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Research on Graph Energies in 2019

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Abstract

This survey lists all papers on graph energies, known to the authors, published in 2019, and summarizes their main collective characteristics. In addition to new mathematical results, in 2019 several noteworthy and somewhat unexpected non-mathematical applications of graph energies have been proposed. The general conclusion of our analysis is that research on graph energies is as active as ever, showing no sign of attenuation.

1 Introduction

In the book [R1], a detailed account of the research on graph energies is given, covering the period between 1931, when Erich Hückel first time used the total π-electron energy [R2], 1978, when one of the present authors introduced the concept of graph energy [R3], until the first few months of 2019. At this point two other recent reviews of the theory of graph energy deserve to be mentioned [R4,R5].

In the present times, research on graph energies is in great expansion. Thus, according to our records, in 2015, 2016, 2017, and 2018, not less than 114, 121, 146, and 138 articles on graph energies have been published, more than two each week. The same happened in 2019, with the following recorded publications:

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In this article we analyze the publications [1]– [151] and point out their main collective characteristics.

2 Census

1.

In the above list are the publications that have appeared in a journal or a book, and are dated by year 2019. Articles in press, and those published on the internet (with a DOI), but not assigned to a 2019-volume of a journal are not included. Also ignored are papers published in "*arXiv*", Ph.D., M.Sc., master, and diploma theses, articles in conference proceedings, as well as unpublished texts that one can find on internet.

The above list contains articles that came to the attention of the present authors. It may well be that some relevant publications are missing. What must be noticed is that our list contains just a single paper published in China [32], and none published in Chinese language. It is absolutely certain that numerous such papers exist, but are difficult to find by non-Chinese.

2.

The majority of authors who published on graph energies are from India and China. This time, in contrast to what was found in the book [R1], the number of Indian authors (109) significantly exceeds that of Chinese authors (65). In addition, contributions to the theory and applications of graph energies were done by scholars from Argentina (1), Bahrain (1), Brazil (2), Colombia (2), France (3), Germany (1), Iran (30), Iraq (3), Malaysia (4), Mexico (3), Morocco (1), Nigeria (1), Pakistan (17), Russia (1), Saudi Arabia (2), Serbia (11), Slovenia (4), South Korea (1), Thailand (1), Taiwan (3), Turkey (12), United Arab Emirates (1), and USA (9), a total of 285 authors. One should recall that many of them published more than one paper.

It should be noted that only a small fraction of papers (about 15%) was produced by non-Asian authors, i.e., authors from Europe and the two Americas, and that just two authors are from Africa. In 2019, there were no authors from Australia, whereas authors from Antarctica are missing, same as in previous times.

Thus, we see that the present-day centers of research of graph energies are in Asia (in India, China, Iran, Pakistan).

3.

Publications concerned with graph energies appeared in a great variety of journals and edited books. Many of these journals/books are of low or suspicious quality. Yet, papers on graph energies are published also in high–standard journals, in which refereeing is rigorous. We mention here "*Linear Algebra and Its Applications*" (9 papers), "*Discrete Applied Mathematics*" (6 papers), "*Applied Mathematics and Computation*" (4 papers), "*Linear and Multilinear Algebra*" (3 papers). As one could expect, the greatest number of published papers (14) is in "*MATCH Communications in Mathematical and in Computer Chemistry*".

4.

The original version of graph energy from year 1978 is based on the eigenvalues of the $(0,1)$-adjacency matrix [R3]. Countless results on this graph energy have been obtained so far, see [R5,R6]. One could get the impression that by now, this field of research is completely exhausted. In contrary, papers published in the recent years clearly demonstrate that this is not the case. Of the numerous papers concerned with ordinary graph energy, we point out Refs. [1,13–15,50,79,100,116,119,132] as well as some of the works published after 2019 [R7–R11]. Most of the newly obtained results on ordinary graph energy are lower and upper bounds. Two noteworthy exceptions are Ref. [13], on the application of the Coulson integral formula, and Ref. [119], on computing local contributions by the Estrada–Benzi method [R12].

Motivated by the great success of the theory of ordinary graph energy [R5,R6], the idea to consider energies based on other graph matrices soon came to mind. The first such new graph energies were the *Laplacian energy* [R13] and the *distance energy* [R14], eventually followed by more than 170 analogous species; for details see in [R1]. Also the papers published in 2019 deal with a great variety of graph energies. In particular, these are:

1. ABC energy
2. Albertson energy
3. average degree energy
4. block energy
5. c-dominating energy
6. color energy
7. complementary distance energy
8. complementary distance signless Laplacian energy
9. distance energy
10. dominating energy
11. dominating Laplacian energy
12. eccentricity energy
13. eccentricity extended energy
14. efficient dominating energy

15. energy of degree product adjacency matrix
16. geometric-arithmetic energy
17. Harary energy
18. harmonic energy
19. incidence energy
20. inverse sum indeg energy
21. iota energy
22. Laplacian energy
23. Laplacian-energy-like invariant
24. Laplacian incidence energy
25. Laplacian partition energy
26. Laplacian resolvent energy
27. matching energy
28. minimum covering Gutman energy
29. minimum efficient dominating energy
30. minimum Laplacian efficient dominating energy
31. minimum total dominating energy
32. minimum vertex-block dominating energy
33. mixed energy
34. normalized signless Laplacian energy
35. path energy
36. quasi-Laplacian energy
37. Randić energy
38. Randić color energy
39. reciprocal distance signless Laplacian energy
40. resolvent energy
41. Seidel energy
42. signless Laplacian resolvent energy
43. skew energy
44. skew distance energy
45. skew Laplacian energy
46. status sum adjacency energy
47. sum-connectivity energy
48. trace norm
49. vertex energy
50. Zagreb energy

Among these energies, the following are new, i.e., are not listed in the book [R1]: block energy [126], c-dominating energy [58], eccentricity energy [140], eccentricity extended energy [128], efficient dominating energy [72], energy of degree product adjacency matrix [94], minimum covering Gutman energy [104], minimum total dominating energy [83, 84], minimum vertex-block dominating energy [133], mixed energy [16], quasi-Laplacian energy [80], skew distance energy [19], and status sum adjacency energy [131].

What some authors call "*trace norm*" [95] is same as the ordinary graph energy. This change of terminology was proposed by Nikiforov [R15]–[R17], but has not been accepted by the majority of the scientific community.

3 Applications of graph energies

The graph energy based on the adjacency matrix, as well as its variant based on other graph matrices were all conceived as mathematical object, and their mathematical properties were investigated and established. However, relatively recently, applications of graph energies begun to appear in the literature. Also in 2019 a variety of such applications were published. These are concerned with the following diverse topics:

- data mining [45]
- quality assessment [109]
- spreading rate of virus [34, 41]
- face recognition [11]
- ecosystem response to climate change [105]
- chakras [101]

Note that the "spreading rate" in [34,41] pertains to computer viruses. The vertices of the graphs considered in [105] are "soil", "climate", "hydrogeomorphic features", "biotic features", and similar, connected by directed or undirected edges. The respective energy "*indicates the overall strength of positive and negative feedbacks present*".

In [11], the facial image is described as a collection of three complete connected graphs. Their spectra and energies are then used to describe the global features of the face image.

For an European reader, the idea to connect (ordinary) graph energy with the six chakras of the esoteric tradition of Hinduism and yoga [101] might look a bit bizarre and farfetched.

In Indian alternative medical therapies, one considers the energy vortices known as chakras (from Snaskrit, meaning "wheel"). These are believed to be linked to a physiological endocrine gland via nerves. According to vedas, these energy points are visualized as wheels with different number of petals. In [101], the ordinary, Laplacian, signless Laplacian, and Randić energies of these chakras are computed and compared.

4 Concluding remarks

The purpose of the present survey is to document and analyze the present-day researches on graph energies. In the same time, we intended to give the readers a general idea on what is currently happening in the theory of graph energies.

We believe that we can safely claim that the general conclusion of our study is that research on graph energies is as active as ever, showing no sign of attenuation.

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