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Correcting the Number of Borderenergetic Graphs of Order 10

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Abstract

In the paper "A Computer Search for the Borderenergetic Graphs of Order 10", *MATCH Commun. Math. Comput. Chem.* **74** (2015) 321–332, 47 non-isomorphic non-complete borderenergetic graphs are reported. By computer search, we find that their total number is 49. The two missing graphs are now determined. In addition, we establish the number of borderenergetic graphs on 11 vertices, and provide their classification.

1 Two new borderenergetic graphs on 10 vertices

A graph of order n is said to be *borderenergetic* [1] if its energy is equal to the energy of the complete graph of order n, i.e., if it is equal to $2n-2$. For more notation consult [2].

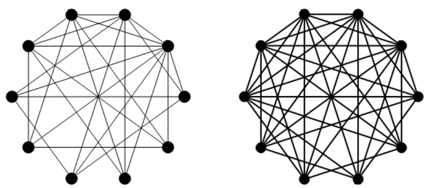
A graph G is said to be *integral* if all its eigenvalues are integers. We say that a polynomial with integer coefficients is *quadratic* (resp. *cubic*, *quartic*) if the highest degree of all its irreducible factors in rational field is two (resp. three, four). In accordance with this, a borderenergetic graph is said to be a *quadratic* (resp. *cubic*, *quartic*) if its characteristic polynomial is quadratic (resp. cubic, quartic).

In [2], it was reported that there are exactly 47 non-isomorphic non-complete borderenergetic graphs on 10 vertices. Based on the software package *nauty*, developed by McKay [3] and the *The GNU MPFR library* [4], we developed a program to search for borderenergetic graphs on 10 and 11 vertices. As a result, we found two additional borderenergetic graphs on 10 vertices and all borderenergetic graphs on 11 vertices.

The adjacency matrices of the missing two graphs P_1 and P_2 are as follows:

$$\mathbf{A}(P_1) = \begin{bmatrix} 0&0&0&1&0&1&0&0&1&1 \\ 0&0&0&0&1&0&1&1&0&1 \\ 0&0&0&0&0&0&1&1&1&1 \\ 1&0&0&0&0&1&0&0&1&1 \\ 0&1&0&0&0&0&1&1&0&1 \\ 1&0&0&1&0&0&0&0&1&1 \\ 0&1&1&0&1&0&0&1&1&1 \\ 0&1&1&0&1&0&1&0&1&1 \\ 1&0&1&1&0&1&1&1&0&1 \\ 1&1&1&1&1&1&1&1&1&0 \end{bmatrix} \quad \text{and} \quad \mathbf{A}(P_2) = \begin{bmatrix} 0&0&1&0&1&1&1&0&1&1 \\ 0&0&0&1&0&1&1&1&1&1 \\ 1&0&0&0&1&1&0&1&1&1 \\ 0&1&0&0&0&1&1&1&1&1 \\ 1&0&1&0&0&0&1&1&1&1 \\ 1&1&1&1&0&0&1&1&1&1 \\ 1&1&0&1&1&1&0&1&1&1 \\ 0&1&1&1&1&1&1&0&1&1 \\ 1&1&1&1&1&1&1&1&0&1 \\ 1&1&1&1&1&1&1&1&1&0 \end{bmatrix}.$$

The respective graphs are depicted in Figure 1.



(a) (b)

Figure 1: (a) The graph P_1, (b) The graph P_2

The characteristic polynomial and the spectrum of P_1 are

$$\phi(P_1, \lambda) = \lambda^{10} - 26\lambda^8 - 58\lambda^7 + 70\lambda^6 + 392\lambda^5 + 490\lambda^4 + 134\lambda^3 - 203\lambda^2 - 180\lambda - 44$$

and

$$Sp(P_1) = \{x_1, x_2, x_3, -1, -1, -1, -1, -1, -2, -2\}$$

where x_1, x_2, x_3 are the three distinct roots of the equation

$$x^3 - 9x^2 + 21x - 11 = 0\,.$$

The characteristic polynomial and the spectrum of P_2 are

$$\phi(P_2, \lambda) = \lambda^{10} - 36\lambda^8 - 126\lambda^7 - 105\lambda^6 + 198\lambda^5 + 476\lambda^4 + 360\lambda^3 + 96\lambda^2$$

and

$$Sp(P_2) = \left\{ \frac{9+\sqrt{33}}{2}, \frac{9-\sqrt{33}}{2}, 0, 0, -1, -1, -1, -2, -2, -2 \right\}.$$

It can be verified that neither P_1 nor P_2 is isomorphic to any graph reported in [2], and that their energies are both equal to 18.

2 Borderenergetic graphs on 11 vertices

Among the graphs on 11 vertices, we find that there are no borderenergetic graphs with less than or equal to 24 edges. There are no non-complete borderenergetic graphs with $|E|$ edges, where $|E| \in \{42, 46, 48, 50, 51, 52, 53, 54\}$. We found that there are four types of borderenergetic graphs on 11 vertices, and these are integral, quadratic, cubic, and quartic. More precisely, there are 24, 78, 51, and 5 integral, quadratic, cubic, and quartic borderenergetic graphs on 11 vertices, respectively.

The statistics of borderenergetic graphs on 11 vertices with different number of edges is summarized in Table 1. The column $|E|$ indicates the number of edges of the set of graphs, e.g., there are 2, 2, 3, 0 integral, quadratic, cubic, and quartic borderenergetic graphs on 11 vertices with 26 edges.

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[4] GNU MPFR Library, `http://www.mpfr.org/mpfr-current/mpfr.html`.

$\|E\|$	integral	quadratic	cubic	quartic	total
25	2	0	1	0	3
26	2	2	3	0	7
27	0	2	3	0	5
28	0	14	1	0	15
29	0	4	10	0	14
30	2	5	1	2	10
31	5	4	7	0	16
32	1	4	4	0	9
33	0	8	0	3	11
34	0	4	2	0	6
35	3	2	0	0	5
36	0	5	1	0	6
37	4	3	5	0	12
38	0	3	7	0	10
39	0	6	0	0	6
40	0	8	0	0	8
41	0	2	4	0	6
43	2	0	0	0	2
44	1	0	2	0	3
45	0	1	0	0	1
47	0	1	0	0	1
49	2	0	0	0	2
total	24	78	51	5	158

Table 1. Statistics of non-complete borderenergetic graphs on 11 vertices with different number of edges, $|E|$.