# Dynamic Representations of Biological Sequences 

Dorota Bielińska-Waż ${ }^{a}$, Damian Panas ${ }^{a}$, Piotr Waż ${ }^{b}$<br>${ }^{a}$ Department of Radiological Informatics and Statistics, Medical University of Gdańsk, Gdańsk, Poland<br>djwaz@gumed.edu.pl, dpanas@gumed.edu.pl<br>${ }^{b}$ Department of Nuclear Medicine, Medical University of Gdańsk, Gdańsk, Poland<br>phwaz@gumed.edu.pl

(Received January 6, 2019)


#### Abstract

Methods of bioinformatics in which the biological sequences (DNA, RNA, protein) are represented by sets of material points in 2D, 3D, or 20D space, and described by values analogous to the ones used in the dynamics, as e.g. moments of inertia, are reviewed. A new application of the 3D method, called by us 3D-Dynamic Representation of DNA/RNA Sequences is proposed. It is shown that the method is useful for a description of complete genome sequences of dengue virus.


## 1 Introduction

In 2007, we developed a new bioinformatics method and invented its descriptive name 2DDynamic Representation of $D N A / R N A$ Sequences. At present the name refers to a class of bioinformatics methods, but originally it has been assigned to the two-dimensional version only [1-3]. The method has been extended to three-dimensions (3D-Dynamic Representation of DNA/RNA Sequences) [4] and a twenty-dimensional version of the method (20D-Dynamic Representation of Protein Sequences) has also been designed [5]. In the present work, we demonstrate the practical power and usefulness of this approach by its application to a description of the complete genome sequences of dengue virus.

The main idea of the Dynamic Representations of Biological Sequences (DRBS) is transforming the sequences to sets of material points in 2D, 3D, or 20D space. We call
these methods dynamic, because we treat the biochemical objects as rigid bodies and characterize them by numerical characteristics (descriptors), analogous to the ones used in the classical dynamics as, for example, moments of inertia.

In this paper, after reviewing in Section 2 the present status of the DRBS methods, we discuss, in Section 3, an application of the 3D version of the method to the description of the dengue virus genome.

## 2 DRBS Methods

In the DRBS methods the biological sequences (DNA, RNA, protein) are represented by dynamic graphs. The graphs are composed of point-masses in a properly defined Euclidean space. The shape and the location of a dynamic graph in the space is characteristic for the sequence described by this graph. So far, DRBS methods utilizing dynamic graphs defined in 2D and 3D spaces (applied to DNA/RNA sequences) $[1,4]$ and in 20D space (applied to protein sequences) [5] have been designed.

The DNA/RNA sequence is a sequence composed of four letters A, C, G and T/U corresponding to four bases - Adenine, Cytosine, Guanine, and Thymine/Uracil, respectively. In DBRS methods each base is represented by a unit basis vector.

A dynamic graph is defined by the method of its construction. The graph is developed recursively, performing a series of shifts (walks) along the basis vectors representing the bases, starting from the origin of the Cartesian coordinate system in the pertinent Euclidean space. Techniques using the idea of shifts (or walks) have been introduced as the first graphical representations of the DNA sequences - in three dimensions by Hamori and Ruskin [6] and in two dimensions by Gates [7], Nandy [8], Leong and Morgenthaler [9]. In two dimensions, the three mentioned above methods use four orthogonal directions for the representations of the four bases. They differ by the assignment of the basis vectors to particular bases. The most popular diagrams are the Nandy plots [8]. The two-dimensional methods are the most convenient techniques for the visualization. But, in their original form, the methods are not free from non-uniqueness, referred to as degeneracy (in some cases different sequences are represented by the same diagrams). In order to reduce or to remove the degeneracy, many new Graphical Representation (GR) methods have been designed, including the DRBS ones. One of the most important features of the graphical representations of DNA/RNA sequences is their usefulness in both graphical and numeri-
cal revealing different aspects of similarity of the objects they describe [10-26] (for reviews see [27,28]).

The range of applicability of the GR methods have been also extended to a description of protein sequences, e.g. [29, 30]. A numerical representation of proteins as walks in a 20D space, introduced by Novič and Randić [31], has been applied in 20D DRBS [5].

The DRBS graph is composed of a set mass-one point masses. The biological sequence is mapped onto the distribution of the point masses forming the dynamic graph. The graph is constructed by properly locating a point-mass at the end of each vector representing a base. In particular, in the 2D-Dynamic Representation of DNA/RNA Sequences the bases are represented by the basis vectors, the same as in the Nandy plots: $\mathrm{A}=(-1,0)$, $\mathrm{G}=(1,0), \mathrm{C}=(0,1)$ and $\mathrm{T} / \mathrm{U}=(0,-1)$. If, for example, the first base is G , then the vector representing the first shift is $(1,0)$ and the coordinates of the first point-mass are $(1,0)$. This point-mass is a starting point for the second shift representing the second base in the sequence. At the end of the second vector the second point-mass is located. The procedure is repeated until the end of the sequence. If the ends of some vectors meet several times at the same point, then the mass of this point increases: it is the sum of masses corresponding to all vectors ending at this point. In the Nandy plots, if the shifts are performed back and forth along the same trace then some parts of the sequence are hidden. As a consequence, different sequences may be represented by the same plot. In the 2D-Dynamic Representation of DNA/RNA Sequences, by summing up masses at the points visited by several vectors, this kind of degeneracy is removed. A drawback of this approach is that the history of emergence of a graph is lost - the graphs self-overlap. This feature has been corrected in the 3D-Dynamic Representation of DNA/RNA Sequences. In the 3D method we represent the bases as follows: $\mathrm{A}=(-1,0,1), \mathrm{G}=(1,0,1), \mathrm{C}=$ $(0,1,1)$, and $\mathrm{T} / \mathrm{U}=(0,-1,1)$. The construction of the dynamic graph is here analogous to the one in the 2 D method. The shifts start from the origin of the 3 D coordinate system and continue using subsequent bases in the sequence. Similarly as in the 2D case, to obtain the dynamic graph, at the end of each vector we locate a point-mass with mass equal to 1 .

For the representation of protein sequences we choose a 20D space. The assignment of the axes of the 20 -dimensional coordinate system to the amino acids has been described in [5]. The construction of a 20D-dynamic graph is the same as in the 2D and the 3D
methods, i.e. we start shifts at the origin of the 20D coordinate system, and at the end of the vectors we locate the point-mass with mass equal to 1 . As a consequence of the choice of the basis vectors, the masses of all the points in the 3 D and in the 20 D method are equal to 1 . The 20 D method for protein sequences can easily be extended to more dimensions, if more than 20 amino acids were present in the sequence. New basis vectors can be introduced in the same way as it has been done for 20 dimensions, and the upper limit in all the summations present in the method can be modified accordingly.

For a numerical characterization of the 2D-dynamic graphs we have selected the coordinates of the center of mass and the principal moments of inertia of the graph in the 2D space [1]. Using the 2D-dynamic graphs we have created the corresponding mass-density distributions and took moments of these distributions as descriptors [2]. We also have considered the angles between the $x$ axis and the principal axis of the 2D-dynamic graph as descriptors. Good descriptors are also coordinates of the center of mass divided by the principal moments of inertia [32]. We have also described the 2D-dynamic graphs using matrix elements of the tensor of inertia and the graph radius, i.e. the length of the position vector of the mass center [33].

In the case of the 3D-dynamic graphs, analogously as in the 2D method, for the characterization of the graphs we have used coordinates of the center of mass, and the principal moments of inertia in the 3D space [4]. We have also used as descriptors the coordinates of the center of mass divided by the principal moments of inertia or divided by the normalized principal moments of inertia, and the cosines of the angles between different planes [4].

For the characterization of the 20D-dynamic graphs representing the protein sequences, we have chosen the 20D normalized principal moments of inertia, and the sum of all 20 normalized principal moments of inertia [5].

The 2D and the 3D methods allow for a direct visualization of the objects. In order to visualize the 20D graphs we have projected them onto 2D or 3D spaces.

In order to study the similarity/dissimilarity between sequences one has to define a similarity measures. In particular, for comparing sequences we have introduced a new normalized similarity measure [34]. We have also introduced mass overlaps as another similarity measure [3]. In similarity studies, the 2D-dynamic graphs have been treated as rigid bodies. In order to find the maximum of the overlap of masses of a pair of graphs,
the graphs are shifted and rotated. The numerical procedures are based on the genetic algorithms [3].

It is worth to notice, that the idea of characterizing biological sequences by the moments of inertia, introduced by us [1], has already been adopted by other authors. Yao et al. applied this idea for the representation of protein sequences by 2D moments of inertia [35] and by 3D moments of inertia [36]. Hou et al. applied 3D moments of inertia to characterize graphs representing protein sequences [37].

The method introduced by us, 2D-Dynamic Representation of DNA Sequences, has also been generalized to three dimensions by Aram and Iranmanesh [38]. As a consequence, two different 3D methods derived from the same method and having the same name are present in the literature $[4,38]$.

In the present work, for the characterization of the sequences we apply the 3D-Dynamic Representation of DNA/RNA Sequences in the sense introduced earlier by us: the sequences are represented by the material points and described by values used in classical dynamics [4]. The following descriptors are used in this work:

- Coordinates of the centers of mass of the 3D-dynamic graphs $\mu_{a}$, where $a=x, y, z$;
- Matrix elements of the tensor of the moment of inertia of the 3D-dynamic graphs $I_{a b}$, where $b=x, y, z$;
- Values $D_{k}^{a}=\mu_{a} / I_{k}$, where $I_{k}$ are the principal moments of inertia of the 3D-dynamic graphs and $k=1,2,3$.

The coordinates of the center of mass of the 3D-dynamic graph, in the $X Y Z$ coordinate system are defined as

$$
\begin{equation*}
\mu_{a}=\frac{\sum_{i=1}^{N} m_{i} a_{i}}{\sum_{i=1}^{N} m_{i}} \tag{1}
\end{equation*}
$$

where $\left(x_{i}, y_{i}, z_{i}\right)$ are the coordinates of the mass $m_{i}$. We assume $m_{i}=1$ for all the points. Then, the length of the sequence $N$ is equal to the total mass of the 3D-dynamic graph

$$
\begin{equation*}
N=\sum_{i=1}^{N} m_{i} \tag{2}
\end{equation*}
$$

and the coordinates of the center of mass of the 3D-dynamic graph are

$$
\begin{equation*}
\mu_{a}=\frac{1}{N} \sum_{i=1}^{N} a_{i} . \tag{3}
\end{equation*}
$$

The tensor of the moment of inertia is defined by the matrix

$$
\hat{I}=\left(\begin{array}{ccc}
I_{x x} & I_{x y} & I_{x z}  \tag{4}\\
I_{y x} & I_{y y} & I_{y z} \\
I_{z x} & I_{z y} & I_{z z}
\end{array}\right)
$$

with the matrix elements

$$
\begin{equation*}
I_{a a}=\sum_{i=1}^{N} m_{i}\left[\left(b_{i}^{\prime}\right)^{2}+\left(c_{i}^{\prime}\right)^{2}\right], \quad I_{a b}=I_{b a}=-\sum_{i=1}^{N} m_{i} a_{i}^{\prime} b_{i}^{\prime}, \tag{5}
\end{equation*}
$$

where $\{a, b, c\}=\{x, y, z\}, a \neq b \neq c$ and $\left(x_{i}^{\prime}, y_{i}^{\prime}, z_{i}^{\prime}\right)$ are the coordinates of $m_{i}$ in the Cartesian coordinate system with the origin selected at the center of mass. The principal moments of inertia $I_{k}$ are defined as the eigenvalues of $\hat{I}$

$$
\begin{equation*}
\hat{I} \omega_{k}=I_{k} \omega_{k} \tag{6}
\end{equation*}
$$

where $\omega_{k}$ are the eigenvectors of the problem.
The eigenvalues are obtained by solving the following set of equations:

$$
\left|\begin{array}{llr}
I_{x x}-I & I_{x y} & I_{x z}  \tag{7}\\
I_{y x} & I_{y y}-I & I_{y z} \\
I_{z x} & I_{z y} & I_{z z}-I
\end{array}\right|=0
$$

The eigenvectors $\omega_{1}, \omega_{2}, \omega_{3}$ form the basis for a new coordinate system. The corresponding axes of this new system are denoted $\Omega_{1}, \Omega_{2}, \Omega_{3}$ and referred to as the principal axes. The eigenvalues $I_{1}, I_{2}, I_{3}$, are called the principal moments of inertia and are equal to the moments of inertia associated with the rotations around the principal axes.

The moment of inertia of an object about a rotational axis describes how difficult is to induce the rotation of the object around this axis. If the mass is concentrated close to the axis, it is easier to accelerate into spinning fast and the moment of inertia is smaller. As a consequence, the values of these descriptors reflect the concentrations of masses around the axes.

In our previous work we also used the square roots of the normalized principal moments of inertia [4]:

$$
\begin{equation*}
r_{1}=\sqrt{\frac{I_{1}}{N}}, \quad r_{2}=\sqrt{\frac{I_{2}}{N}}, \quad r_{3}=\sqrt{\frac{I_{3}}{N}} . \tag{8}
\end{equation*}
$$

The relative orientation of the new and old coordinate system may be described by the cosines of properly defined angles. Let $M_{1}, M_{2}$ and $M_{3}$ denote, respectively, the planes $(X, Y),(X, Z)$ and $(Y, Z)$. Similarly, $Q_{1}, Q_{2}, Q_{3}$ stand for the planes $\left(\Omega_{1}, \Omega_{2}\right),\left(\Omega_{1}, \Omega_{3}\right)$,
$\left(\Omega_{2}, \Omega_{3}\right)$, respectively. For the characterization of the 3D-dynamic graphs we used the cosines of the angles between the planes of the two systems of coordinates (Fig. 1) [4]:

$$
\begin{equation*}
C_{i j} \equiv \cos \left(M_{i}, Q_{j}\right), \quad i, j=1,2,3 . \tag{9}
\end{equation*}
$$



Figure 1. Planes $M_{1}:(X, Y), M_{2}:(X, Z), M_{3}:(Y, Z), Q_{1}:\left(\Omega_{1}, \Omega_{2}\right), Q_{2}:\left(\Omega_{1}, \Omega_{3}\right), Q_{3}:\left(\Omega_{2}, \Omega_{3}\right)$.

## 3 Results and Discussion

Recently we applied the 2D version of the DRBS approach to the description of Zika [33] and influenza [39] virus genomes. In the present work, complete strains of the dengue virus genome are studied using the 3D method. Each year dengue virus (DENV) causes dengue fever in about 390 million individuals in over 100 countries [40-42]. DENV exists in four serotypes (DENV-1 - DENV-4). ${ }^{1}$

In the present calculations, 2712 sequences have been used: 822 sequences of serotype 1 (DENV-1), 827 of serotype 2 (DENV-2), 829 of serotype 3 (DENV-3), and 234 of serotype 4 (DENV-4). The representative examples of the results (the principal moments of inertia) for 25 sequences for each serotype are listed in Table 1. The complete set of results is available from the authors upon request.

The distribution of the $x$ coordinate of the center of mass of the 3D-dynamic graphs $\left(\mu_{x}\right)$ representing all 2712 sequences is visualized in Fig. 2. As one can see, the distribution is composed of 3 separate sub-distributions. The values of $\mu_{x}$ cluster around 3 values.

[^0]Table 1. Selected sequence data and the principal moments of inertia.

| Serotype 1 |  |  |  |  |  | Serotype 2 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. | Accession | $N$ | $I_{1} / 10^{10}$ | $I_{2} / 10^{10}$ | $I_{3} / 10^{6}$ | No. | Accession | $N$ | $I_{1} / 10^{10}$ | $I_{2} / 10^{10}$ | $I_{3} / 10^{6}$ |
| 1 | LC335877 | 10179 | 8.823 | 8.822 | 10.32 | 26 | AF489932 | 10722 | 10.34 | 10.34 | 5.255 |
| 2 | LC335871 | 10179 | 8.823 | 8.822 | 9.932 | 27 | AJ487271 | 10597 | 9.979 | 9.979 | 4.361 |
| 3 | LC335872 | 10179 | 8.823 | 8.822 | 10.16 | 28 | LC367234 | 10723 | 10.33 | 10.33 | 5.096 |
| 4 | LC335873 | 10179 | 8.823 | 8.822 | 10.02 | 29 | FJ390389 | 10632 | 10.07 | 10.07 | 6.343 |
| 5 | LC335879 | 10179 | 8.823 | 8.822 | 10.41 | 30 | AJ968413 | 10723 | 10.34 | 10.34 | 4.236 |
| 6 | AB608789 | 10677 | 10.18 | 10.18 | 9.393 | 31 | AB543624 | 10731 | 10.37 | 10.37 | 2.385 |
| 7 | AB608788 | 10693 | 10.23 | 10.23 | 10.64 | 32 | AB189122 | 10723 | 10.33 | 10.33 | 4.123 |
| 8 | AB608786 | 10749 | 10.39 | 10.39 | 9.838 | 33 | AB189123 | 10723 | 10.33 | 10.33 | 4.252 |
| 9 | AB608787 | 10749 | 10.39 | 10.39 | 11.13 | 34 | AB189124 | 10723 | 10.33 | 10.33 | 4.311 |
| 10 | HE795086 | 10179 | 8.826 | 8.825 | 9.930 | 35 | AB122020 | 10723 | 10.34 | 10.34 | 6.044 |
| 11 | AB519681 | 10735 | 10.35 | 10.35 | 5.922 | 36 | AB122022 | 10723 | 10.34 | 10.34 | 5.942 |
| 12 | AB189120 | 10735 | 10.35 | 10.35 | 5.904 | 37 | LC129169 | 10723 | 10.34 | 10.34 | 3.162 |
| 13 | AB189121 | 10735 | 10.35 | 10.35 | 7.190 | 38 | LC121816 | 10685 | 10.22 | 10.22 | 4.539 |
| 14 | AB195673 | 10718 | 10.30 | 10.30 | 6.101 | 39 | AB479041 | 10647 | 10.11 | 10.11 | 6.136 |
| 15 | AB204803 | 10706 | 10.27 | 10.27 | 6.357 | 40 | LC111438 | 10678 | 10.21 | 10.21 | 3.842 |
| 16 | LC011948 | 10693 | 10.23 | 10.23 | 11.30 | 41 | KF744400 | 10176 | 8.832 | 8.832 | 6.189 |
| 17 | LC016760 | 10693 | 10.23 | 10.23 | 11.10 | 42 | KF744401 | 10176 | 8.832 | 8.832 | 6.138 |
| 18 | LC128301 | 10693 | 10.23 | 10.23 | 9.953 | 43 | KF744402 | 10176 | 8.834 | 8.834 | 5.044 |
| 19 | LC011945 | 10693 | 10.23 | 10.23 | 11.12 | 44 | KY474330 | 10648 | 10.13 | 10.13 | 5.436 |
| 20 | LC011949 | 10693 | 10.23 | 10.23 | 11.54 | 45 | KY474334 | 10460 | 9.602 | 9.601 | 5.137 |
| 21 | HM469966 | 10735 | 10.35 | 10.35 | 12.47 | 46 | KY474325 | 10648 | 10.13 | 10.13 | 5.601 |
| 22 | JX669462 | 10737 | 10.35 | 10.35 | 6.281 | 47 | KY474324 | 10648 | 10.13 | 10.13 | 5.535 |
| 23 | JX669463 | 10735 | 10.35 | 10.35 | 5.948 | 48 | KY474318 | 10648 | 10.13 | 10.13 | 5.527 |
| 24 | JX669464 | 10736 | 10.35 | 10.35 | 6.809 | 49 | KY474315 | 10648 | 10.13 | 10.13 | 5.496 |
| 25 | JX669465 | 10736 | 10.36 | 10.36 | 7.453 | 50 | KY474316 | 10647 | 10.13 | 10.13 | 6.654 |
| Serotype 3 |  |  |  |  |  | Serotype 4 |  |  |  |  |  |
| No. | Accession | $N$ | $I_{1} / 10^{10}$ | $I_{2} / 10^{10}$ | $I_{3} / 10^{6}$ | No. | Accession | $N$ | $I_{1} / 10^{10}$ | $I_{2} / 10^{10}$ | $I_{3} / 10^{6}$ |
| 51 | LT898451 | 10707 | 10.28 | 10.28 | 7.102 | 76 | KY474335 | 10642 | 10.07 | 10.07 | 12.55 |
| 52 | LT898452 | 10707 | 10.28 | 10.28 | 6.848 | 77 | KR922405 | 10164 | 8.769 | 8.768 | 10.13 |
| 53 | AB189125 | 10707 | 10.28 | 10.28 | 7.118 | 78 | KP188557 | 10425 | 9.467 | 9.466 | 13.51 |
| 54 | AB189126 | 10707 | 10.28 | 10.28 | 7.210 | 79 | KP188558 | 10572 | 9.873 | 9.872 | 12.08 |
| 55 | AB189127 | 10707 | 10.28 | 10.28 | 7.321 | 80 | KP188560 | 10576 | 9.884 | 9.883 | 12.11 |
| 56 | AB189128 | 10707 | 10.27 | 10.27 | 5.750 | 81 | KP188562 | 10649 | 10.09 | 10.09 | 12.11 |
| 57 | KY794787 | 10617 | 10.02 | 10.02 | 7.380 | 82 | KP188563 | 10654 | 10.11 | 10.10 | 12.12 |
| 58 | KY794788 | 10590 | 9.945 | 9.945 | 7.588 | 83 | KP188564 | 10618 | 10.00 | 10.00 | 13.80 |
| 59 | KY794789 | 10634 | 10.07 | 10.07 | 7.540 | 84 | KP188566 | 10426 | 9.471 | 9.470 | 11.56 |
| 60 | KY794790 | 10634 | 10.07 | 10.07 | 7.666 | 85 | JN983813 | 10649 | 10.09 | 10.09 | 12.73 |
| 61 | KY794786 | 10643 | 10.10 | 10.10 | 8.330 | 86 | MG272272 | 10652 | 10.10 | 10.10 | 12.83 |
| 62 | JX669489 | 10709 | 10.28 | 10.28 | 8.546 | 87 | MG272274 | 10652 | 10.10 | 10.10 | 11.96 |
| 63 | JX669498 | 10707 | 10.28 | 10.28 | 8.109 | 88 | KF041260 | 10652 | 10.10 | 10.09 | 15.06 |
| 64 | KF954946 | 10686 | 10.21 | 10.21 | 6.968 | 89 | FJ024476 | 10606 | 9.970 | 9.968 | 14.63 |
| 65 | KF954947 | 10675 | 10.18 | 10.18 | 7.054 | 90 | GQ868579 | 10593 | 9.932 | 9.931 | 14.05 |
| 66 | KF954948 | 10678 | 10.19 | 10.19 | 7.187 | 91 | GQ868580 | 10598 | 9.946 | 9.945 | 11.41 |
| 67 | KF954949 | 10677 | 10.19 | 10.19 | 7.076 | 92 | GQ868581 | 10592 | 9.929 | 9.929 | 11.41 |
| 68 | KF954945 | 10686 | 10.21 | 10.21 | 7.250 | 93 | GQ868582 | 10606 | 9.969 | 9.968 | 11.43 |
| 69 | JX669495 | 10707 | 10.28 | 10.27 | 7.997 | 94 | GQ868583 | 10551 | 9.814 | 9.813 | 11.03 |
| 70 | JX669499 | 10707 | 10.28 | 10.28 | 7.715 | 95 | GQ868584 | 10552 | 9.817 | 9.816 | 11.19 |
| 71 | KY863456 | 10707 | 10.28 | 10.28 | 7.414 | 96 | GQ868585 | 10606 | 9.970 | 9.969 | 14.51 |
| 72 | JX669492 | 10709 | 10.28 | 10.28 | 8.423 | 97 | JQ922560 | 10739 | 10.34 | 10.34 | 13.04 |
| 73 | JX669500 | 10707 | 10.28 | 10.28 | 8.180 | 98 | JQ922558 | 10626 | 10.03 | 10.02 | 12.56 |
| 74 | JX669497 | 10709 | 10.28 | 10.28 | 7.798 | 99 | JQ922559 | 10616 | 9.992 | 9.991 | 10.21 |
| 75 | JX669501 | 10707 | 10.28 | 10.28 | 8.953 | 100 | KP406806 | 10664 | 10.13 | 10.13 | 13.48 |

The left sub-distribution, with the smallest values of $\mu_{x}$, is formed by serotype 2 , the right sub-distribution is formed by serotype 4 , and the middle one by both 1 and 3 serotypes.


Figure 2. Distribution of $\mu_{x}$ for 2712 3D-dynamic graphs representing complete genome sequences of dengue virus.

Examples of 3D-dynamic graphs are shown in Fig. 3. The sequence of serotype 1 is there compared with other serotypes. Two sequences of the same serotype (top, left panel) are represented by similar graphs. The differences between sequences are graphically clearly visible: serotypes 1 and 2 (top, right panel) and serotypes 1 and 4 (bottom, right panel). The differences between the graphs representing serotypes 1 and 3 (bottom, left panel) are smaller.


Figure 3. 3D-dynamic graphs representing the complete genome sequences of dengue virus.


Figure 4. Two-dimensional classification diagrams.


Figure 5. Three-dimensional classification diagrams.

The classification of all 2712 sequences is clearly seen in two- and three-dimensional classification diagrams (Figs. 4 and 5, respectively). In the two-dimensional diagrams (Fig. 4) relations between pairs $D_{1}^{x}$ and $D_{3}^{z}$ (top, left panel), $I_{x x}$ and $I_{z z}$ (top, right panel), $I_{x z}$ and $I_{y y}$ (bottom, left panel), $I_{y z}$ and $I_{z z}$ (bottom, right panel) are displayed. Similarly, in the three-dimensional diagrams (Fig. 5) relations between triples of quantities $D_{1}^{x}-D_{1}^{z}-D_{1}^{y}$ (top panel), $I_{x x}-I_{y z}-I_{z z}$ (middle panel), and $\mu_{z}-\mu_{y}-\mu_{x}$ (bottom panel) are displayed. Different serotypes are represented by different symbols in the plots. The symbols are the same in all diagrams. Similarly as in the images presented in Figures 2 and 3 , also here, serotypes 1 and 3 cluster, and as a consequence we observe three separate groups: serotypes 1 and 3 (group 1), serotype 2 (group 2), and serotype 4 (group $3)$.

Our results (clustering to 3 groups) are consistent with the known antigenic relationships between DENV serotypes: DENV-1 and DENV-3 share common epitopes that are not present in DENV-2 or DENV-4 [43].

In the present work, we used in the calculations a large number of sequences (several thousands). As the result, the sequences have been correctly classified. As one can see, the descriptors characterizing the sequences are located in different parts of the classification diagrams (Figs. 4 and 5). Additionally, the 3D-dynamic graphs proved to be a simple intuitive tool which may easily be applied to a graphical comparison of the sequences (Fig. 3). Consequently, this approach may be used in both numerical and graphical studies of biomedical sciences. Therefore, the creation of a public server available online for the analysis of the sequences based on the DRBS-based approaches is an important aim of our future work.

Summarizing, the present results supply another confirmation of the usefulness of DRBS-based approaches to the characterization of the genome sequences of viruses. In particular, as one can see, the sensitivity of the methods is high and the set of descriptors is adequate for the detection of different serotypes of a virus.

## References

[1] D. Bielińska-Waż, T. Clark, P. Waż, W. Nowak, A. Nandy, 2D-dynamic representation of DNA sequences, Chem. Phys. Lett. 442 (2007) 140-144.
[2] D. Bielińska-Waż, W. Nowak, P. Waż, A. Nandy, T. Clark, Distribution moments of 2D-graphs as descriptors of DNA sequences, Chem. Phys. Lett. 443 (2007) 408-413.
[3] D. Bielińska-Waż, P. Waż, T. Clark, Similarity studies of DNA sequences using genetic methods, Chem. Phys. Lett. 445 (2007) 68-73.
[4] P. Waż, D. Bielińska-Waż, 3D-dynamic representation of DNA sequences, J. Mol. Model. 20 (2014) \#2141.
[5] A. Czerniecka, D. Bielińska-Waż, P. Waż, T. Clark, 20D-dynamic representation of protein sequences, Genomics 107 (2016) 16-23.
[6] E. Hamori, J. Ruskin, H Curves, a novel method of representation of nucleotide series especially suited for long DNA sequences, J. Biol. Chem. 258 (1983) 1318-1327.
[7] M. A. Gates, Simpler DNA sequence representations, Nature 316 (1985) 219-219.
[8] A. Nandy, A new graphical representation and analysis of DNA sequence structure. I: Methodology and application to globin genes, Curr. Sci. 66 (1994) 309-314.
[9] P. M. Leong, S. Morgenthaler, Random walk and gap plots of DNA sequences, Comput. Appl. Biosci. 11 (1995) 503-507.
[10] M. Randić, M. Vračko, A. Nandy, S. Basak, On 3-D graphical representation of DNA primary sequences and their numerical characterization, J. Chem. Inf. Comp. Sci. 40 (2000) 1235-1244.
[11] C. Li, J. Wang, On a 3-D representation of DNA primary sequences, Comb. Chem. High Throughput Screen. 7 (2004) 23-27.
[12] Y. Yao, X. Nan, T. Wang, Analysis of similarity/dissimilarity of DNA sequences based on a 3-D graphical representation, Chem. Phys. Lett. 411 (2005) 248-255.
[13] Y. Yang, Y. Zhang, M. Jia, C. Li, L. Meng, Non-degenerate graphical representation of DNA sequences and its applications to phylogenetic analysis, Comb. Chem. High Throughput Screen. 16 (2013) 585-589.
[14] C. Yuan, B. Liao, T. Wang, New 3D graphical representation of DNA sequences and their numerical characterization, Chem. Phys. Lett. 379 (2003) 412-417.
[15] C. T. Zhang, R. Zhang, H. Y. Ou, The Z curve database: a graphic representation of genome sequences, Bioinformatics 19 (2003) 593-599.
[16] B. Liao, T. Wang, 3-D graphical representation of DNA sequences and their numerical characterization, J. Mol. Struct. (Theochem) 681 (2004) 209-212.
[17] B. Liao, T. Wang, Analysis of similarity/dissimilarity of DNA sequences based on 3-D graphical representation, Chem. Phys. Lett. 388 (2004) 195-200.
[18] B. Liao, Y. Zhang, K. Ding, T. J. Wang, Analysis of similarity/dissimilarity of DNA sequences based on a condensed curve representation, J. Mol. Struct. (Theochem) 717 (2005) 199-203.
[19] Z. Cao, B. Liao, R. Li, A group of 3D graphical representation of DNA sequences based on dual nucleotides, Int. J. Quantum Chem. 108 (2008) 1485-1490.
[20] I. Pesek, J. Žerovnik, A numerical characterization of modified Hamori curve representation of DNA sequences, MATCH Commun. Math. Comput. Chem. 60 (2008) 301-312.
[21] W. Chen, B. Liao, X. Xiang, W. Zhu, An improved binary representation of DNA sequences and its applications, MATCH Commun. Math. Comput. Chem. 61 (2009) 767-780.
[22] Z. Cao, R. Li, W. Chen, A 3D graphical representation of DNA sequence based on numerical coding method, Int. J. Quantum Chem. 110 (2010) 975-985.
[23] J. F. Yu, J. H. Wang, X. Sun, Analysis of similarities/dissimilarities of DNA sequences based on a novel graphical representation, MATCH Commun. Math. Comput. Chem. 63 (2010) 493-512.
[24] Y. Li, Y. Qin, X. Zheng, Y. Zhang, Three-unit semicircles curve: A compact 3D graphical representation of DNA sequences based on classifications of nucleotides, Int. J. Quantum Chem. 112 (2012) 2330-2335.
[25] N. Jafarzadeh, A. Iranmanesh, C-curve: A novel 3D graphical representation of DNA sequence based on codons, Math. Biosci. 241 (2013) 217-224.
[26] Z. Y. Mo, W. Zhu, Y. Sun, Q. L. Xiang, M. Zheng, M. Chen, Z. J. Li, One novel representation of DNA sequence based on the global and local position information, Sci. Rep. 8 (2018) \#7592.
[27] D. Bielińska-Waż, Graphical and numerical representations of DNA sequences: Statistical aspects of similarity, J. Math. Chem. 49 (2011) 2345-2407.
[28] M. Randić, M. Novič, D. Plavšić, Milestones in graphical bioinformatics, Int. J. Quantum Chem. 113 (2013) 2413-2446.
[29] M. Mahmoodi-Reihani, F. Abbasitabar, V. Zare-Shahabadi, A novel graphical representation and similarity analysis of protein sequences based on physicochemical properties, Physica A 510 (2018) 477-485.
[30] C. Y. Wu, R. Gao, Y. De Marinis, Y. Zhang, A novel model for protein sequence similarity analysis based on spectral radius, J. Theor. Biol. 446 (2018) 61-70.
[31] M. Novič, M. Randić, Representation of proteins as walks in 20-D space, SAR QSAR Environ. Res. 19 (2008) 317-337.
[32] P. Waż, D. Bielińska-Waż, A. Nandy, Descriptors of 2D-dynamic graphs as a classification tool of DNA sequences, J. Math. Chem. 52 (2014) 132-140.
[33] D. Panas, P. Waż, D. Bielińska-Waż, A. Nandy, S. Basak, 2D-dynamic representation of DNA/RNA sequences as a characterization tool of the zika virus genome, MATCH Commun. Math. Comput. Chem. 77 (2017) 321-332.
[34] P. Waż, D. Bielińska-Waż, Non-standard similarity/dissimilarity analysis of DNA, Genomics 104 (2014) 464-471.
[35] Y. H. Yao, Q. Dai, Ch. Li, P. A. He, X. Y. Nan, Y. Z. Zhang, Analysis of similarity/dissimilarity of protein sequences, Proteins Struct. Funct. Bioinf. 73 (2008) 864-871.
[36] Y. H. Yao, S. Yan, J. Han, Q. Dai, P. A. He, A novel descriptor of protein sequences and its application, J. Theor. Biol. 347 (2014) 109-117.
[37] W. Hou, Q. Pan, M. He, A new graphical representation of protein sequences and its applications, Physica A 444 (2016) 996-1002.
[38] V. Aram, A. Iranmanesh, 3D-dynamic representation of DNA sequences, MATCH Commun. Math. Comput. Chem. 67 (2012) 809-816.
[39] D. Panas, P. Waż, D. Bielińska-Waż, A. Nandy, S. Basak, An application of the 2D-dynamic representation of DNA/RNA sequences to the prediction of influenza A virus subtypes, MATCH Commun. Math. Comput. Chem. 80 (2018) 295-310.
[40] M. G. Guzman, S. B. Halstead, H. Artsob, P. Buchy, J. Farrar, D. J. Gubler, E. Hunsperger, A. Kroeger, H. S. Margolis, E. Martínez, M. B. Nathan, J. L. Pelegrino, C. Simmons, S. Yoksan, R. W. Peeling, Dengue: a continuing global threat, Nat. Rev. Microbiol. 8 (2010) 7-16.
[41] T. K. Mackey, B. A. Liang, Threats from emerging and re-emerging neglected tropical diseases (NTDs), Infect. Ecol. Epidemiol. 2 (2012) \#18667.
[42] S. Bhatt, P. W. Gething, O. J. Brady, J. P. Messina, A. W. Farlow, C. L. Moyes, J. M. Drake, J. S. Brownstein, A. G. Hoen, O. Sankoh, M. F. Myers, D. B. George, T. Jaenisch, G. R. Wint, C. P. Simmons, T. W. Scott, J. J. Farrar, S. I. Hay, The global distribution and burden of dengue, Nature 496 (2013) 504-507.
[43] R. de Alwis, K. L. Williams, M. A. Schmid, C. Y. Lai, B. Patel, S. A. Smith, J. E. Crowe, W. K. Wang, E. Harris, A. M. de Silva, Dengue viruses are enhanced by distinct populations of serotype cross-reactive antibodies in human immune sera, PLoS Pathog 10 (2014) \#e1004386.


[^0]:    ${ }^{1}$ In 2013 the discovery of the fifth serotype was reported: Normile, D. (2013) Tropical medicine. Surprising new dengue virus throws a spanner in disease control efforts, Science. 342: 415. DOI: 10.1126/science.342.6157.415.

