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# BOOK REVIEW

## Matrix Inequalities for Iterative Systems

by

Hanjo Täubig

CRC Press, Boca Raton, 2017, XV+202 pp., ISBN 978-1-498777773, 117£

In spite of its title, this book is mainly concerned with the number of walks in graphs and digraphs, and the related matrix–power–based and vertex–degree–based mathematical results. In the *Preface*, the author explains that this book is based on his habilitation thesis (defended at the München Technical University in 2015), and therefore the text is written in such a way that it can be "*used by the reader in the easiest possible way*". As a consequence of such an approach, the material presented in the book is easy to follow, and will be useful and suitable for those who have little previous knowledge of the mathematical research done in this area. Mathematical chemists will particularly benefit from reading this book.

The book is divided into four parts:

**1** *Introduction*

**2** *Undirected Graphs*

**3** *Directed Graphs*

**4** *Applications*

In Part **1** the mathematical apparatus used is briefly outlined, among which are matrices and vectors, graphs, number of walks in graphs, entry sums and matrix powers, diagonalization of matrices and spectral decomposition.

In the chapter *Motivations* are listed and briefly presented the scientific disciplines in which walk counts and related concepts play role. These include combinatorics,

graph theory, automata and formal languages, random walks and Markov chains, population genetics and evolution theory, theoretical chemistry. In fact, a large amount of material presented in the book comes from, and is strongly related with, chemical graph theory.

Part **2** is concerned with undirected graphs and the Hermitian matrices associated with them, especially with the adjacency matrix. It consists of two chapters: *General Results* and *Restricted Graph Classes*. The former outlines all relevant results on and related to the number of walks in graphs, most of which being inequalities. As characteristic examples, we mention here Theorem 4.2: $w_{a+b} \leq \sqrt{w_{2a}\, w_{2b}}$, Theorem 4.13: $w_k \geq n \left(\frac{2m}{n}\right)^k$, and Theorem 4.32: $w_k \leq \sum_v d(v)^k$, valid for undirected $(n, m)$-graphs where $w_k$ is the number of walks of length $k$, and $d(v)$ is the degree of the vertex $v$. In this chapter are collected numerous results on first and second Zagreb indices, as well as on their multiplicative versions, but are mentioned also the harmonic and hyper–Zagreb indices.

The chapter *Restricted Graph Classes* begins with the examination of the inequality $M_1/n \leq M_2/m$, where $M_1$ is the first and $M_2$ the second Zagreb index. Needless to say, that this part of Täubig's book will be most attractive to the scholars interested in topological indices. In this chapter, a number of other results are outlined, valid for special classes of graphs and walks of special length.

As its title says, Part **3** is concerned with directed graphs and nonsymmetric matrices associated with them.

Part **4** is mainly concerned with bounds for the largest eigenvalue of the adjacency matrix and its relation to walk counts and vertex degrees. However, a short text (on five pages) is devoted to eigenvalue moments and graph energy. It is somewhat mysterious why spectral moments, which are just the number of closed walks (and which have been extensively studied in the past, including chemical applications) have not been considered in greater detail.

The book ends with an extensive and impressive *Bibliography* containing around 400 references, and a short *Index*

In summary: The book *Matrix Inequalities for Iterative Systems* is a valuable, state-of-the-art treatise of walk–related research in graph theory and linear algebra. It will be useful for both experts and beginners, including researchers in mathematical chemistry.

Ivan Gutman