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A Topological Approach to Assembling Strands–Based DNA Triangular Prisms

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Abstract

DNA triangular prisms with one helical turn on each edge has been realized recently by one or five synthetic DNA single strands with rationally designed sequences. In the present paper, we determine all possible existing topological structures for DNA triangular prisms with such double-helical edges. Here triangular prism links are assembled as the mathematical models for DNA triangular prisms by using six oriented twist tangles as basic building blocks. In this process, we firstly show that only 22 orientations are allowed to exist in triangular prism links. Then there are 366 link types of triangular prism links identified from all generated link diagrams by considering the symmetry of triangular prism. We note that each type includes infinite triangular prism links by changing the number of building blocks on each edge. Our work provides a list of candidates for further synthesized DNA triangular prisms with required topological structures.

1 Introduction

DNA, as an information-coding polymer capable of programming nanostructure assembly, has been programmed to assemble into a range of well-defined nanostructures [1–5]. Among these, hollow polyhedra [6–9] have attracted the attentions of many researchers owing to their resemble natural structures such as viral capsids as well as tremendous potential for scaffolding and encapsulating functional materials [10–16]. Triangular Prisms, as the simplest prisms, have been assembled by deliberately designing different size and component DNA strands or DNA titles [17–24]. More recently, DNA triangular prisms,

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with one helical turn on each edge, were assembled by one or five synthetic DNA single strands with rationally designed sequences [25]. These two DNA prisms both have nontrivial topological structures, which play the important roles on the control of the size, the stability of structure and synthetic strategy for nanoparticles. For example, the single-stranded DNA triangular prism has more compact structure than the five-stranded one, and also is the smallest 3D DNA polyhedron ($\sim$ 3.4 nm) ever reported. However, it is possible to synthesize a DNA triangular prism with a smaller size by reducing the twist number of the single-stranded DNA knot from theoretical viewpoints. Thus, it is natural to ask that which knots or links as the topological structures of DNA triangular prisms are allowed to exist. The answer for this question has a fundamental impact on the theoretical rules for the design and construction of a serial of DNA prism molecules. We address it in the present paper.

Polyhedral links are the interlocked and interlinked architectures formed by treating DNA as a thin strand, which are presented as mathematical models to explain the topological structures of DNA polyhedra [26–29]. To date, there are a lot of works devoted to the research of polyhedra links, including the construction method and obtaining the related link invariants [30–40]. In particular, some prism links with even crossing number on each edge are constructed from prismatic plane graphs [41]. However, these works are mainly used to characterize and explain the structures of the synthesized DNA polyhedra with complete helix turns on each edge. There are very little paper devoted to the study of the ones with complete or incomplete helix turns on each edge due to their unpredictable orientation [42, 43]. Recently, a general method is supposed to assemble DNA tetrahedra with such structures by constructing six twisted tangles as basic building blocks, where each tangle is oriented and composed of two twisted stands with odd or even crossing number [44]. This work affords an expectation to solve the same problem on DNA triangular prism.

In the present paper, six twisted tangles are taken as basic building blocks to assemble triangular prism links. However, triangle prism have lower symmetry and more complex structures than tetrahedra. Hence it becomes more difficult to determine the orientation of triangular prism links and also to further identify the same link type in the construction process. Here there are 22 different orientations identified from the initial 55 orientations obtained by considering all possible orientations of each edge. And then there are triangular prism links of almost one thousand determined directly by these 22 orientations, which are further classified into 366 link types under considering the same topological structures produced by the construction method and symmetry of triangular prism. Our research provide a general classification for the topological structures for triangle prism links, and also can be served as a list of candidates for further synthesized DNA triangular prisms with required topological structures.

2 The construction of triangular prism links

2.1 Graphs and link diagrams

Some notations and basic definitions [45, 46] are introduced in this section.

In graph theory, *a planar graph* G is a graph that can be drawn in the plane with no edge crossings. Such a drawing is called *a plane graph* of G. Since all convex polyhedrons are 3-connected planar graphs, and each of them has an embedding on the plane. Such an embedding is called *a polyhedral graph*.

A link L is a collection of circles which may be linked or knotted together without intersections. *A knot* is taken as a special link only with a circle. A link can be oriented by giving one of the two directions along each circle, and its reverse $-L$ is formed by reversing the orientation on each circle. *A link diagram* D of L is a regular projection of L onto a plane such that the corresponding space curve crosses over or under at each crossing is indicated by creating broken strands. Similarly, D also can be oriented, and its reverse is denoted by $-D$.

Two links L_1 and L_2 are *equivalent*, denoted by $L_1 = L_2$, if there exists an ambient isotopy that maps one to the other. It is well-known that ambient isotopy is an equivalence relation on links. Each equivalence class of links is called a link type, and the link type of a link diagram means the equivalence class of the link represented by this diagram.With some abuse of terminology, the word 'link' is applied to mean a whole equivalence class (a knot type) or a particular representative member.

2.2 The construction of triangular prism links

A twist tangle of length m, denoted by T, is two parallel strands with m half-twists for any positive integer m (Fig. 1(b)). Four endpoints of T are marked by NW, NE, SW and SE, as shown in Fig. 1(a). T allow four orientations α, β, γ and $-\alpha$ since the orientation $-\beta$ (or $-\gamma$) overlaps the orientation β (or γ) for T by rotating it by 180 degrees in the plane. There are six types of oriented twist tangles such that each type has an antiparallel orientation on its two strands, denoted by a^{α}_{2n}, $a^{-\alpha}_{2n}$, a^{β}_{2n-1}, b^{α}_{2n}, $b^{-\alpha}_{2n}$ and b^{γ}_{2n-1} (Fig. 1(b)). Here the twist tangle $a^{-\alpha}_{2n}$ (or $b^{-\alpha}_{2n}$) can be obtained from a^{α}_{2n} (or b^{α}_{2n}) by reversing the orientations on two strands. In fact, any other oriented twist tangle with such antiparallel orientation must be one of the mirror images of the above six types [44].

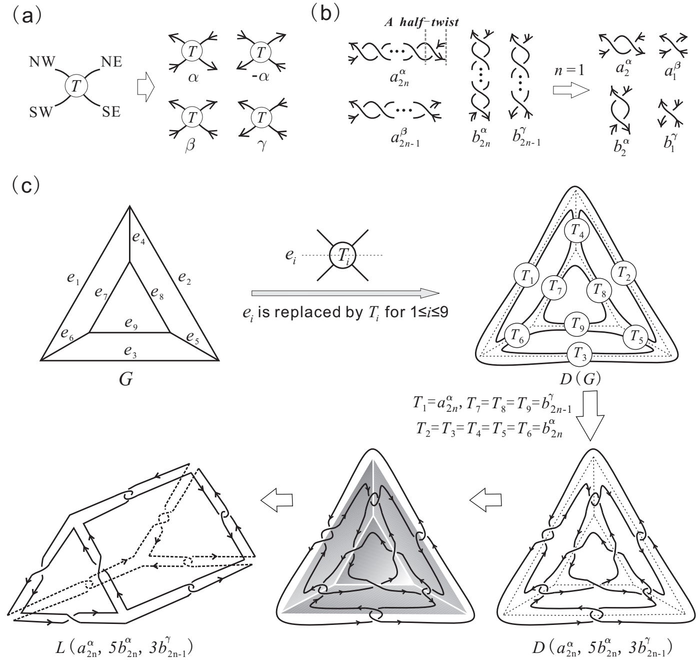

Figure 1. (a) Four orientations of twist tangle T: α, $-\alpha$, β, γ; (b) Four types of oriented twist tangles: a^{α}_{2n}, a^{β}_{2n-1}, b^{α}_{2n} and b^{γ}_{2n-1}; (c) The construction of $D(G)$, $D(a^{\alpha}_{2n}, 5b^{\alpha}_{2n}, 3b^{\gamma}_{2n-1})$ and $L(a^{\alpha}_{2n}, 5b^{\alpha}_{2n}, 3b^{\gamma}_{2n-1})$.

Let G be a plane graph of triangular prism P. An oriented link diagram $D(G)$ can be obtained from G by replacing each edge e_i with an oriented twist tangle T_i for $1 \leq i \leq 9$ and then connecting the endpoints of two twist tangles along the boundary of each face (See Fig. 1(c)). This diagram $D(G)$ is called an oriented triangular prism link diagram or *OTP link diagram*. Here for convenience, $D(G)$ is denoted as $D(T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9)$ by recording the twist tangle on each edge in a sequence from outside to inside and in the clockwise direction, and the orientation of $D(G)$ is denoted accordingly by $o(\tau_1,\tau_2,\tau_3,\tau_4,\tau_5,\tau_6,\tau_7,\tau_8,\tau_9)$, where $\tau_i \in \{\alpha,-\alpha,\beta,\gamma\}$ is the orientation of T_i for $1 \le i \le 9$. The reverse of $o(\tau_1,\tau_2,\tau_3,\tau_4,\tau_5,\tau_6,\tau_7,\tau_8,\tau_9)$, denoted by $-o(\tau_1,\tau_2,\tau_3,\tau_4,\tau_5,\tau_6,\tau_7,\tau_8,\tau_9)$, is obtained by reversing the orientation of each τ_i for $1 \le i \le 9$. Each twist tangle T_i is called a twist edge of $D(G)$. In this paper, $D(G)$ corresponds to a triangular prism link $L(T_1,T_2,T_3,T_4,T_5,T_6,T_7,T_8,T_9)$ in $\mathbb{R}^3$ by considering it as a spherical embedding of this link. This is illustrated in Fig. 1(c) by taking $D(a^{\alpha}_{2n}, 5b^{\alpha}_{2n}, 3b^{\gamma}_{2n-1})$ for example.

Note. In this paper, all OTP link diagrams constructed by our method are all alternating. In fact, these link diagram, their mirror images and reverse are composed of all OTP link diagrams. Here we only consider these OTP link diagrams in the present paper.

3 Results

In this section, let G be a triangular prism graph with each edge labeled by e_i, and $D(G)$ be any OTP link diagram obtained from G by replacing each edge e_i with a twist tangle T_i oriented by τ_i for $1 \le i \le 9$ (Fig. 1(c)). In construction process of $D(G)$, we must keep a nonconflict orientation between any two adjacent twist tangles connected. Hence all possible orientations of $D(G)$ are determined in the following lemma.

Lemma 3.1. *The OTP link diagram $D(G)$ must has one of the 22 orientations described in Fig. 2 and their reverses.*

Proof. There are three cases we need to consider according to the orientation of $D(G)$.

Case 1. At least one of three twist tangles T_1, T_2 and T_3 has the orientation α. Without loss of generality, assume that T_1 is oriented with α. Then the adjacent twist tangle T_4 has three possible orientations α, β and γ shown in Fig. 3(a). When T_4 is oriented with α or β, the adjacent twist tangle T_2 is oriented with α or β. When T_4 is oriented with γ, T_2 will be oriented with γ. Similarly, repeatedly consider the orientations of such twist tangles adjacent to the oriented twist tangle until the orientation of $D(G)$ is determined. In this case, there are 29 orientations described from Fig. 3(a).

Case 2. At least one of T_1, T_2 and T_3 has the orientation β, but none of them is oriented with α. Without loss of generality, assume that T_1 has the orientation β, then

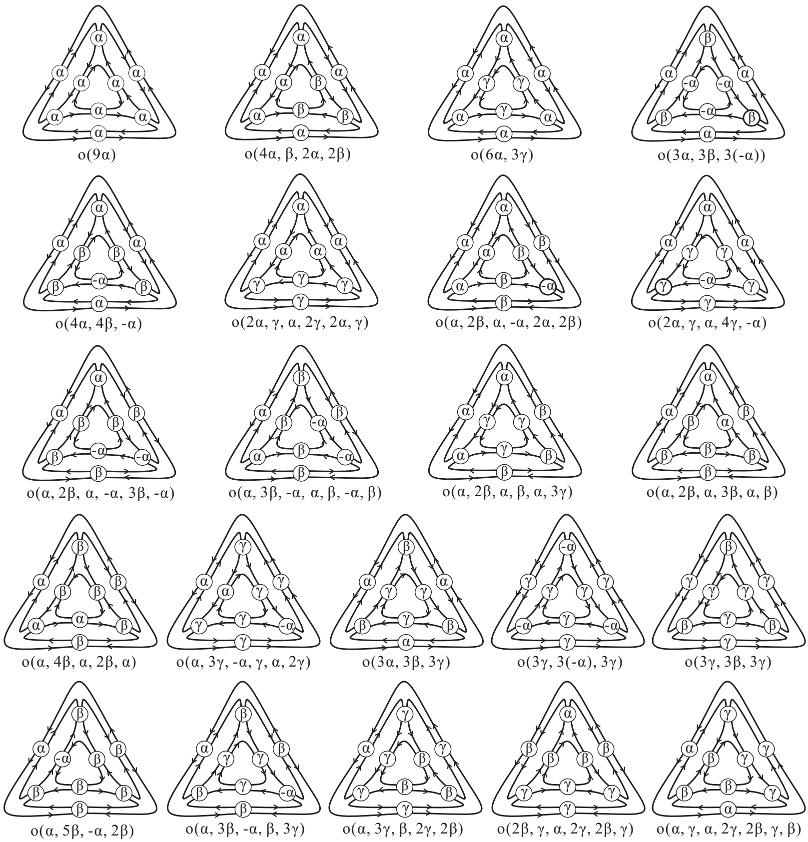

Figure 2. 22 orientations of OTP link diagrams.

T_4 has three possible orientations α, β and γ. Accordingly, T_2 is oriented with β or γ, as described in Fig. 3(b). Similarly, repeatedly consider the orientations of such adjacent twist tangles until $D(G)$ is oriented. In this case, there are 16 orientations described from Fig. 3(b).

Case 3. T_1, T_2 and T_3 are each oriented with γ. Then T_4 adjacent to T_1 and T_2 only has two possible orientations $-\alpha$ or β. Similarly, repeatedly consider the orientations of the adjacent twist tangles until $D(G)$ is oriented. In this case, there are 10 orientations described from Fig. 3(c).

According to the above construction process, there are 55 possible orientations to exist

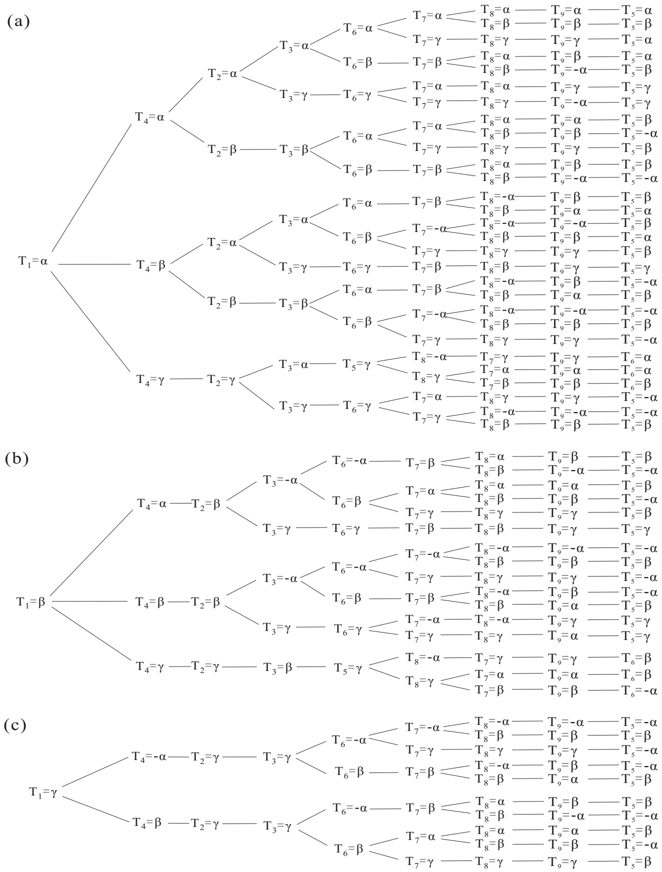

Figure 3. The straight line between two twist tangles indicates their adjacent relationship, and the equal sign means the twist tangle having the orientation. (a) case 1; (b) case 2; (c) case 3.

in $D(G)$. However, there are the same or reversed ones among these orientations when $D(G)$ is embedded into $\mathbb{R}^3$ as an OTP link $L(G)$. These cases are described in table 1. These 17 different orientations in the table 1, together with the remaining five orientations $o(9\alpha)$, $o(3\alpha, 3\beta, 3(-\alpha))$, $o(\alpha, 3\gamma, -\alpha, \gamma, \alpha, 2\gamma)$, $o(3\gamma, 3(-\alpha), 3\gamma)$ and $o(3\gamma, 3\beta, 3\gamma)$, form 22 different orientations for $D(G)$. Hence $D(G)$ must have one of the above 22 orientations or their reverses. ■

$o(D(G))$	The same orientations as $o(D(G))$
$o(4\alpha,\beta,2\alpha,2\beta)$	$o(5\alpha,2\beta,\alpha,\beta)$, $o(\alpha,2\beta,\alpha,\beta,4\alpha)$, $o(3\alpha,\beta,2\alpha,2\beta,\alpha)$, $-o(2\beta,-\alpha,\beta,5(-\alpha))$
$o(6\alpha,3\gamma)$	$-o(3\gamma,6(-\alpha))$
$o(4\alpha,4\beta,-\alpha)$	$o(3\alpha,2\beta,\alpha,\beta,-\alpha,\beta)$, $o(3\alpha,\beta,\alpha,\beta,-\alpha,2\beta)$, $o(2\beta,-\alpha,\alpha,2\beta,3\alpha)$, $-o(\alpha,3\beta,-\alpha,\beta,3(-\alpha))$
$o(2\alpha,\gamma,\alpha,2\gamma,2\alpha,\gamma)$	$o(\alpha,\gamma,\alpha,2\gamma,2\alpha,\gamma,\alpha)$
$o(\alpha,2\beta,\alpha,-\alpha,2\alpha,2\beta)$	$-o(2\beta,-\alpha,\alpha,2(-\alpha),2\beta,-\alpha)$
$o(2\alpha,\gamma,\alpha,4\gamma,-\alpha)$	$o(\alpha,\gamma,\alpha,2\gamma,\alpha,\gamma,-\alpha,\gamma)$, $-o(\alpha,3\gamma,-\alpha,2\gamma,2(-\alpha))$
$o(\alpha,2\beta,\alpha,-\alpha,3\beta,-\alpha)$	$o(2\beta,-\alpha,\alpha,\beta,-\alpha,\beta,\alpha,\beta)$
$o(\alpha,3\beta,-\alpha,\alpha,\beta,-\alpha,\beta)$	$o(2\beta,-\alpha,\alpha,-\alpha,\beta,\alpha,2\beta)$
$o(\alpha,2\beta,\alpha,\beta,\alpha,3\gamma)$	$-o(3\gamma,-\alpha,\beta,2(-\alpha),2\beta)$, $-o(3\gamma,2(-\alpha),2\beta,-\alpha,\beta)$, $-o(3\gamma,\beta,2(-\alpha),2\beta,-\alpha)$, $-o(2\beta,-\alpha,\beta,2(-\alpha),3\gamma)$
$o(\alpha,2\beta,\alpha,3\beta,\alpha,\beta)$	$-o(2\beta,-\alpha,2\beta,2(-\alpha),2\beta)$
$o(\alpha,4\beta,\alpha,2\beta,\alpha)$	$-o(2\beta,-\alpha,\beta,-\alpha,2\beta,-\alpha,\beta)$
$o(3\alpha,3\beta,3\gamma)$	$o(3\gamma,3\beta,3\alpha)$
$o(\alpha,5\beta,-\alpha,2\beta)$	$o(2\beta,-\alpha,5\beta,\alpha)$
$o(\alpha,3\beta,-\alpha,\beta,3\gamma)$	$o(3\gamma,-\alpha,4\beta,\alpha)$, $o(3\gamma,2\beta,-\alpha,\beta,\alpha,\beta)$, $o(3\gamma,\beta,-\alpha,\beta,\alpha,2\beta)$, $-o(2\beta,-\alpha,\alpha,2\beta,3\gamma)$
$o(\alpha,3\gamma,\beta,2\gamma,2\beta)$	$o(2\beta,\gamma,\beta,4\gamma,\alpha)$, $-o(\beta,\gamma,\beta,2\gamma,\beta,\gamma,-\alpha,\gamma)$
$o(2\beta,\gamma,\alpha,2\gamma,2\beta,\gamma)$	$-o(\beta,\gamma,\beta,2\gamma,-\alpha,\beta,\gamma,\beta)$
$o(\alpha,\gamma,\alpha,2\gamma,2\beta,\gamma,\beta)$	$o(2\alpha,\gamma,\beta,2\gamma,2\beta,\gamma)$, $o(\beta,\gamma,\beta,2\gamma,\beta,\alpha,\gamma,\alpha)$, $-o(2\beta,\gamma,\beta,2\gamma,2(-\alpha),\gamma)$

Table 1. 17 different orientations of $D(G)$.

Using the above lemma, we can obtain the following theorems.

Theorem 3.2. *There are 104 link types of OTP link diagrams with the orientation $o(9\alpha)$, which are numbered from 1 to 104 in table 2 (Appendix A).*

Proof. For the orientation $o(9\alpha)$, each twist tangle oriented with α must be a^{α}_{2n} or b^{α}_{2n}, then the number of the resulting OTP link diagrams can be calculated by the following formula

$$2C^0_9 + 2C^1_9 + 2C^2_9 + 2C^3_9 + 2C^4_9 = 512.$$

In fact, among these diagrams, many diagrams are equivalent since they are corresponding to the same link in $\mathbb{R}^3$. Hence there are five cases to be considered in the following.

Case 1. When all twist tangles of $D(G)$ are all a^{α}_{2n} or all b^{α}_{2n}, two OTP link diagrams $D(9a^{\alpha}_{2n})$ and $D(9b^{\alpha}_{2n})$ can be obtained.

Case 2. When a twist tangle of $D(G)$ is a^{α}_{2n} and the remaining twist tangles are all b^{α}_{2n}, the resulting OTP link diagrams of C^1_9 can be divided into two equivalence classes such that all diagrams in each class are corresponding to the same link in $\mathbb{R}^3$. One class has six members, including $D(a^{\alpha}_{2n}, 8b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, 7b^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 6b^{\alpha}_{2n})$, $D(6b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$, $D(7b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$ and $D(8b^{\alpha}_{2n}, a^{\alpha}_{2n})$, where each diagram has a a^{α}_{2n} as an edge of any triangular face of $D(G)$. The remaining link diagrams consist of the other class, including $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, 5b^{\alpha}_{2n})$, $D(4b^{\alpha}_{2n}, a^{\alpha}_{2n}, 4b^{\alpha}_{2n})$ and $D(5b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$, where each diagram has a a^{α}_{2n} as the lateral edge. Accordingly, we use $D(a^{\alpha}_{2n}, 8b^{\alpha}_{2n})$ and $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, 5b^{\alpha}_{2n})$ to represent these two link types respectively. Similarly, when a and b are exchanged in the case 2, we obtain two link types of OTP link diagrams $D(b_{2n}^{\alpha}, 8a_{2n}^{\alpha})$ and $D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, 5a_{2n}^{\alpha})$.

Case 3. When two twist tangles of $D(G)$ are both a_{2n}^{α} and the remaining twist tangles are all b_{2n}^{α}, the resulting OTP link diagrams of C_9^2 can be divided into four subcases as below.

Subcase 3.1. There are six OTP link diagrams such that each diagram has two a_{2n}^{α} as the edges of a triangular face of $D(G)$. These diagrams include $D(2a_{2n}^{\alpha}, 7b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 6b_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 6b_{2n}^{\alpha})$, $D(6b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(6b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$ and $D(7 b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$. Since these six link diagrams are corresponding to the same OTP link in $\mathbb{R}^3$, we use $D(2a_{2n}^{\alpha}, 7b_{2n}^{\alpha})$ to denote this link type.

Subcase 3.2. There are nine OTP link diagrams such that each triangular face of each diagram has a a_{2n}^{α} as an edge. These diagrams are further identified as three equivalence classes, and each class has three members. The first class includes $D(a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$ and $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, a_{2n}^{\alpha})$. The second class includes $D(a_{2n}^{\alpha}, 6b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 6b_{2n}^{\alpha}, a_{2n}^{\alpha})$ and $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$. The third class includes $D(a_{2n}^{\alpha}, 7b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$ and $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$. Accordingly, we use $D(a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 6b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$ and $D(a_{2n}^{\alpha}, 7b_{2n}^{\alpha}, a_{2n}^{\alpha})$ to represent these three link types respectively.

Subcase 3.3. There are 18 OTP link diagrams such that each diagram has one a_{2n}^{α} as the lateral edge and the other one a_{2n}^{α} as an edge of a triangular face of $D(G)$. These diagrams are further identified as three equivalence classes, and each class has six members. The first class includes $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 5b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$, $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(4b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha})$ and $D(5b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$. The second class includes $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 5b_{2n}^{\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$, $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$, $D(4b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$ and $D(5b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$. The third class includes $D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 5b_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$, $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(4b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$ and $D(5b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$. These three link types are represented respectively by $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 5b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 5b_{2n}^{\alpha})$ and $D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 5b_{2n}^{\alpha})$.

Subcase 3.4. There are three OTP link diagrams such that each diagram has two a_{2n}^{α} as the lateral edges. These diagrams includes $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$, $D(3b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 4b_{2n}^{\alpha})$ and $D(4b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$, which are corresponding to the same OTP link in $\mathbb{R}^3$. Hence we use $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$ to represent this link type.

Similarly, when a and b are exchanged in the case 3, the resulting OTP link diagrams of C^2_9 can be identified into eight equivalence classes, that are represented by the link diagrams numbered by $8, 10, ..., 22$ in table 2.

Case 4. When three twist tangles of $D(G)$ are all a^{α}_{2n} and the remaining twist tangles are all b^{α}_{2n}, the resulting OTP link diagrams of C^3_9 are divided into six subcases as below.

Subcase 4.1. There are two OTP link diagrams $D(3a^{\alpha}_{2n}, 6b^{\alpha}_{2n})$ and $D(6b^{\alpha}_{2n}, 3a^{\alpha}_{2n})$ such that a triangular face of each diagram is composed by three a^{α}_{2n}. Since these two link diagrams are corresponding to the same link in $\mathbb{R}^3$, we use $D(3a^{\alpha}_{2n}, 6b^{\alpha}_{2n})$ to represent this link type.

Subcase 4.2. There are 18 OTP link diagrams such that each diagram has a a^{α}_{2n} as an edge of one triangular face and has two a^{α}_{2n} as the edges of the other triangular face. These diagrams are further identified as three equivalence classes, and each class has six members. The first class includes $D(a^{\alpha}_{2n}, 5b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, 5b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$, $D(2a^{\alpha}_{2n}, 5b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$ and $D(b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 5b^{\alpha}_{2n}, a^{\alpha}_{2n})$. The second class includes $D(a^{\alpha}_{2n}, 5b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, 4b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 4b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(2a^{\alpha}_{2n}, 4b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 4b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$ and $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 5b^{\alpha}_{2n}, a^{\alpha}_{2n})$. The third class includes $D(a^{\alpha}_{2n}, 6b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, 4b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(2a^{\alpha}_{2n}, 6b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 4b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$ and $D(b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$. Accordingly, we use $D(a^{\alpha}_{2n}, 5b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 5b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$ and $D(a^{\alpha}_{2n}, 6b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$ to represent these three link types respectively.

Subcase 4.3. There are 18 OTP link diagrams such that each diagram has one a^{α}_{2n} as the lateral edge and two a^{α}_{2n} as the edges of a triangular face. These diagrams are identified as three equivalent classes, and each class has six members. The first class includes $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(4b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 5b^{\alpha}_{2n})$, $D(5b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 4b^{\alpha}_{2n})$ and $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$. The second class includes $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(4b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(5b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, 5b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 4b^{\alpha}_{2n})$ and $D(2a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$. The third class includes $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(4b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 4b^{\alpha}_{2n})$, $D(5b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 5b^{\alpha}_{2n})$ and $D(b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$. Accordingly, we use $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$ and $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$ to represent these three link types respectively.

Subcase 4.4. There are 27 OTP link diagrams such that each diagram has one a_{2n}^{α} as the lateral edge and each triangular face has a a_{2n}^{α} as an edge. These diagrams are identified as six equivalent classes. The first class has six members, including $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$ and $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha})$. The second class has six members, including $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha})$ and $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$. The third class has six members, including $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$ and $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$. The fourth class has three members, including $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha})$ and $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$. The fifth class has three members, including $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$ and $D(a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$. The sixth class has three members, including $D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$ and $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$. These six link types are represented respectively by $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$ and $D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha})$.

Subcase 4.5. There are 18 OTP link diagrams such that each diagram has two a_{2n}^{α} as the lateral edges and a a_{2n}^{α} as an edge of a triangular face. These diagrams are identified as three equivalent classes, and each class has six members. The first class includes $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 4b_{2n}^{\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$, $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$, $D(3b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$ and $D(4b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$. The second class includes $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$, $D(2b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 4b_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$, $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(3b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$ and $D(4b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$. The third class includes $D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 4b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$, $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(3b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha})$ and $D(4b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$. Accordingly, we use $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$ and $D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$ to represent these three link types respectively.

Subcase 4.6. There is only one OTP link diagram $D(3b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$ which has three a_{2n}^{α} as the lateral edges.

Similarly, when a and b are exchanged in the case 4, the resulting OTP link diagrams of C_9^3 can be identified into 17 equivalence classes. These 17 link types are represented by the link diagrams numbered by $24, 26, ..., 56$ in table 2.

Case 5. When four twist tangles of $D(G)$ are all a_{2n}^{α} and the remaining tangles are all b_{2n}^{α}, the resulting OTP link diagrams of C_9^4 can be divided into seven subcases as below.

Subcase 5.1. There are six OTP link diagrams such that each diagram has three a_{2n}^{α} as the edges of a triangular face and a a_{2n}^{α} as an edge of the other triangular face. These six link diagrams include $D(3a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$, $D(3a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(3a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$ and $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$, which are corresponding to the same OTP link in $\mathbb{R}^3$. Then we use $D(3a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$ to represent this link type.

Subcase 5.2. There are nine OTP link diagrams such that each triangular face of each diagram has two a_{2n}^{α} as the edges. These diagrams are identified into three equivalent classes, and each class has three members. The first class includes $D(2a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$ and $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$. The second class includes $D(2a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$ and $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$. The third class includes $D(2a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$ and $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$. Accordingly, these three link types are represented by $D(2a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(2a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$ and $D(2a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$.

Subcase 5.3. There are six OTP link diagrams such that each diagram has three a_{2n}^{α} as the edges of a triangular face and a a_{2n}^{α} as the lateral edge. These diagrams include $D(4a_{2n}^{\alpha}, 5b_{2n}^{\alpha})$, $D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha})$, $D(3a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$, $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$, $D(4b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$ and $D(5b_{2n}^{\alpha}, 4a_{2n}^{\alpha})$, which are corresponding to the same OTP link in $\mathbb{R}^3$. Hence we use $D(4a_{2n}^{\alpha}, 5b_{2n}^{\alpha})$ to represent this link type.

Subcase 5.4. There are 54 OTP link diagrams such that each diagram has one a_{2n}^{α} as the lateral edge, one a_{2n}^{α} as an edge of the one triangular face and two a_{2n}^{α} as the edges of the other triangular face. These diagrams are identified as nine equivalence classes, and each class has six members. The first class includes $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$, $D(2a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$ and $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha})$. The second class includes $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$, $D(a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$ and $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$. The third class includes $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2

$b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n})$ and $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$. The fourth class includes $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 4b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$ and $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n})$. The fifth class includes $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n})$ and $D(b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$. The sixth class includes $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 4b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$ and $D(2a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n})$. The seventh class includes $D(2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(2a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$ and $D(b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, 4b^{\alpha}_{2n}, a^{\alpha}_{2n})$. The eighth class includes $D(2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 4b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$ and $D(b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$. The ninth class includes $D(2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$ and $D(2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 4b^{\alpha}_{2n}, a^{\alpha}_{2n})$. Accordingly, we use $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$ and $D(2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$ to represent the nine link types.

Subcase 5.5. There are 18 OTP link diagrams such that each diagram has two a^{α}_{2n} as the lateral edges and two a^{α}_{2n} as the edges of a triangular face of $D(G)$. These diagrams are further identified as three equivalent classes, and each class has six members. The first class includes $D(2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, 4a^{\alpha}_{2n}, 4b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$, $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(3b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n})$ and $D(4b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$. The second class includes $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$, $D(2a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 4b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$, $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(3b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$ and $D(4b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$. The third class includes $D(b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, 4b^{\alpha}_{2n})$, $D(2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$, $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n})$, $D(3b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n})$ and $D(4b^{\alpha}_{2n}, 4a^{\alpha}_{2n}, b^{\alpha}_{2n})$. Accordingly, we use $D(2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$ and $D(b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$ to represent these three link types.

Subcase 5.6. There are 27 OTP link diagrams such that each diagram has two a^{α}_{2n} as the lateral edges and also its each triangular face has a a^{α}_{2n} as an edge. These diagrams are further identified into six equivalence classes. The first class has three members, including $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$ and $D(2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n})$. The second class has six members, including $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n})$ and $D(2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$. The third class has six members, including $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n})$ and $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$. The fourth class has six members, including $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n})$ and $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$. The fifth class has 3 members, including $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$ and $D(a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$. The sixth class has three members, including $D(2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\alpha}_{2n})$ and $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$. Accordingly, we use $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n})$ and $D(2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$ to represent these six link types.

Subcase 5.7. There are six OT-link diagrams such that each diagram has three a^{α}_{2n} as the lateral edges and one a^{α}_{2n} as an edge of a triangular face. These diagrams include $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$, $D(2b^{\alpha}_{2n}, 4a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$, $D(3b^{\alpha}_{2n}, 4a^{\alpha}_{2n}, 2b^{\alpha}_{2n})$, $D(3b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n})$ and $D(3b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n})$, which are corresponding to the same OTP link in $\mathbb{R}^3$. Hence we use $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, 3b^{\alpha}_{2n})$ to represent this link type.

Similarly, when a and b are exchanged in the case 5, the resulting OTP link diagrams of C^4_9 can be identified into 24 equivalence classes. These link types are represented by 24 link diagrams numbered by $58, 60, ..., 104$ in table 2. ■

Theorem 3.3. *There are 64 link types of OTP link diagrams with the orientation* $o(4\alpha, \beta, 2\alpha, 2\beta)$*, which are numbered from 105 to 168 in table 2 (Appendix A).*

Proof. For the orientation $o(4\alpha, \beta, 2\alpha, 2\beta)$, each twist tangle oriented with β must be a^{β}_{2n-1}, and each twist tangle oriented with α will be a^{α}_{2n} or b^{α}_{2n}, then the number of the resulting OTP link diagrams can be calculated by the following formula

$$2C_6^0 + 2C_6^1 + 2C_6^2 + C_6^3 = 64.$$

We have four cases by only considering six twist tangles oriented with α as below.

Case 1. When all twist tangles oriented with α are all a_{2n}^{α} or all b_{2n}^{α}, two OTP link diagrams $D(4a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ and $D(4b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ can be obtained.

Case 2. When a twist tangle is a_{2n}^{α} and the remaining twist tangles are all b_{2n}^{α}, the resulting OTP link diagrams of C_6^1 are divided into two subcases as below.

Subcase 2.1. There are four link types of OTP link diagrams such that each diagram has a a_{2n}^{α} as an edge of triangular faces. These diagrams include $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$, $D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ and $D(4b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$.

Subcase 2.2. There are two link types of OTP link diagrams $D(4b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ and $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ such that each diagram has a a_{2n}^{α} as the lateral edge.

Similarly, when a and b are exchanged in the case 2, the resulting OTP link diagrams of C_6^1 are identified into six equivalence classes. These link types are represented by six link diagrams numbered by $108, 110, ..., 118$ in table 2.

Case 3. When two twist tangles of $D(G)$ are both a_{2n}^{α} and the remaining twist tangles are all b_{2n}^{α}, the resulting OTP link diagrams of C_6^2 are divided into four cases.

Subcase 3.1. There are three link types of OTP link diagrams such that each diagram has two a_{2n}^{α} as the edges of a triangular face. These diagrams include $D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$, $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ and $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$.

Subcase 3.2. There are three link types of OTP link diagrams such that each triangular face of each diagram has a a_{2n}^{α} as an edge. These diagrams include $D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ and $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$.

Subcase 3.3. There are eight link types of OTP link diagrams such that each diagram has one a_{2n}^{α} as the lateral edge and the other a_{2n}^{α} as an edge of triangular faces. These diagrams include $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$, $D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$, $D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$, $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ and $D(4b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$.

Subcase 3.4. There is one OTP link diagram $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, a^{\beta}_{2n-1}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$ which has two a^{α}_{2n} as the lateral edges.

Similarly, when a and b are exchanged in the case 3, the resulting OTP link diagrams of C^2_6 can be identified into 15 equivalence classes. These link types are represnted by 15 link diagrams numbered by $120, 122, ..., 148$ in table 2.

Case 4. When three twist tangles of $D(G)$ are all a^{α}_{2n} and the remaining twist tangles are all b^{α}_{2n}, the resulting OTP link diagrams of C^3_6 are divided into five subcases to be considered as below.

Subcase 4.1. There is one OTP link diagram $D(3a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\beta}_{2n-1}, 2b^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$ whose three a^{α}_{2n} forms a triangular face.

Subcase 4.2. There are three link types of OTP link diagrams such that each diagram has a a^{α}_{2n} as an edge of one triangular face and has two a^{α}_{2n} as the edges of the other triangular face. These diagrams include $D(2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\beta}_{2n-1}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\beta}_{2n-1}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$ and $D(b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\beta}_{2n-1}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$.

Subcase 4.3. There are six link types of OTP link diagrams such that each diagram has one a^{α}_{2n} as the lateral edge and two a^{α}_{2n} as the edges of a triangular face. These diagrams include $D(2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, a^{\beta}_{2n-1}, 2b^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\beta}_{2n-1}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, a^{\beta}_{2n-1}, 2b^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$, $D(2a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\beta}_{2n-1}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$, $D(b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\beta}_{2n-1}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$ and $D(b^{\alpha}_{2n}, 3a^{\alpha}_{2n}, a^{\beta}_{2n-1}, 2b^{\alpha}_{2n}, a^{\beta}_{2n-1})$.

Subcase 4.4. There are six link types of OTP link diagrams such that each diagram has one a^{α}_{2n} as the lateral edge and its each triangular face has a a^{α}_{2n} as an edge. These diagrams include $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, a^{\beta}_{2n-1}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$, $D(a^{\alpha}_{2n}, 3b^{\alpha}_{2n}, a^{\beta}_{2n-1}, 2a^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, a^{\beta}_{2n-1}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$, $D(2b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\beta}_{2n-1}, 2a^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$, $D(2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, a^{\beta}_{2n-1}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$ and $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\beta}_{2n-1}, 2a^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$.

Subcase 4.5. There are four link types of OTP link diagrams such that each diagram has two a^{α}_{2n} as the lateral edges and a a^{α}_{2n} as an edge of a triangular face. These diagrams include $D(a^{\alpha}_{2n}, 2b^{\alpha}_{2n}, a^{\alpha}_{2n}, a^{\beta}_{2n-1}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$, $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, a^{\beta}_{2n-1}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$, $D(2b^{\alpha}_{2n}, 2a^{\alpha}_{2n}, a^{\beta}_{2n-1}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$ and $D(3b^{\alpha}_{2n}, a^{\alpha}_{2n}, a^{\beta}_{2n-1}, 2a^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$. ■

Theorem 3.4. *There are 24 link types of OTP link diagrams with the orientation $o(6\alpha, 3\gamma)$, which are numbered from 169 to 192 in table 2 (Appendix A).*

Proof. For the orientation $o(6\alpha, 3\gamma)$, each twist tangle oriented with γ must be b^{γ}_{2n-1}, then the number of the resulting OTP link diagrams can be calculated by the following formula

$$2C_6^0 + 2C_6^1 + 2C_6^2 + C_6^3 = 64.$$

We have four cases by only considering six twist tangles oriented with α for $D(G)$.

Case 1. When these six twist tangles are all a_{2n}^{α} or all b_{2n}^{α}, two OTP link diagrams $D(6a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(6b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ can be obtained.

Case 2. When a twist tangle is a_{2n}^{α} and the remaining twist tangles oriented with α are all b_{2n}^{α}, the resulting OTP link diagrams of C_6^1 can be divided into two equivalence classes. One class has three members, including $D(a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, where each diagram has a a_{2n}^{α} as an edge of a triangular face. The remaining link diagrams consist of the other class, including $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(4b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(5b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, where each diagram has a a_{2n}^{α} as the lateral edge. Accordingly, we use $D(a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ to represent these two link types respectively. Similarly, when a and b are exchanged in the case 2, we obtain two OTP link diagrams $D(b_{2n}^{\alpha}, 5a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$.

Case 3. When two twist tangles are both a_{2n}^{α} and the remaining twist tangles oriented with α are all b_{2n}^{α}, the resulting OTP link diagrams of C_6^2 can be divided into three subcases as below.

Subcase 3.1. There are three OTP link diagrams such that each diagram has two a_{2n}^{α} as the edges of a triangular face of $D(G)$. These diagrams include $D(2a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$. Since these three link diagrams are corresponding to the same OTP link in $\mathbb{R}^3$, we use $D(2a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ to denote this link type.

Subcase 3.2. There are nine OTP link diagrams such that each diagram has one a_{2n}^{α} as the lateral edge and the other a_{2n}^{α} as an edge of a triangular face. These diagrams are identified into three equivalent classes, and each class has three members. The first class includes $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$. The second class includes $D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$. The third class includes $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$. Accordingly, these three link types are represented respectively by $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha},$

$2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$.

Subcase 3.3. There are three OTP link diagrams such that each diagram has two a_{2n}^{α} as the lateral edges. These diagrams includes $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(4b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(3b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, which are corresponding to the same OTP link in $\mathbb{R}^3$. Hence we use $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ to represent this link type.

Similarly, when a and b are exchanged in the case 3, the resulting OTP link diagrams of C_6^2 can be identified into five equivalence classes, that are $D(2b_{2n}^{\alpha}, 4a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$.

Case 4. When three twist tangles of $D(G)$ are all a_{2n}^{α} and the remaining twist tangles oriented with α are all b_{2n}^{α}, the resulting OTP link diagrams of C_6^3 can be divided into four subcases as below.

Subcase 4.1. There is only one OTP link diagram $D(3a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ such that its three twist tangle a_{2n}^{α} form a triangular face.

Subcase 4.2. There are nine OTP link diagrams such that each diagram has one a_{2n}^{α} as the lateral edge and two a_{2n}^{α} as the edges of a triangular face. These diagrams are identified into three equivalent classes, and each class has three members. The first class includes $D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$. The second class includes $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$. The third class includes $D(b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(2a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$. Accordingly, we use $D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ to represent these three link types.

Subcase 4.3. There are nine OTP link diagrams such that each diagram has two a_{2n}^{α} as the lateral edges and a a_{2n}^{α} as an edge of any triangular face. These diagrams are identified into three equivalent classes, and each class has three members. The first class includes $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$. The second class includes $D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(2b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$. The third class includes $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$. Accordingly, we use $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$, $D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ and $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ to represent these three link types.

Subcase 4.4 There is only one OTP link diagram $D(3b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$ which has three a_{2n}^{α} as the lateral edges. ■

Theorem 3.5. *There are 16 link types of OTP link diagrams with the orientation $o(3\alpha, 3\beta, 3(-\alpha))$, which are numbered from 193 to 208 in table 2 (Appendix A).*

Proof. For the orientation $o(3\alpha, 3\beta, 3(-\alpha))$, each twist tangle oriented with β must be a_{2n-1}^{β}, and each twist tangle oriented with α (or $-\alpha$) will be a_{2n}^{α} (or $a_{2n}^{-\alpha}$) or b_{2n}^{α} (or $b_{2n}^{-\alpha}$), then the number of the resulting OTP link diagrams can be calculated by the following formula

$$2C_6^0 + 2C_6^1 + 2C_6^2 + C_6^3 = 64.$$

We have four cases by only considering six twist tangles without the orientation β as below.

Case 1. When each twist tangles oriented with α is all a_{2n}^{α} and each oriented with $-\alpha$ is all $a_{2n}^{-\alpha}$, an OTP link diagram $D(3a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3a_{2n}^{-\alpha})$ can be obtained. Similarly, when a is replaced by b in the case 1, we obtain the OTP link diagram $D(3b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$.

Case 2. When a twist tangle is a_{2n}^{α} or $a_{2n}^{-\alpha}$ and the remaining twist tangle each is b_{2n}^{α} or $b_{2n}^{-\alpha}$ according to the related orientation α or $-\alpha$, the resulting OTP link diagrams of C_6^1 include $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$, $D(3b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, 2b_{2n}^{-\alpha})$, $D(3b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$ and $D(3b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$. Since these six link diagrams are corresponding to the same OTP link or its reverse in $\mathbb{R}^3$, we use $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$ to represent this link type. Similarly, when a and b are exchanged in the case 2, we obtain the OTP link diagram $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3a_{2n}^{-\alpha})$.

Case 3. When two twist tangles each is a_{2n}^{α} or $a_{2n}^{-\alpha}$ and the remaining twist tangles each is b_{2n}^{α} or $b_{2n}^{-\alpha}$ according to the related orientation α or $-\alpha$, the resulting OTP link diagrams of C_6^2 can be divided into two subcases as below.

Subcase 3.1 There are six OTP link diagrams such that each diagram has two a_{2n}^{α} or two $a_{2n}^{-\alpha}$ as the edges of a triangular face. These diagrams include $D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$, $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$, $D(3b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$, $D(3b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$ and $D(3b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, 2a_{2n}^{-\alpha})$. Since these six link diagrams are corresponding to the same OTP link or its reverse in $\mathbb{R}^3$, we use $D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$ to denote this link type.

Subcase 3.2. There are nine OTP link diagrams such that each triangular face of each diagram has a a_{2n}^{α} or a $a_{2n}^{-\alpha}$ as an edge. These diagrams are further identified into three equivalent classes, and each class has three members. The first class includes $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, 2b_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$ and $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$. The second class includes $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, 2b_{2n}^{-\alpha})$ and $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$. The third class includes $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, 2b_{2n}^{-\alpha})$ and $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$. Accordingly, we use $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, 2b_{2n}^{-\alpha})$, $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$ and $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$ to represent these three link types respectively.

Similarly, when a and b are exchanged in the case 3, the resulting OTP link diagrams of C_6^2 can be identified into four equivalence classes, that are $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3a_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, 2a_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$ and $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$.

Case 4. When three twist tangles each is a_{2n}^{α} or $a_{2n}^{-\alpha}$ and the remaining twist tangles each is b_{2n}^{α} or $b_{2n}^{-\alpha}$ according to the related orientation α or $-\alpha$, the resulting OTP link diagrams of C_6^3 can be divided into two subcases as below.

Subcase 4.1. There are two OTP link diagrams $D(3a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$ and $D(3b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3a_{2n}^{-\alpha})$ such that each diagram has three a_{2n}^{α} or three $a_{2n}^{-\alpha}$ as the edges of a triangular face of $D(G)$. Since these two link diagrams are reverse to each other as an OTP link in $\mathbb{R}^3$, we use $D(3a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$ to represent this link type.

Subcase 4.2. There are 18 OTP link diagrams such that each diagram has a a_{2n}^{α} (or a $a_{2n}^{-\alpha}$) as an edge of one triangular face and has two $a_{2n}^{-\alpha}$ (or two a_{2n}^{α}) as the edges of the other triangular face of $D(G)$. These diagrams are further identified as three equivalent classes, and each class has six members. The first class includes $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, 2a_{2n}^{-\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$, $D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$ and $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, 2b_{2n}^{-\alpha})$. The second class includes $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, 2a_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$, $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$ and $D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, 2b_{2n}^{-\alpha})$. The third class includes $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, 2a_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$, $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, 2b_{2n}^{-\alpha})$, $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$ and $D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$. Accordingly, we use $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$, $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha}$

) and $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, 2a_{2n}^{-\alpha})$ to represent these three link types respectively. ■

Theorem 3.6. *There are 32 link types of OTP link diagrams with the orientation $o(4\alpha, 4\beta, -\alpha)$, which are numbered from 209 to 240 in table 2 (Appendix A).*

Proof. The number of the OTP link diagrams oriented with $o(4\alpha, 4\beta, -\alpha)$ can be calculated by the following formula

$$2(2C_4^0 + 2C_4^1 + C_4^2) = 32.$$

We have two cases by only considering five twist tangles without the orientation β as below.

Case 1: When the twist tangle oriented with $-\alpha$ is $a_{2n}^{-\alpha}$, we have three subcases to be considered in the following.

Subcase 1.1. When all twist tangles oriented with α are all a_{2n}^{α} or all b_{2n}^{α}, two OTP link diagrams $D(4a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$ and $D(4b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$ are obtained.

Subcase 1.2. When a twist tangle is a_{2n}^{α} and the remaining three twist tangles oriented with α are all b_{2n}^{α}, the resulting OTP link diagrams of C_4^1 can be identified into four equivalence classes. These diagrams include $D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha}$), $D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$ and $D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$. Similarly, when a and b are exchanged in the subcase 1.2, the resulting OTP link diagrams of C_4^1 can be identified into four equivalence classes. We use $D(b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$, $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$, $D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$ and $D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$ to represent the four link types.

Subcase 1.3. When two twist tangles are both a_{2n}^{α} and the remaining two twist tangles oriented with α are both b_{2n}^{α}, we obtain six OTP link diagrams. There are three link types of OTP link diagrams such that each diagram has two a_{2n}^{α} as the edges of a triangular face. These diagrams include $D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$, $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$ and $D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$. There are another three link types of OTP link diagrams such that each diagram has one a_{2n}^{α} as the lateral edge and the other a_{2n}^{α} as an edge of a triangular face. These diagrams include $D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$ and $D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$.

Case 2: The twist tangle oriented with $-\alpha$ is $b_{2n}^{-\alpha}$. Similarly to the case 1, the resulting OTP link diagrams can be identified into 16 equivalence classes, which are represented by these 16 link diagrams numbered by 210, 212, ..., 240 in table 2. ■

Theorem 3.7. *There are 20 link types of OTP link diagrams with the orientation $o(2\alpha, \gamma, \alpha, 2\gamma, 2\alpha, \gamma)$, which are numbered from 241 to 260 table 2 (Appendix A).*

Proof. For the orientation $o(2\alpha, \gamma, \alpha, 2\gamma, 2\alpha, \gamma)$, the number of the resulting OTP link diagrams can be calculated by the following formula

$$2C_5^0 + 2C_5^1 + 2C_5^2 = 32.$$

We have three cases by only considering five twist tangles oriented with α as below.

Case 1. When all twist tangles oriented with α are all a_{2n}^{α} or all b_{2n}^{α}, two OTP link diagrams $D(2a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2a_{2n}^{\alpha}, b_{2n-1}^{\gamma})$ and $D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$ can be obtained.

Case 2. When a twist tangle of $D(G)$ is a_{2n}^{α} and the remaining four twist tangles are all b_{2n}^{α}, the resulting OTP link diagrams of C_5^1 are divided into two subcases as below.

Subcase 2.1. There are four OTP link diagrams such that each diagram has a a_{2n}^{α} as an edge of any triangular face. These diagrams are further identified into two equivalence classes, and each class has two members. The first class includes $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$ and $D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma})$. The second class includes $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$ and $D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$. Accordingly, these two link types are represented respectively by $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$ and $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$.

Subcase 2.2. There is only one OTP link diagram $D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$ which has a a_{2n}^{α} as the lateral edge.

Similarly, when a and b are exchanged in the case 2, the resulting OTP link diagrams of C_5^1 can be identified into three equivalent classes. These three link types are represented by three link diagrams numbered by 244, 246 and 248 in table 2.

Case 3. When two twist tangles are both a_{2n}^{α} and the remaining three twist tangles are all b_{2n}^{α}, the resulting OTP link diagrams of C_5^2 can be divided into three subcases as below.

Subcase 3.1. There are two OTP link diagrams $D(2a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$ and $D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2a_{2n}^{\alpha}, b_{2n-1}^{\gamma})$ such that each diagram has two a_{2n}^{α} as the edges of a triangular face. Since these two link diagrams are corresponding to the same OTP link in $\mathbb{R}^3$, we use $D(2a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$ to represent this link type.

Subcase 3.2. There are four OTP link diagrams such that each triangular face for each diagram has a a^{α}_{2n} as an edge. These diagrams are further identified into three equivalent classes. The first class has two members, including $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, b^{\gamma}_{2n-1})$ and $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\gamma}_{2n-1})$. The second class has one diagram $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\gamma}_{2n-1})$. The third class also has one diagram $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, b^{\gamma}_{2n-1})$. Accordingly, we use $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, b^{\gamma}_{2n-1})$, $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\gamma}_{2n-1})$ and $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, b^{\gamma}_{2n-1})$ to represent these three link types respectively.

Subcase 3.3. There are four OTP link diagrams such that each diagram has one a^{α}_{2n} as the lateral edge and the other a^{α}_{2n} as an edge of a triangular face. These diagrams are further identified into two equivalence classes. Also, each class has two members. The first class includes $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, 2b^{\alpha}_{2n}, b^{\gamma}_{2n-1})$ and $D(2b^{\alpha}_{2n}, b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\gamma}_{2n-1})$. The second class includes $D(2b^{\alpha}_{2n}, b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, b^{\gamma}_{2n-1})$ and $D(b^{\alpha}_{2n}, a^{\alpha}_{2n}, b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, 2b^{\alpha}_{2n}, b^{\gamma}_{2n-1})$. Accordingly, we use $D(a^{\alpha}_{2n}, b^{\alpha}_{2n}, b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, 2b^{\alpha}_{2n}, b^{\gamma}_{2n-1})$ and $D(2b^{\alpha}_{2n}, b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, b^{\alpha}_{2n}, b^{\gamma}_{2n-1})$ to represent these two link types respectively.

Similarly, when a and b are exchanged in the case 3, the resulting OTP link diagrams of C^2_5 can be identified into six equivalence classes. These link type are represented by six link digrams numbered by 250, 252, ..., 260 in table 2. ■

Theorem 3.8. *There are 20 link types of OTP link diagrams with the orientation $o(\alpha, 2\beta, \alpha, -\alpha, 2\alpha, 2\beta)$, which are numbered from 261 to 280 in table 2 (Appendix A).*

Proof. For the orientation $o(\alpha, 2\beta, \alpha, -\alpha, 2\alpha, 2\beta)$, the number of the resulting OTP link diagrams are calculated by the following formula

$$2(2C^0_4 + 2C^1_4 + C^2_4) = 32.$$

We have two cases by only considering five twist tangles oriented with α or $-\alpha$ for $D(G)$ as below.

Case 1. When the twist tangle oriented with $-\alpha$ is $a^{-\alpha}_{2n}$, we have three subcases as below.

Subcase 1.1. When all twist tangles oriented with α are all a^{α}_{2n} or all b^{α}_{2n}, two OTP link diagrams $D(a^{\alpha}_{2n}, 2a^{\beta}_{2n-1}, a^{\alpha}_{2n}, a^{-\alpha}_{2n}, 2a^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$ and $D(b^{\alpha}_{2n}, 2a^{\beta}_{2n-1}, b^{\alpha}_{2n}, a^{-\alpha}_{2n}, 2b^{\alpha}_{2n}, 2a^{\beta}_{2n-1})$ can be obtained.

Subcase 1.2. When a twist tangle is a_{2n}^{α} and the remaining three twist tangles oriented with α are all b_{2n}^{α}, the resulting OTP link diagrams of C_4^1 are divided into two equivalence classes. One class includes $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ and $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$, where each diagram has a a_{2n}^{α} as an edge of a triangular face. The other class includes $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ and $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$, where each diagram has a a_{2n}^{α} as the lateral edge. Accordingly, we use $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ and $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ to represent these two link types respectively. Similarly, when a and b are exchanged in the subcase 1.2, we obtain two link types represented by the OTP link diagrams $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ and $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$.

Subcase 1.3 When two twist tangles of $D(G)$ are both a_{2n}^{α} and the remaining two twist tangles oriented with α are both b_{2n}^{α}, the resulting OTP link diagrams of C_4^2 can be also divided into three subcases in the following.

Subcase 1.3.1. There is one OTP link diagram $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ whose each triangular face has one a_{2n}^{α} as an edge.

Subcase 1.3.2. There are four OTP link diagrams such that each diagram has one a_{2n}^{α} as the lateral edge and the other a_{2n}^{α} as an edge of a triangular face. These diagrams are further identified into two equivalence classes, and each class has two members. The first class includes $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ and $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$. The second class includes $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ and $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$. Accordingly, we use $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ and $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ to represent these two link types respectively.

Subcase 1.3.3. There is one OTP link diagram $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$ which has two a_{2n}^{α} as the lateral edges.

Case 2. The twist tangle oriented with $-\alpha$ of $D(G)$ is $b_{2n}^{-\alpha}$. Similarly to the case 1, the resulting OTP link diagrams are identified into ten equivalence classes, which are represented by ten link diagrams numbered by 262, 264, ..., 280 in table 2. ■

Theorem 3.9. *There are 16 link types of OTP link diagrams with the orientation $o(2\alpha, \gamma, \alpha, 4\gamma, -\alpha)$, which are numbered from 281 to 296 in table 2 (Appendix A).*

Proof. The number of OTP link diagrams with the orientation $o(2\alpha, \gamma, \alpha, 4\gamma, -\alpha)$ can be calculated by the following formula

$$2(2C_3^0 + 2C_3^1) = 16.$$

If the twist tangle oriented with $-\alpha$ of $D(G)$ is $a_{2n}^{-\alpha}$, we have two cases as below.

Case 1. When all twist tangles oriented with α of $D(G)$ are all a_{2n}^{α} or all b_{2n}^{α}, two OTP link diagrams $D(2a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$ and $D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$ can be obtained.

Case 2. When a twist tangle of $D(G)$ is a_{2n}^{α} and the remaining two twist tangles oriented with α are both b_{2n}^{α}, the resulting OTP link diagrams of C_3^1 are identified into three equivalent classes. These diagrams include $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$, $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$ and $D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$. Similarly, when a and b are exchanged in the case 2, we obtain three link types of OTP link diagrams $D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$, $D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$ and $D(2a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$.

If the twist tangle oriented with $-\alpha$ of $D(G)$ is $b_{2n}^{-\alpha}$. Similarly to the above cases, the resulting OTP link diagrams are identified into eight equivalence classes, which are represented by eight link diagrams numbered by 282, 284, ..., 296 in table 2. ■

Theorem 3.10. *There are ten link types of OTP link diagrams with the orientation $o(\alpha, 2\beta, \alpha, -\alpha, 3\beta, -\alpha)$, which are numbered from 297 to 306 in table 2 (Appendix A).*

Proof. The number of OTP link diagrams with the orientation $o(\alpha, 2\beta, \alpha, -\alpha, 3\beta, -\alpha)$ are calculated by the following formula

$$2C_4^0 + 2C_4^1 + C_4^2 = 16.$$

We have three cases as below by only considering four twist tangles oriented with α or $-\alpha$ for G.

Case 1. When these four twist tangles each is a_{2n}^{α} or $a_{2n}^{-\alpha}$ according to the related orientation α or $-\alpha$ respectively, one OTP link diagram $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$ can be obtained. Similarly, when a is replaced by b in the case 1, we obtain an OTP link diagram $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$.

Case 2. When a twist tangle is a_{2n}^{α} or $a_{2n}^{-\alpha}$ and the remaining twist tangles each is b_{2n}^{α} or $b_{2n}^{-\alpha}$, the resulting OTP link diagrams of C_4^1 are divided into two equivalence classes such that two OTP link diagrams in each class are reverse to each other as an OTP link in $\mathbb{R}^3$. One class includes $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$ and $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$, where each diagram has a a_{2n}^{α} or $a_{2n}^{-\alpha}$ as an edge of any triangular face of $D(G)$. The other class includes $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$ and $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, 3a_{2n-1}^{\beta},$

$b_{2n}^{-\alpha}$), where each diagram has a a_{2n}^{α} or $a_{2n}^{-\alpha}$ as the lateral edge. Accordingly, we use $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$ and $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$ to represent these two link types respectively. Similarly, when a and b are exchanged in the case 2, the resulting OTP link diagrams of C_4^1 can be identified into two equivalence classes. We use $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$ and $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$ to represent these two link types.

Case 3. When two twist tangles each is a_{2n}^{α} or $a_{2n}^{-\alpha}$ and the remaining twist tangles each is b_{2n}^{α} or $b_{2n}^{-\alpha}$ according to the related orientation α or $-\alpha$ respectively, the resulting OTP link diagrams of C_4^2 can be divided into three subcases as below.

Subcase 3.1. There is one OTP link diagram $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$ whose each triangular face has a a_{2n}^{α} or a $a_{2n}^{-\alpha}$ as an edge.

Subcase 3.2. There are four OTP link diagrams such that each diagram has one a_{2n}^{α} or $a_{2n}^{-\alpha}$ as the lateral edge and the other a_{2n}^{α} or $a_{2n}^{-\alpha}$ as an edge of a triangular face. These diagrams are further identified into two equivalence classes such that two OTP link diagrams in each class are reverse to each other as a OTP link in $\mathbb{R}^3$. The first class includes $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$ and $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$. The second class includes $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$ and $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$. Accordingly, these two link types are represented by $D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$ and $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$.

Subcase 3.3. There is one OTP link diagram $D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$ which has a a_{2n}^{α} and a $a_{2n}^{-\alpha}$ as the lateral edges. ■

Theorem 3.11. *There are ten link types of OTP link diagrams with the orientation $o(\alpha, 3\beta, -\alpha, \alpha, \beta, -\alpha, \beta)$, which are numbered from 307 to 316 in table 2 (Appendix A).*

Proof. The number of the resulting OTP link diagrams can be calculated by the following formula

$$2C_4^0 + 2C_4^1 + C_4^2 = 16.$$

We have three cases as below by only considering four twist tangles each oriented with α or $-\alpha$ for $D(G)$.

Case 1. When these four twist tangles each is a_{2n}^{α} or $a_{2n}^{-\alpha}$ according to the related orientation α or $-\alpha$ respectively, one OTP link diagram $D(a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n-1}^{\beta})$ can be obtained. Similarly, when a is replaced by b in the case 1, we obtain one OTP link diagram $D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$.

Case 2. When a twist tangle is a_{2n}^{α} or $a_{2n}^{-\alpha}$ and the remaining twist tangles each is b_{2n}^{α} or $b_{2n}^{-\alpha}$, the resulting OTP link diagrams of C_4^1 can be divided into two equivalence classes such that to OTP link diagrams in each class are reverse to each other as an OTP link in $\mathbb{R}^3$. One class includes $D(a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$ and $D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n-1}^{\beta})$, where each diagram has a a_{2n}^{α} or $a_{2n}^{-\alpha}$ as an edge of any triangular face of $D(G)$. The other class includes $D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$ and $D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$, where each diagram has a a_{2n}^{α} or $a_{2n}^{-\alpha}$ as the lateral edge. Accordingly, we use $D(a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$ and $D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$ to represent these two link types respectively. Similarly, when a and b are exchanged in the case 2, we obtain two link types represented by $D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n-1}^{\beta})$ and $D(a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n-1}^{\beta})$.

Case 3. When two twist tangles each is a_{2n}^{α} or $a_{2n}^{-\alpha}$ and the remaining twist tangles each is b_{2n}^{α} or $b_{2n}^{-\alpha}$ according to the related orientations α and $-\alpha$ respectively, the resulting OTP link diagrams of C_4^2 are divided into three subcases.

Subcase 3.1. There is one OTP link diagram $D(a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n-1}^{\beta})$ whose each triangular face has a a_{2n}^{α} or $a_{2n}^{-\alpha}$ as an edge.

Subcase 3.2. There are four OTP link diagrams such that each diagram has one a_{2n}^{α} or $a_{2n}^{-\alpha}$ as the lateral edge and the other a_{2n}^{α} or $a_{2n}^{-\alpha}$ as an edge of a triangular face. These diagrams are further identified into two equivalence classes such that two OTP link diagrams in each class are reverse to each other as an OTP link in $\mathbb{R}^3$. The first class includes $D(a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$ and $D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n-1}^{\beta})$. The second class includes $D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n-1}^{\beta})$ and $D(a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$. Accordingly, we use $D(a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$ and $D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n-1}^{\beta})$ to represent these two link types respectively.

Subcase 3.3. There is one OTP link diagram $D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$ which has a a_{2n}^{α} and a $a_{2n}^{-\alpha}$ as the lateral edge. ■

For the remaining twelve orientations, in each orientation there are at most eight link diagrams produced by using the twist tangle to replace the related orientation on each edge, and hence it is easy to identify their link types. Here the proof of the following theorem is omitted .

Theorem 3.12. *There are 50 link types of OTP link diagrams obtained from the remaining 12 orientations, which are numbered from 317 to 366 in table 2 (Appendix A).*

The following theorem can be directly obtained from the above theorems.

Theorem 3.13. *Let G be a triangular prism graph. Then there are 366 link types of OTP link diagrams constructed from G.*

Conjecture 3.14. *These 366 link types of OTP link diagrams constructed from G as above are different to each other.*

For each OTP link with each edge having even crossing number, this conjecture may be proved by calculating their HOMFLY polynomials based on this relation between the polyhedral graph and these links [31, 32]. However, this method will lost its value for the remaining links since it is difficult to obtain the HOMFLY polynomials of these links.

4 Conclusion

In this paper, triangular prism links are constructed to give all topological structures of DNA triangular prisms with double-helical edges. All OTP links are determined by using 22 different orientations and further identified into 366 link types by considering the same topological structures produced by the construction method and symmetry of triangular prism in the construction process. For each OTP link, each edge is two twisted stranded with the crossing number $2n$ or $2n-1$ and each vertex is a 'hole' formed by connecting any two adjacent edges. These links are numbered sequentially from 1 to 366 in table 2 (Appendix A). Also, we note that each OTP link has the same parameter n for each edge, which is approximate to a regular triangular prism with equal edge lengths. In fact, the parameter n can be different for each edge.

On the other hand, it is worth noting that there are 104 OTP links with the same orientation $o(9\alpha)$ (called as 'even' links), where their each edge have even crossing number. And there is only one OTP link $D(3b^{\gamma}_{2n-1}, 3a^{\beta}_{2n-1}, 3b^{\gamma}_{2n-1})$ with the orientation $o(3\gamma, 3\beta, 3\gamma)$ (called as 'odd' link), where its each edge have odd crossing number. The remaining each OTP link have at least two edges having even and odd crossing number respectively. Also, each OTP link is alternating. The link $D(9a^{\alpha}_{2n})$ covers the topological structure of five-strands DNA prism [25], which have the largest crossing number 18 for $n = 1$ in table 2 (Appendix A). In contrast, the single-stranded DNA triangular prism is the smallest 3D DNA polyhedron ever reported, whose topological structure is ambient isotopic to a alternating OTP link. However, the knot $D(3b^{\gamma}_{2n-1}, 3a^{\beta}_{2n-1}, 3b^{\gamma}_{2n-1})$ numbered as 351 has only half twist on each edge for $n = 1$, which may be the minimum crossing number on each edge. This result provides a possibility to create a smaller-sized triangular prism knot from theoretical viewpoints. Thus, our work provides an insight deeply into the possible existing topological structures for DNA triangular prisms, and also gives a list of candidates for further synthesizing these DNA molecules with required topological structures.

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Appendix A The list of 366 OTP links

$N(D)$	$D(G)$	$N(D)$	$D(G)$
1	$D(9b_{2n}^{\alpha})$	2	$D(9a_{2n}^{\alpha})$
3	$D(a_{2n}^{\alpha}, 8b_{2n}^{\alpha})$	4	$D(b_{2n}^{\alpha}, 8a_{2n}^{\alpha})$
5	$D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 5b_{2n}^{\alpha})$	6	$D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, 5a_{2n}^{\alpha})$
7	$D(2a_{2n}^{\alpha}, 7b_{2n}^{\alpha})$	8	$D(2b_{2n}^{\alpha}, 7a_{2n}^{\alpha})$
9	$D(a_{2n}^{\alpha}, 7b_{2n}^{\alpha}, a_{2n}^{\alpha})$	10	$D(b_{2n}^{\alpha}, 7a_{2n}^{\alpha}, b_{2n}^{\alpha})$
11	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 5b_{2n}^{\alpha})$	12	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 5a_{2n}^{\alpha})$
13	$D(a_{2n}^{\alpha}, 6b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$	14	$D(b_{2n}^{\alpha}, 6a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$
15	$D(a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$	16	$D(b_{2n}^{\alpha}, 5a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$
17	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 5b_{2n}^{\alpha})$	18	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 5a_{2n}^{\alpha})$
19	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 5b_{2n}^{\alpha})$	20	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 5a_{2n}^{\alpha})$
21	$D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$	22	$D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$
23	$D(3a_{2n}^{\alpha}, 6b_{2n}^{\alpha})$	24	$D(3b_{2n}^{\alpha}, 6a_{2n}^{\alpha})$
25	$D(a_{2n}^{\alpha}, 6b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$	26	$D(b_{2n}^{\alpha}, 6a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$
27	$D(3b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$	28	$D(3a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$
29	$D(a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$	30	$D(b_{2n}^{\alpha}, 5a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$
31	$D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$	32	$D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$
33	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha})$	34	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 4a_{2n}^{\alpha}, b_{2n}^{\alpha})$
35	$D(a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$	36	$D(b_{2n}^{\alpha}, 5a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$
37	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha})$	38	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n}^{\alpha}, b_{2n}^{\alpha})$
39	$D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$	40	$D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$
41	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$	42	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$
43	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha})$	44	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n}^{\alpha}, b_{2n}^{\alpha})$
45	$D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$	46	$D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$
47	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$	48	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$
49	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$	50	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$
51	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$	52	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$
53	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$	54	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$
55	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$	56	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$
57	$D(4a_{2n}^{\alpha}, 5b_{2n}^{\alpha})$	58	$D(4b_{2n}^{\alpha}, 5a_{2n}^{\alpha})$
59	$D(2a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$	60	$D(2b_{2n}^{\alpha}, 5a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$
61	$D(3a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$	62	$D(3b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$
63	$D(2a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$	64	$D(2b_{2n}^{\alpha}, 4a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$
65	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$	66	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$
67	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$	68	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$
69	$D(2a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$	70	$D(2b_{2n}^{\alpha}, 4a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$
71	$D(b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$	72	$D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$
73	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$	74	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$
75	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$	76	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$
77	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$	78	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$
79	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$	80	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$
81	$D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$	82	$D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$
83	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n}^{\alpha})$	84	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n}^{\alpha})$
85	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$	86	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$
87	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha})$	88	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha})$
89	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$	90	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$
91	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$	92	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$
93	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$	94	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$
95	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$	96	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$

97	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$	98	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$
99	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$	100	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$
101	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha})$	102	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha})$
103	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha})$	104	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha})$
105	$D(4b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	106	$D(4a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
107	$D(4b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	108	$D(4a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
109	$D(4b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	110	$D(4a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
111	$D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	112	$D(b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
113	$D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	114	$D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
115	$D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	116	$D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
117	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	118	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
119	$D(4b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	120	$D(4a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
121	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	122	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
123	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	124	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
125	$D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	126	$D(b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
127	$D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	128	$D(b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
129	$D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	130	$D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
131	$D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	132	$D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
133	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	134	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
135	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	136	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
137	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	138	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
139	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	140	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
141	$D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	142	$D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
143	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	144	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
145	$D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	146	$D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
147	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	148	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
149	$D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	150	$D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
151	$D(b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	152	$D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
153	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	154	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
155	$D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	156	$D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
157	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	158	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n-1}^{\beta}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
159	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	160	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
161	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	162	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
163	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	164	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
165	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	166	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
167	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	168	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
169	$D(6b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	170	$D(6a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
171	$D(a_{2n}^{\alpha}, 5b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	172	$D(b_{2n}^{\alpha}, 5a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
173	$D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	174	$D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
175	$D(2a_{2n}^{\alpha}, 4b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	176	$D(2b_{2n}^{\alpha}, 4a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
177	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	178	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
179	$D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	180	$D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
181	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	182	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
183	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	184	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
185	$D(3a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	186	$D(3b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
187	$D(b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	188	$D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
189	$D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	190	$D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
191	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	192	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
193	$D(3b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$	194	$D(3a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3a_{2n}^{-\alpha})$
195	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$	196	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3a_{2n}^{-\alpha})$
197	$D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$	198	$D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3a_{2n}^{-\alpha})$
199	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, 2b_{2n}^{-\alpha})$	200	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, 2a_{2n}^{-\alpha})$
201	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$	202	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$
203	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$	204	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$
205	$D(3a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n}^{-\alpha})$	206	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, 2a_{2n}^{-\alpha})$
207	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 2a_{2n}^{-\alpha}, b_{2n}^{-\alpha})$	208	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, b_{2n}^{-\alpha}, a_{2n}^{-\alpha})$
209	$D(4b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	210	$D(4b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
211	$D(4a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	212	$D(4a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
213	$D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	214	$D(a_{2n}^{\alpha}, 3b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
215	$D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	216	$D(3b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
217	$D(b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	218	$D(b_{2n}^{\alpha}, 3a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
219	$D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	220	$D(3a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
221	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	222	$D(2a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$

223	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	224	$D(2b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
225	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	226	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
227	$D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	228	$D(2b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
229	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	230	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
231	$D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	232	$D(2a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
233	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	234	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
235	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	236	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
237	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	238	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
239	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	240	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
241	$D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$	242	$D(2a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2a_{2n}^{\alpha}, b_{2n-1}^{\gamma})$
243	$D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$	244	$D(2a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2a_{2n}^{\alpha}, b_{2n-1}^{\gamma})$
245	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$	246	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2a_{2n}^{\alpha}, b_{2n-1}^{\gamma})$
247	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$	248	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2a_{2n}^{\alpha}, b_{2n-1}^{\gamma})$
249	$D(2a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$	250	$D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2a_{2n}^{\alpha}, b_{2n-1}^{\gamma})$
251	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$	252	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, 2a_{2n}^{\alpha}, b_{2n-1}^{\gamma})$
253	$D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$	254	$D(2a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma})$
255	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$	256	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma})$
257	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma})$	258	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$
259	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma})$	260	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma})$
261	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	262	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, b_{2n}^{-\alpha}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
263	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	264	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
265	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	266	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, b_{2n}^{-\alpha}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
267	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	268	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, 2b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
269	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	270	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, 2a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
271	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	272	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, b_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
273	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	274	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
275	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	276	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, b_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
277	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	278	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, b_{2n}^{-\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
279	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$	280	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, a_{2n}^{\alpha}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta})$
281	$D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$	282	$D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, b_{2n}^{-\alpha})$
283	$D(2a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$	284	$D(2a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, b_{2n}^{-\alpha})$
285	$D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$	286	$D(2b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, b_{2n}^{-\alpha})$
287	$D(2a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$	288	$D(2a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, b_{2n}^{-\alpha})$
289	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$	290	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, b_{2n}^{-\alpha})$
291	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$	292	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, b_{2n}^{-\alpha})$
293	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$	294	$D(b_{2n}^{\alpha}, a_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, b_{2n}^{-\alpha})$
295	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, a_{2n}^{-\alpha})$	296	$D(a_{2n}^{\alpha}, b_{2n}^{\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 4b_{2n-1}^{\gamma}, b_{2n}^{-\alpha})$
297	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$	298	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$
299	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$	300	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$
301	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$	302	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$
303	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$	304	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$
305	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha})$	306	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, b_{2n}^{-\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha})$
307	$D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$	308	$D(a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n-1}^{\beta})$
309	$D(a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$	310	$D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n-1}^{\beta})$
311	$D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$	312	$D(a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n-1}^{\beta})$
313	$D(a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n-1}^{\beta})$	314	$D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$
315	$D(a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n-1}^{\beta})$	316	$D(b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{-\alpha}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{-\alpha}, a_{2n-1}^{\beta})$
317	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	318	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
319	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	320	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
321	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	322	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
323	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n-1}^{\beta}, a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$	324	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n-1}^{\beta}, b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma})$
325	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n-1}^{\beta})$	326	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n-1}^{\beta})$
327	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n-1}^{\beta})$	328	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n-1}^{\beta})$
329	$D(b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, b_{2n}^{\alpha}, a_{2n-1}^{\beta})$	330	$D(a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, a_{2n}^{\alpha}, a_{2n-1}^{\beta})$
331	$D(b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha})$	332	$D(a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha})$
333	$D(a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha})$	334	$D(b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha})$
335	$D(b_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, a_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, b_{2n}^{\alpha})$	336	$D(a_{2n}^{\alpha}, 4a_{2n-1}^{\beta}, b_{2n}^{\alpha}, 2a_{2n-1}^{\beta}, a_{2n}^{\alpha})$
337	$D(b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma}, b_{2n}^{-\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma})$	338	$D(a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma}, a_{2n}^{-\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma})$
339	$D(a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma}, b_{2n}^{-\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma})$	340	$D(b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma}, a_{2n}^{-\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma})$
341	$D(b_{2n}^{\alpha}, 3b_{2n-1}^{\gamma}, a_{2n}^{-\alpha}, b_{2n-1}^{\gamma}, b_{2n}^{\alpha}, 2b_{2n-1}^{\gamma})$	342	$D(a_{2n}^{\alpha}, 3b_{2n-1}^{\gamma}, b_{2n}^{-\alpha}, b_{2n-1}^{\gamma}, a_{2n}^{\alpha}, 2b_{2n-1}^{\gamma})$
343	$D(3b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n-1}^{\gamma})$	344	$D(3a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n-1}^{\gamma})$
345	$D(a_{2n}^{\alpha}, 2b_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n-1}^{\gamma})$	346	$D(b_{2n}^{\alpha}, 2a_{2n}^{\alpha}, 3a_{2n-1}^{\beta}, 3b_{2n-1}^{\gamma})$
347	$D(3b_{2n-1}^{\gamma}, 3b_{2n}^{-\alpha}, 3b_{2n-1}^{\gamma})$	348	$D(3b_{2n-1}^{\gamma}, 3a_{2n}^{-\alpha}, 3b_{2n-1}^{\gamma})$

349	$D(3b^{\gamma}_{2n-1}, a^{-\alpha}_{2n}, 2b^{-\alpha}_{2n}, 3b^{\gamma}_{2n-1})$	350	$D(3b^{\gamma}_{2n-1}, b^{-\alpha}_{2n}, 2a^{-\alpha}_{2n}, 3b^{\gamma}_{2n-1})$
351	$D(3b^{\gamma}_{2n-1}, 3a^{\beta}_{2n-1}, 3b^{\gamma}_{2n-1})$	352	$D(b^{\alpha}_{2n}, 5a^{\beta}_{2n-1}, b^{-\alpha}_{2n}, 2a^{\beta}_{2n-1})$
353	$D(a^{\alpha}_{2n}, 5a^{\beta}_{2n-1}, a^{-\alpha}_{2n}, 2a^{\beta}_{2n-1})$	354	$D(a^{\alpha}_{2n}, 5a^{\beta}_{2n-1}, b^{-\alpha}_{2n}, 2a^{\beta}_{2n-1})$
355	$D(b^{\alpha}_{2n}, 3a^{\beta}_{2n-1}, b^{-\alpha}_{2n}, a^{\beta}_{2n-1}, 3b^{\gamma}_{2n-1})$	356	$D(a^{\alpha}_{2n}, 3a^{\beta}_{2n-1}, a^{-\alpha}_{2n}, a^{\beta}_{2n-1}, 3b^{\gamma}_{2n-1})$
357	$D(a^{\alpha}_{2n}, 3a^{\beta}_{2n-1}, b^{-\alpha}_{2n}, a^{\beta}_{2n-1}, 3b^{\gamma}_{2n-1})$	358	$D(b^{\alpha}_{2n}, 3a^{\beta}_{2n-1}, a^{-\alpha}_{2n}, a^{\beta}_{2n-1}, 3b^{\gamma}_{2n-1})$
359	$D(b^{\alpha}_{2n}, 3b^{\gamma}_{2n-1}, a^{\beta}_{2n-1}, 2b^{\gamma}_{2n-1}, 2a^{\beta}_{2n-1})$	360	$D(a^{\alpha}_{2n}, 3b^{\gamma}_{2n-1}, a^{\beta}_{2n-1}, 2b^{\gamma}_{2n-1}, 2a^{\beta}_{2n-1})$
361	$D(2a^{\beta}_{2n-1}, b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, 2a^{\beta}_{2n-1}, b^{\gamma}_{2n-1})$	362	$D(2a^{\beta}_{2n-1}, b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, 2a^{\beta}_{2n-1}, b^{\gamma}_{2n-1})$
363	$D(b^{\alpha}_{2n}, b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, 2a^{\beta}_{2n-1}, b^{\gamma}_{2n-1}, a^{\beta}_{2n-1})$	364	$D(a^{\alpha}_{2n}, b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, 2a^{\beta}_{2n-1}, b^{\gamma}_{2n-1}, a^{\beta}_{2n-1})$
365	$D(a^{\alpha}_{2n}, b^{\gamma}_{2n-1}, b^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, 2a^{\beta}_{2n-1}, b^{\gamma}_{2n-1}, a^{\beta}_{2n-1})$	366	$D(b^{\alpha}_{2n}, b^{\gamma}_{2n-1}, a^{\alpha}_{2n}, 2b^{\gamma}_{2n-1}, 2a^{\beta}_{2n-1}, b^{\gamma}_{2n-1}, a^{\beta}_{2n-1})$

Table 2. The list of 366 OTP links (the link D is labeled by the number $N(D)$).

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