $p > q$ and In has a Latimer diagram with condition $p < q$. Thus, the mathematical system $(X, \odot)$ have $s.f.d(X) = 2$, and $l.i.f.d(X) = u.i.f.d(X) = 1$ and the mathematical system $(Y, \odot)$ have $l.i.f.d(Y) = u.i.f.d(Y) = 3$.

3.2 Chemical hyperstructures for element with four oxidation states

This notion was generalized by Al-Jinani in 2019 and Al-Tahan in 2022 [18-19]. In general, we have the Latimer diagram as follows

$$A \xrightarrow{p} B \xrightarrow{q} C \xrightarrow{r} D$$

with A, B, C, and D are elements, p, q, and r are potentials difference (in volts). Based on Al-Jinani and Al-Tahan [18-19], there are five cases for hyperstructures that have been discussed for this case, that is, when $p > q > r$, $p = q = r$, $p > r > q$, $p > r = q$, and $p = r > q$. However, this study is a little mistaken because based on Douglas in Appendix E, there is no Latimer diagram with the condition $p = q = r$ does not exist [42]. Therefore, it is sufficient to discuss when the conditions $p > q > r$, $p > r > q$, $p > r = q$, and $p = r > q$. Next, let s, t, u, v be the number of equivalents oxidized or reduced of A, B, C, and D respectively with condition $s > t > u > v$. Let $m_1 = s - t$, $m_2 = t - u$, and $m_3 = u - v$ be the electronic difference. We have the following potential reduction.

1. $A \to C$, $E_1 = \frac{pm_1+qm_2}{m_1+m_2}$
2. $A \to D$, $E_2 = \frac{pm_1+qm_2+rm_3}{m_1+m_2+m_3}$
3. $B \to D$, $E_3 = \frac{qm_2+rm_3}{m_2+m_3}$

with E_1, E_2, and E_3 be the potentials difference of $A \to C$, $A \to D$, and $B \to D$. Based on Al-Tahan [18], for case $p > q > r$ we have the following results.

3.2.1 Case 1 : $E_2 < q$ and $E_2 > q$

Let $M = \{A, B, C, D\}$ and define a hyperoperation $\odot$ as a product of all spontaneous reactions between two elements in M. Based on Al-Tahan [18], for case $E_2 < q$, we have $(M, \odot)$ as follows.

Table 8. $(M, \odot)$

$\odot$	A	B	C	D
A	$\{A\}$	$\{A, B\}$	H	H
B	$\{A, B\}$	$\{B\}$	$\{B, C\}$	$\{B, C, D\}$
C	H	$\{B, C\}$	$\{C\}$	$\{C, D\}$
D	H	$\{B, C, D\}$	$\{C, D\}$	$\{D\}$

For case $E_2 < q$, $(M, \odot)$ is a hypergroup. Then, we have following results.

Proposition 4. *Let $M = \{A, B, C, D\}$, then $s.f.d(M) = 3$*

Proof. Simple calculation gives that

$$\mu(A) = \tfrac{9}{21} \quad \mu(B) = \tfrac{14}{27} \quad \mu(C) = \tfrac{14}{33} \quad \mu(D) = \tfrac{11}{21}$$

We represent $({}_1M, \odot)$ by the following table:

Table 9. $({}_1M, \odot)$

$\odot$	A	B	C	D
A	$\{A\}$	$\{A, B, C\}$	$\{A, C\}$	H
B	$\{A, B, C\}$	$\{B\}$	$\{B, C\}$	$\{B, D\}$
C	$\{A, C\}$	$\{B, C\}$	$\{C\}$	$\{B, C, D\}$
D	H	$\{B, D\}$	$\{B, C, D\}$	$\{D\}$

Based on Table 9, simple calculation gives the following results.

$$\mu(A) = \tfrac{19}{42} \; \mu(B) = \tfrac{29}{66} \; \mu(C) = \tfrac{29}{66} \; \mu(D) = \tfrac{19}{42}$$

We represent $({}_2M, \odot)$ by the following table:

Table 10. $({}_2M, \odot)$

$\odot$	A	B	C	D
A	$\{A, D\}$	H	H	$\{A, D\}$
B	H	$\{B, C\}$	$\{B, C\}$	H
C	H	$\{B, C\}$	$\{B, C\}$	H
D	$\{A, D\}$	H	H	$\{A, D\}$

Based on Table 10, it is easily to see that $\mu(A) = \mu(B) = \mu(C) = \mu(D) = \frac{1}{3}$. Then, $({}_3M, \odot)$ is a total hypergroup. Therefore, $s.f.d(M) = 3$. ∎

Proposition 5. *Let $M = \{A, B, C, D\}$, then $l.i.f.d(M) = u.i.f.d(M) = 2$*

Proof. simple calculation gives that:

$$\mu(A) = \tfrac{3}{16} \; \mu(B) = \mu(C)\tfrac{7}{24} \; \mu(D) = \tfrac{11}{48}$$
$$\lambda(A) = \tfrac{17}{48} \; \lambda(B) = \lambda(C) = \tfrac{1}{4} \; \lambda(D) = \tfrac{5}{16}$$

Based on above results, we have

$$(\mu \wedge \lambda)(A) = \tfrac{3}{16} \; (\mu \wedge \lambda)(B) = (\mu \wedge \lambda)(C) = \tfrac{1}{4} \; (\mu \wedge \lambda)(D) = \tfrac{11}{48}$$

We represent $({}_0H, \odot_{\mu\wedge\lambda})$ by the following table:

Table 11. $({}_0H, \odot_{\mu\wedge\lambda})$

$\odot_{\mu\wedge\lambda}$	A	B	C	D
A	$\{A\}$	H	H	$\{A, D\}$
B	H	$\{B, C\}$	$\{B, C\}$	$\{B, C, D\}$
C	H	$\{B, C\}$	$\{B, C\}$	$\{B, C, D\}$
D	$\{A, D\}$	$\{B, C, D\}$	$\{B, C, D\}$	$\{D\}$

Based on Table 11, simple calculation gives that:

$$\mu_2(A) = \tfrac{3}{16}\ \mu_2(B) = \mu_2(C) = \mu_2(D) = \tfrac{13}{48}$$
$$\lambda_2(A) = \tfrac{13}{48}\ \lambda_2(B) = \lambda_2(C) = \lambda_2(D) = \tfrac{9}{48}$$

Based on the above results, we get $(\mu \wedge \lambda)(A) = (\mu \wedge \lambda)(B) = (\mu \wedge \lambda)(C) = (\mu \wedge \lambda)(D) = \frac{9}{48}$. Then, $({}_1H, \odot_{\mu_1\wedge\lambda_1})$ is a total hypergroup. Since $({}_1H, \odot_{\mu_1\wedge\lambda_1})$ and $({}_2H, \odot_{\mu_2\wedge\lambda_2})$ isomorphic, then $l.i.f.d(H) = 2$ and in the similar way as finding $l.i.f.d(H)$, we get $u.i.f.d(H) = 2$ ■

Remark. Let $M = \{A, B, C, D\}$ and define a hyperoperation $\odot^*$ as a product of all spontaneuos reactions in M with condition $E_2 > q$. Based on Al-Tahan [18], $(M, \odot)$ is isomorphic with $(M, \odot^*)$. Thus, the *s.f.d*, *l.i.f.d*, and *u.i.f.d* of M for case $E_2 > q$ is same with *s.f.d*, *l.i.f.d*, and *u.i.f.d* of M for case $E_2 < q$, that is $s.f.d(M) = 3$, $l.i.f.d(M) = u.i.f.d(M) = 2$.

Example 2. Based on [43] and [44], chemical hyperstructures for Tellurium(Te), Bismuth(Bi), and Titanium(Ti) have condition $E_2 < q$. Thus, the *s.f.d*, *l.i.f.d*, and *u.i.f.d* of this chemical hyperstructures is 3, 2, and 2 respectively.

Example 3. Based on [18], chemical hyperstructures for Silver (Ag) have condition $E_2 > q$. Thus, the *s.f.d*, *l.i.f.d*, and *u.i.f.d* of this chemical hyperstructures is 3, 2, and 2 respectively.

3.2.2 Case 2 : $E_2 = q$

Let $M = \{A, B, C, D\}$ and define a hyperoperation $\boxdot$ as a product of all spontaneous reactions between two element in M for case $E_2 = q$. Based on [18], we have $(M, \boxdot)$ as follows.

Table 12. $(M, \boxdot)$

$\boxdot$	A	B	C	D
A	$\{A\}$	$\{A,B\}$	$\{A,B,C\}$	H
B	$\{A,B\}$	$\{B\}$	$\{B,C\}$	$\{B,C,D\}$
C	$\{A,B,C\}$	$\{B,C\}$	$\{C\}$	$\{C,D\}$
D	H	$\{B,C,D\}$	$\{C,D\}$	$\{D\}$

Proposition 6. *Let* $M = \{A, B, C, D\}$*, then* $s.f.d(M) = 2$

Proof. Simple calculation gives that: $\mu(A) = \frac{19}{42}$ $\mu(B) = \mu(C) = \frac{29}{66}$ $\mu(D) = \frac{19}{42}$ We represent $({}_1M, \boxdot)$ by the following table:

Table 13. $({}_1M, \boxdot)$

$\boxdot$	A	B	C	D
A	$\{A,D\}$	H	H	$\{A,D\}$
B	H	$\{B,C\}$	$\{B,C\}$	H
C	H	$\{B,C\}$	$\{B,C\}$	H
D	$\{A,D\}$	H	H	$\{A,D\}$

Based on Table 13, we get $\mu(A) = \mu(B) = \mu(C) = \mu(D) = \frac{1}{3}$. Then, $({}_2M, \boxdot)$ is a total hypergroup. Thus, $s.f.d(M) = 2$. ∎

Proposition 7. *Let* $M = \{A, B, C, D\}$*, then* $l.i.f.d(M) = u.i.f.d(M) = 2$

Proof. The proof is similar to that Proposition 5. ∎

Example 4. Based on [18], chemical hyperstructures for Americium (Am) have condition $E_2 = q$. Thus, the $s.f.d$, $l.i.f.d$, and $u.i.f.d$ are same, that is 2.

4 Chemical hyperstructures for electrochemical cells

Electrochemical Processes are reduction-oxidation reactions (redox) in which the energy release by spontaneous reaction is converted to electricity (Chang, 2010). In 2018, Al-Tahan study about hyperstructures in electrochemical cells [21]. There are three cases for this hyperstructures, that is in Galvanic Cells, Electrolysis Cells, and Galvanic/Electrolysis Cells.

4.1 Hyperstructures on galvanic cells

In Galvanic Cells, there is two half-cells, first cell, called X is a metal with larger electronegativity and second cell, called Y is a metal with smaller electronegativity. The redox reactions for the two separate half-cells are provided as follows:

$$X^{m+} + me^- \to X$$
$$Y \to Y^{n+} + ne^-$$

with $m+$ and $n+$ is a oxidation number of X and Y respectively and e is an electron. If metal X and Y react, we have the following balanced reaction:

$$mY + nX^{m+} \to nX + mY^{n+}$$

Now, let $H = \{X, Y, X^{m+}, Y^{n+}\}$ and suppose $X = u$, $Y = v$, $X^{m+} = w$, and $Y^{n+} = x$ and define a hyperoperation $\boxdot$ as follows.

$$x \boxdot y = \begin{cases} \text{Possible reaction between } x \text{ and } y \text{ in a Galvanic Cell} \\ \{x, y\} \text{ If } x \text{ and } y \text{ do not react in Galvanic Cell} \end{cases}$$

Based on [21], we have the following table.

Table 14. $(H, \boxdot)$

$\boxdot$	u	v	w	x
u	$\{u\}$	$\{u, v\}$	$\{u, w\}$	$\{u, x\}$
v	$\{u, v\}$	$\{v\}$	$\{u, x\}$	$\{v, x\}$
w	$\{u, w\}$	$\{u, x\}$	$\{w\}$	$\{w, x\}$
x	$\{u, x\}$	$\{v, w\}$	$\{w, x\}$	$\{x\}$

Then, we have the following results.

Proposition 8. *Let $H = \{u, v, w, x\}$, then $s.f.d(H) = 1$.*

Proof. Simple calculation gives that:

$$\mu(u) = \tfrac{5}{9}\ \mu(v) = \tfrac{3}{5}\ \mu(w) = \tfrac{3}{5}\ \mu(x) = \tfrac{5}{9}$$

We represent $({}_0H, \boxdot)$ by the following table:

Table 15. $({}_0H, \boxdot)$

$\boxdot$	u	v	w	x
u	$\{u,x\}$	H	H	$\{u,x\}$
v	H	$\{v,w\}$	$\{v,w\}$	H
w	H	$\{v,w\}$	$\{v,w\}$	H
x	$\{u,x\}$	H	H	$\{u,x\}$

Based on Table 15, simple calculation gives that $\mu(u) = \mu(v) = \mu(w) = \mu(x) = \frac{1}{3}$. Then, $({}_1H, \boxdot)$ is a total hypergroup. Thus, we have $s.f.d(H) = 1$. ■

Proposition 9. *Let $H = \{u, v, w, x\}$, then $l.i.f.d(H) = u.i.f.d(H) = 2$.*

Proof. Simple calculation gives that:

$$\mu(u) = \mu(x) = \tfrac{5}{16}\ \mu(v) = \mu(w) = \tfrac{3}{16}$$
$$\lambda(u) = \lambda(d) = \tfrac{5}{16}\ \lambda(b) = \tfrac{8}{16}\ \lambda(c) = \tfrac{7}{16}$$

Based on above results, we have

$$(\mu \wedge \lambda)(u) = \tfrac{5}{16}\ (\mu \wedge \lambda)(v) = \tfrac{3}{16}\ (\mu \wedge \lambda)(w) = \tfrac{3}{16}\ (\mu \wedge \lambda)(x) = \tfrac{5}{16}$$

We represent $({}_0H, \boxdot_{\mu\wedge\lambda})$ by the following table:

Table 16. $({}_0H, \boxdot_{\mu\wedge\lambda})$

$\boxdot_{\mu\wedge\lambda}$	u	v	w	x
u	$\{u,x\}$	H	H	$\{u,x\}$
v	H	$\{v,w\}$	$\{v,w\}$	H
w	H	$\{v,w\}$	$\{v,w\}$	H
x	$\{u,x\}$	H	H	$\{u,x\}$

Based on Table 16, simple calculation gives that $\mu_1(u) = \mu_1(v) = \mu_1(w) = \mu_1(x) = \frac{4}{16}$ and $\lambda_1(u) = \lambda_1(v) = \lambda_1(w) = \lambda_1(x) = \frac{2}{16}$. Then, we get $(\mu_1 \wedge \lambda_1)(u) = (\mu_1 \wedge \lambda_1)(v) = (\mu_1 \wedge \lambda_1)(w) = (\mu_1 \wedge \lambda_1)(x) = \frac{2}{16}$. Then, $({}_1H, \boxdot_{\mu_1\wedge\lambda_1})$ is a total hypergroup. Therefore, $l.i.f.d(H) = u.i.f.d(H) = 2$ ■

4.2 Hyperstructures on electrolysis cells

Electrolysis Cells has two half-cells, first cell, called X is a metal with larger electronegativity and second cell, called Y is a metal with smaller electronegativity. The redox reactions for the two separate half-cells are provided as follows:

$$X \rightarrow X^{m+} + me^-$$
$$Y^{n+} + ne^- \rightarrow Y$$

with $m+$ and $n+$ is a oxidation number of X and Y respectively and e is an electron. If metal X and Y react, we have the following balanced reaction:

$$nX + mY^{n+} \rightarrow mY + nX^{m+}$$

Now, let $H = \{X, Y, X^{m+}, Y^{n+}\}$ and suppose $X = u$, $Y = v$, $X^{m+} = w$, and $Y^{n+} = x$ and define a hyperoperation $\boxminus$ as follows.

$$x \boxminus y = \begin{cases} \text{Possible reaction between } x \text{ and } y \text{ in a Galvanic Cell} \\ \{x, y\} \text{ If } x \text{ and } y \text{ do not react in Galvanic Cell} \end{cases}$$

Based on [21], we have the following table.

Table 17. $(H, \boxminus)$

$\boxminus$	u	v	w	x
u	$\{u\}$	$\{u, v\}$	$\{u, w\}$	$\{v, w\}$
v	$\{u, v\}$	$\{v\}$	$\{u, w\}$	$\{v, w\}$
w	$\{u, w\}$	$\{v, w\}$	$\{w\}$	$\{w, x\}$
x	$\{v, w\}$	$\{v, x\}$	$\{w, x\}$	$\{x\}$

Based on [21], the hyperstructures of $(H, \boxminus)$ is isomorphic to $(H, \boxplus)$. Thus, $(H, \boxminus)$ have same $s.f.d$, $l.i.f.d$ and $u.i.f.d$, that is $s.f.d(H) = 1$, $l.i.f.d(H) = u.i.f.d(H) = 2$.

4.3 Hyperstructures on galvanic/electrolysis cells

let $H = \{X, Y, X^{m+}, Y^{n+}\}$ and suppose $X = u$, $Y = v$, $X^{m+} = w$, and $Y^{n+} = x$ and define a hyperoperation $\boxplus$ as follows.

$$x \boxplus y = \begin{cases} \text{Possible reaction between } x \text{ and } y \text{ in a Galvanic Cell or Electrolysis Cell} \\ \{x, y\} \text{ If } x \text{ and } y \text{ do not react in Galvanic Cell or Electrolysis Cell} \end{cases}$$

Based on [21], we have the following table.

Table 18. $(H, \boxminus)$

$\boxminus$	u	v	w	x
u	$\{u\}$	$\{u, v\}$	$\{u, w\}$	$\{v, w\}$
v	$\{u, v\}$	$\{v\}$	$\{u, x\}$	$\{v, x\}$
w	$\{u, w\}$	$\{u, x\}$	$\{w\}$	$\{w, x\}$
x	$\{v, w\}$	$\{v, x\}$	$\{w, x\}$	$\{x\}$

Then, we have the following results.

Remark. $(H, \boxminus)$ not have a fuzzy degree.

Proposition 10. *Let* $H = \{u, v, w, x\}$*, then* $l.i.f.d(H) = u.i.f.d(H) = 1$

Proof. The proof is similar to that Proposition 5. ∎

5 Chemical hyperstructures for equilibrium reactions

Chemical equilibrium reaction is the reaction reached when the rates of the forward and reverse reactions are equal and the concentrations of reactants and products remain constant [40]. There are three cases for this chemical equilbrium reaction. The first case is a reaction that takes the form $P \rightleftarrows Q$, the second case $P + Q \rightleftarrows R$, and the third case $P + Q \rightleftarrows R + S$ where $P, Q, R,$ and S are chemical elements or compounds.

5.1 Case 1 : $P \rightleftarrows Q$

Let $H = \{P, Q\}$ and define a hyperoperation $\odot$ as a reaction that occurs in Case 1. If it do not react, write $s \odot t = \{s, t\}$. Based on [25], we have following results.

Table 19. $(H, \odot)$

$\odot$	P	Q
P	$\{Q\}$	$\{P, Q\}$
Q	$\{P, Q\}$	$\{Q\}$

We have the following results.

Proposition 11. *Let $H = \{P, Q\}$, then $s.f.d(H) = 1$*

Proof. Simple calculation gives that $\mu(P) = \mu(Q) = \frac{2}{3}$. Then, $({}_1H, \odot)$ is a total hypergroup. Then, $s.f.d(H) = 1$. ■

Proposition 12. *Let $H = \{P, Q\}$, then $l.i.f.d(H) = u.i.f.d(H) = 2$*

Proof. Simple calculation gives that $\mu(P) = \mu(Q) = \frac{1}{2}$ and $\lambda(P) = \lambda(Q) = \frac{1}{4}$. We have $(\mu \wedge \lambda)(P) = \frac{1}{4}$. Then, $({}_1H, \odot_{\mu\wedge\lambda})$ is a total hypergroup. Therefore, $l.i.f.d(H) = u.i.f.d(H) = 2$. ■

5.2 Case 2 : $P + Q \rightleftarrows R$

Let $H = \{P, Q, R\}$ and define a hyperoperation $\otimes$ as a reaction that occurs in Case 2. If it do not react, write $s \odot t = \{s, t\}$. Based on [25], we have following results.

Table 20. $(H, \otimes)$

$\odot$	P	Q	R
P	$\{P\}$	$\{R\}$	$\{P, R\}$
Q	$\{R\}$	$\{Q\}$	$\{Q, R\}$
R	$\{P, R\}$	$\{Q, R\}$	$\{P, Q\}$

We have the following results.

Remark. H is not have fuzzy degree.

Proposition 13. *Let $H = \{P, Q, R\}$, then $l.i.f.d(H) = u.i.f.d(H) = 1$*

Proof. The proof is similar to that Proposition 5. ■

5.3 Case 3 : $P + Q \rightleftarrows R + S$

Let $H = \{P, Q, R, S\}$ and define a hyperoperation $\oplus$ as a reaction that occurs in Case 3. If it do not react, write $s \odot t = \{s, t\}$. Based on [25], we have following results.

Table 20. $(H, \oplus)$

$\odot$	P	Q	R	S
P	$\{P\}$	$\{R, S\}$	$\{P, R\}$	$\{P, S\}$
Q	$\{R, S\}$	$\{Q\}$	$\{Q, R\}$	$\{Q, S\}$
R	$\{P, R\}$	$\{Q, R\}$	$\{R\}$	$\{P, Q\}$
S	$\{P, S\}$	$\{Q, S\}$	$\{P, Q\}$	$\{S\}$

We have the following results.

Remark. H is not have fuzzy degree.

Proposition 14. *Let $H = \{P, Q, R, S\}$, then $l.i.f.d(H) = u.i.f.d(H) = 1$*

Proof. The proof is similar to that Proposition 5. ■

6 Chemical hyperstructures for acid rain reactions

Acid rain is a precipitation that occurs because water (H_2O) reacts with sulfur trioxide (SO_3), nitrogen dioxide (NO_2), and carbonate (CO_3) to produce rain with sulfuric acid (H_2SO_4), nitrous acid (HNO_3), and carbonate acid (H_2CO_3) [40]. In this paper, we only analyze fuzzy degree and intuitionistic fuzzy degree for carbonate acid formation reactions. The formation reactions of H_2CO_3 are gives as follows.

$$CO_2 + H_2O \rightarrow H_2CO_3$$

Based on [45], define hyperoperations $\odot$ as a product of reactions that occurs in carbonate acid formation reactions. If it does not react, view as an identity reaction, i.e. for every two elements in the reactants, the products are the reactants themselves. Furthermore, let $A = \{CO_2, H_2O, H_2CO_3\}$ and let $a = CO_2$, $b = H_2O$, and $c = H_2CO_3$. Then, we have the following Table 22.

Table 22. $(A, \odot)$

$\boxdot$	a	b	c
a	$\{a\}$	$\{c\}$	$\{a, c\}$
b	$\{c\}$	$\{b\}$	$\{b, c\}$
c	$\{a, c\}$	$\{b, c\}$	$\{c\}$

Furthermore, Using the same steps as before, the following propositions are obtained.

Proposition 15. *Let* $A = \{a, b, c\}$*, then* $s.f.d(A) = 1$

Proposition 16. *Let* $A = \{a, b, c\}$*, then* $l.i.f.d(A) = u.i.f.d(A) = 1$

7 Conclusions

Based on the explanation above, we have determined the fuzzy degree and intuitionistic fuzzy degree of chemical hyperstructures of Redox Reactions, Electrochemical Cells, Equilbrium reactions, and Acid Rain Formation Reactions. For future research, fuzzy degree and intuitionistic fuzzy degree can be determined on other chemical hyperstructures.

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