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BOOK REVIEW

Handbook of Research on Advanced Applications of Graph Theory in Modern Society

edited by

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The HANDBOOK consists of a Preface (written by the Editors) and 21 chapters, followed by a list of all references quoted in the preceding chapters, short information on every contributor, and a detailed index. Before the Preface there is a Detailed Table of Contents, in which a summary of each 21 chapters can be found. It is interesting that the Preface mentions also a Chapter 22, claimed to be concerned with Zagreb, hyper-Zagreb, and forgotten topological indices; the authors of Chapter 22 are not revealed.

The Editors and the majority of contributors are from India. However, there are also contributors from Bulgaria, Chile, Greece, Malaysia, Pakistan, Russia, South Africa, Spain, Sri Lanka, and USA.

As the title of the HANDBOOK says, it covers a variety of topics, all related to graph theory and its applications. In addition to topics dealing with usual graphs and digraphs, a significant parts of the book (Chapters 16, 18, 19, 20, 21) is devoted to fuzzy graphs. Because this review is aimed at colleagues interested in mathematical chemistry, in what follows we pay attention to Chapters 3, 4, 5, 6, 11, and 20. The other chapters are:

1. *Domination Theory in Graphs*
2. *An Introduction to Intersection Graphs*
7. *$L(h,k)$-Labeling of Intersection Graphs*
8. *Set-Valuations of Graphs and Their Applications.*
9. *Sumset Valuations of Graphs and Their Applications*
10. *Tripartite and Quadpartite Size Ramsey Numbers for All Pairs of Connected Graphs on Four Vertices*
12. *Applying Graph Theory to Detect Cases of Money Laundering and Terrorism Financing*
13. *Formalization and Discrete Modelling of Communication in the Digital Age by Using Graph Theory*
14. *Graph Theory: Novel Multiple-Attribute Decision-Making Effect*
15. *Influential Nodes in Social Networks: Centrality Measures*
16. *Integration of Multiple Cache Server Scheme for User-Based Fuzzy Logic in Content Delivery Networks*
17. *Social Networks and Graph Theory in the Search for Distant Knowledge: Studying the Field of Industrial Engineering*
18. *Recent Developments on the Basics of Fuzzy Graph Theory*
19. *Fuzzy Graphs and Fuzzy Hypergraphs*
21. *Bipolar Neutrosophic Cubic Graphs and Its Applications.*

Of these, the most attractive may be Chapter 12.

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Chapter 3: *Graph Indices* (pp. 66–91), is written by V. R. Kulli. It is a detailed account of the contemporary research on graph indices, with emphasis on those newly introduced. Many among the readers of *MATCH Communications in Mathematical and in Computer Chemistry* are interested in such indices. They will be pleased to learn that in addition to numerous variants of the Zagreb index, there exist Banhatti, Dakshayani, Gourava, Kulli–Basava, and Revan indices, as well as several other less familiar. Chapter 3 offers also an excellent bibliography on this matter, including 55 titles by Kulli himself.

Cahpter 4: *A Novel Weighted First Zagreb Index of Graph* (pp. 92–103), is written by J. Buragohain and A. Bharali. It introduces a weighted version of the first Zagreb

index. Its basic properties are determined, and its correlations with various physico–chemical properties of octane isomers examined.

Chapter 5: *Inverse Sum Indeg Index of Subdivision, t-Subdivision Graphs, and Related Sums* (pp. 104–119), is written by A. Doley, J. Buragohain, and A. Bharali. It reports results on an earlier much studied "inverse sum indeg index". New lower and upper bounds for this index are reported for several classes of graphs, in terms of parameters such as order, size, maximum degree, minimum degree, and the first Zagreb index. Extremal graphs are also characterized. A large number of (earlier known) analytical inequalities are stated and applied. Therefore, reading of Chapter 5 may be recommended to students and beginners in the field of chemical graph theory.

Chapter 6: *The Hyper-Zagreb Index and Some Properties of Graphs* (pp. 120–134), is written by R. Li. In it, the author reports some new results related to the existence of Hamiltonian cycles and paths. In his earlier studies, he found sufficient conditions for their existence, based on the hyper–Zagreb index. Now, he does the same, based on the second Zagreb and forgotten indices.

Chapter 11: *Energy of Graphs* (pp. 267–296) is written by H. Ramane. It is a detailed and comprehensive survey of the theory of graph energy, namely of the sum of absolute values of the eigenvalues of the adjacency matrix of the graph. The text is divided into four subsections:

1. *Introduction*
2. *Bounds for Energy*
3. *Hyperenergertic, Non-Hyperenergetic and Borderenergetic Graphs*
4. *Equienergetic graphs.*

In the first subsection the chemical origins of graph energy are described. Then the Coulson integral formula is deduced, with full details of its proof. In subsection 2 are stated and proved the classical bounds on graph energy - that of McClelland, Gutman, and Koolen–Moulton, followed by several other, more recent ones. Also the problem of graph(s) with greatest energy is discussed. The third subsection presents results on hyperenergetic graphs (those for which $E > 2n - 2$) and borderenergetic graphs (those for which $E = 2n - 2$). Most of the results stated are given with

proofs. Subsection 4 deals with equienergetic graphs. A plethora of such graphs have been designed by now, many of which by Ramane himself. Cahpter 11 ends with an exhaustive bibliography.

Because of its clarity, completeness and reasonable length, Chapter 11 will be a must for all young colleagues (whose number nowadays is large), who are beginning to study graph energy.

Chapter 20: *Energy of m-Polar Fuzzy Digraphs* (pp. 469–491), is written by M. Akram, D. Saleem, and G. Ghorai. In it, after defining the concepts of fuzzy graphs, m-polar fuzzy digraphs, and their energies (ordinary, Laplacian, signless Laplacian), basic properties and bounds for the respective energies are determined. The results are then extensively illustrated by examples. At the end, directions for future research are indicated.

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In conclusion: the HANDBOOK OF RESEARCH ON ADVANCED APPLICATIONS OF GRAPH THEORY IN MODERN SOCIETY presents useful and clear information on various details of graph theory and its applications. Every mathematical library should get a copy of it. Because of Chapters 3, 5, and 11, the HANDBOOK will be of immense value for colleagues working in the area of chemical graph theory, both beginners and experts.

Ivan Gutman