

## On Wiener Index of Common Neighborhood Graphs

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### Abstract

If  $G$  is a simple graph, then  $\text{con}(G)$ , the common neighborhood graph of  $G$ , has the same vertex set as  $G$ , and two vertices of  $\text{con}(G)$  are adjacent if they have a common neighbor in  $G$ . We show that for any bipartite graph  $G$  the Wiener index (i.e., sum of distances between all pairs of vertices) of  $\text{con}(G)$  is always smaller than the Wiener index of  $G$ . For general graphs, however, the Wiener index of common neighbor graphs can be greater. This fact is surprising, since we also show that the diameter of  $\text{con}(G)$  is at most the diameter of  $G$ . We present constructions of two infinite classes of graphs, chemical and unicyclic graphs, which have this property.

# 1 Introduction

Let  $G$  be a simple graph with vertex set  $V(G)$ . The *common neighborhood graph* (or, shorter: *congraph*) of  $G$ , denoted by  $\text{con}(G)$ , is the graph with  $V(\text{con}(G)) = V(G)$ , in which two vertices are adjacent if they have a common neighbor in  $G$ . The motivation for the consideration of congraphs came from the theory of graph energy [2, 18]. In [1, 3] some basic properties of congraphs have been established. Furthermore, common neighbor graphs are implicitly used in the graph coloring theory, Namely, an injective coloring of a graph  $G$  is precisely a proper vertex coloring of  $\text{con}(G)$  (see e.g. [5, 8, 12, 21] for some results on this topic).

One of the obvious questions that can be asked in connection with congraphs is the following: If  $\text{Inv}(G)$  is a numerical invariant of the graph  $G$ , what can be said about  $\text{Inv}(\text{con}(G))$ ? In particular, how  $\text{Inv}(G)$  and  $\text{Inv}(\text{con}(G))$  are related?

In this paper, bearing in mind the extensive recent research activity in this area [10, 11, 13–17, 19, 20, 22, 23], we focus our attention to the Wiener index. For reasons outlined in the subsequent section, it looks plausible (or even, “self-evident”) state the following:

**Conjecture 1.** *Let  $G$  be a graph. Then*

$$W(\text{con}(G)) \leq W(G).$$

*and the equality holds if and only if  $\text{con}(G)$  is isomorphic to  $G$ .*

In what follows, we show that this conjecture is false. Moreover, for every integer  $d$  we construct infinite families of graphs, such that the relation  $W(\text{con}(G)) - W(G) = d$  holds. Furthermore, we also found a construction of an infinite family of chemical graphs for which the common neighborhood graphs have greater value than the original graphs.

At this point we need to slightly re-define the Wiener index.

Let  $G$  be a connected graph and  $u, v \in V(G)$ . The *distance* between the vertices  $u$  and  $v$ , denoted by  $d(u, v|G)$ , is defined as the length of (= number of edges in) a shortest path connecting  $u$  and  $v$  [4]. Then, the *Wiener index* is defined as [6, 7, 9]

$$W = W(G) = \sum_{\{u,v\} \in V(G)} d(u, v|G). \quad (1)$$

For evident reasons, this definition cannot be applied to non-connected graphs.

Let  $d(G, k)$  be the number of vertex pairs of the graph  $G$  whose distance is equal to  $k$ . Then the Wiener index satisfies the relation

$$W = W(G) = \sum_{k \geq 1} k d(G, k) . \tag{2}$$

However, contrary to formula (1), the right-hand side of formula (2) is applicable also to non-connected graphs. Furthermore, if the Wiener index is defined via Eq. (2), and if the graph  $G$  consists of disconnected components  $G_1$  and  $G_2$ , then

$$W(G) = W(G_1) + W(G_2) .$$

In what follows we understand that the Wiener index of a disconnected graph is calculated according to Eq. (2). When speaking of the Wiener indices of congruences, this is important, because of the following result:

**Theorem 2.** [1] *Let  $G$  be a connected bipartite graph, so that its vertex set is partitioned as  $V(G) = V_a \cup V_b$ . Then  $\text{con}(G)$  consists of two disconnected components  $G_a$  and  $G_b$ , whose vertex sets are  $V_a$  and  $V_b$ , respectively. Both graphs  $G_a$  and  $G_b$  are connected.*

By means of Eq. (2), we now have:

**Corollary 3.** *Using the same notation as in Theorem 2,*

$$W(\text{con}(G)) = W(G_a) + W(G_b) .$$

## 2 On Wiener Index of Common Neighborhood Graphs of Bipartite Graphs

Throughout this section we assume that  $G$  is a connected bipartite graph, and that the notation used in Theorem 2 is applicable.

**Lemma 4.** *If  $u, v \in V_a$ , then  $d(u, v|G_a) = 2d(u, v|G)$ . Analogously, if  $u, v \in V_b$ , then  $d(u, v|G_b) = 2d(u, v|G)$ .*

*Proof.* Suppose that  $u, v \in V_a$ . Let a shortest path between the vertices  $u$  and  $v$  in the graph  $G$  embraces the vertices  $u, u_1, v_1, u_2, v_2, \dots, u_{k-1}, v_{k-1}, u_k, v$  (in that order). Then, evidently,  $v_1, v_2, \dots, v_{k-1} \in V_a$  and  $u_1, u_2, \dots, u_k \in V_b$ , and  $d(u, v|G) = 2k$ .

Now, according to the way in which the common neighborhood graphs are constructed,  $u, v_1, v_2, \dots, v_{k-1}, v$  is a shortest path between the vertices  $u$  and  $v$  in  $\text{con}(G)$ . Consequently,  $d(u, v|\text{con}(G)) = k$ . □

Lemma 4 implies the following:

**Theorem 5.** *Using the same notation as in Theorem 2, the Wiener indices of a bipartite graph and its congraph are related as:*

$$W(G) = 2W(\text{con}(G)) + \sum_{u \in V_a; v \in V_b} d(u, v|G) . \tag{3}$$

*Proof.*

$$\begin{aligned} W(G) &= \left[ \sum_{\{u,v\} \in V_a} + \sum_{\{u,v\} \in V_b} + \sum_{u \in V_a; v \in V_b} \right] d(u, v|G) \\ &= 2 \sum_{\{u,v\} \in V_a} d(u, v|G_a) + 2 \sum_{\{u,v\} \in V_b} d(u, v|G_b) + \sum_{u \in V_a; v \in V_b} d(u, v|G) \\ &= 2W(G_a) + 2W(G_b) + \sum_{u \in V_a; v \in V_b} d(u, v|G) \end{aligned}$$

and Eq. (3) follows from Corollary 3. □

Bearing in mind that

$$\sum_{u \in V_a; v \in V_b} d(u, v|G) \geq |V_a| |V_b|$$

we obtain:

**Corollary 6.** *If  $G$  is a connected bipartite graph with bipartition  $V(G) = V_a \cup V_b$ , then*

$$W(G) \geq 2W(\text{con}(G)) + |V_a| |V_b|$$

*with equality if and only if  $G$  is the complete bipartite graph  $K_{a,b}$ .*

**Corollary 7.** *If  $G$  is a bipartite (not necessarily connected) graph with  $m$  edges, then*

$$W(G) \geq 2W(\text{con}(G)) + m$$

*with equality if and only if every component of  $G$  is a complete bipartite graph (which can also be an isolated vertex).*

### 3 On Wiener Index of Common Neighborhood Graphs of Non-bipartite Graphs

From the results in the preceding section, we see that in the case of bipartite graphs, the Wiener index of the congraph is necessarily (much) smaller than the Wiener index of original graph. There are non-bipartite graphs for which  $G \cong \text{con}(G)$ , namely the complete graphs and the odd cycles [1]. For these graphs,  $W(G) = W(\text{con}(G))$ . Bearing this in mind, it appears to be plausible to expect that Conjecture 1 holds. This expectation is further corroborated by Corollary 9.

The fact that the conjecture is false is surprising, since we now show that the diameter of  $G$ , denoted by  $\text{diam}(G)$ , is always at least the diameter of  $\text{con}(G)$ . By  $d_e(u, v)$  we denote the length of a shortest walk of even length between the vertices  $u$  and  $v$ . Notice that  $d_e(u, v)$  in  $G$  is precisely  $d(u, v)$  in  $\text{con}(G)$ .

**Proposition 8.** *Let  $G$  be a connected non-bipartite graph. Then, for every pair of vertices  $u, v \in V(G)$  we have*

$$d_e(u, v) \leq 2 \text{diam}(G).$$

*Proof.* Let  $u, v \in V(G)$  and let  $u = a_0, a_1, \dots, a_k = v$  be the shortest walk of even length between  $u$  and  $v$  (since  $G$  is connected and not bipartite such a walk always exists). If  $d(u, v)$  is even, then  $d_e(u, v) = d(u, v) \leq \text{diam}(G)$ . Thus, suppose that  $d(u, v) < d_e(u, v) = k$ . Let  $j$  be the smallest index such that  $d(u, a_j) < j$ . If  $d(u, a_j) \leq j - 3$ , then  $d(u, a_{j-1}) \leq j - 2$ , which contradicts the choice of  $j$ . On the other hand, if  $d(u, a_j) = j - 2$  then there is an even walk from  $u$  to  $v$  via  $a_j$  of length  $k - 2$ , a contradiction. Hence,  $d(u, a_j) = j - 1$ .

Now, let  $i$  be the biggest index such that  $d(a_i, v) < k - i$ . By analogous argument we obtain that  $d(a_i, v) = k - i - 1$ . In case when  $j \leq i$  then there is an even walk from  $u$  to  $v$  of length  $k - 2$ , a contradiction. Hence,  $i < j$ . Therefore,  $d(u, a_j) = j - 1$  and  $d(a_j, v) = k - j$ , which means that  $d_e(u, v) = d(u, a_j) + 1 + d(a_j, v) \leq 2\text{diam}(G) + 1$ . However, since  $d_e(u, v)$  is even, we infer that  $d_e(u, v) \leq 2\text{diam}(G)$ .  $\square$

Observe that if  $u = a_0, a_1, \dots, a_k = v$  is a shortest walk of even length, then for every even  $i$  the triple  $a_i, a_{i+1}, a_{i+2}$  forms a path of length 2. This, together with Proposition 8, implies the following.

**Corollary 9.** *Let  $G$  be a connected non-bipartite graph. Then*

$$\text{diam}(\text{con}(G)) \leq \text{diam}(G).$$

Let  $G$  be a connected non-bipartite graph. By Corollary 9, we have that the diameter of  $\text{con}(G)$  is at most equal to the diameter of  $G$ . Moreover,  $\text{con}(G)$  has at least as many edges as  $G$ . Therefore, one would expect that  $W(\text{con}(G)) \leq W(G)$ .

In many cases, this inequality is true, but there are also exceptional graphs violating it. In the sequel we describe a class of chemical graphs (i.e., graphs with maximum degree at most 4)  $\{G_k\}_{k=0}^\infty$ , for which  $\lim_{k \rightarrow \infty} [W(G_k) - W(\text{con}(G_k))] = -\infty$ .

Let  $k \geq 0$ . Take a graph consisting of a triangle, to which there is attached (by one of its endpoints) a path of length  $k + 2$ . This graph has exactly one vertex of degree 3, one vertex of degree 1, and all the other vertices have degree 2. Attach to each vertex of degree 2, with the exception of the three neighbors of the vertex of degree 3, two pendent edges, and denote the resulting graph by  $G_k$ , see Fig. 1.

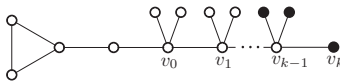


Figure 1: The graph  $G_k$ .

Finally, denote  $\Delta_k = W(G_k) - W(\text{con}(G_k))$ . Then we have:

**Theorem 10.** *For every integer  $i \geq 0$  it holds*

$$\Delta_{2i} = -3i^2 + 9i + 2 \quad \text{and} \quad \Delta_{2i+1} = -3i^2 + 6i + 11.$$

*Proof.* It is easy to compute that  $W(G_0) = 17$  and  $W(\text{con}(G_0)) = 15$  (in Fig. 2 for every vertex the sum of distances to all other vertices is determined), and so  $\Delta_0 = 2$ .

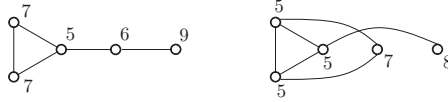


Figure 2: The graphs  $G_0$  and  $\text{con}(G_0)$  together with the sums of all distances for every vertex.

Now, we construct  $G_k$  from  $G_{k-1}$  by attaching three new vertices to the vertex  $v_{k-1}$ . These new vertices are depicted black in Fig. 1. For every other (i.e., white) vertex we compute its contribution to the difference  $\Delta_k - \Delta_{k-1}$  for every black vertex. Since this contribution is the same for every black vertex, we denote it in Fig. 3. For

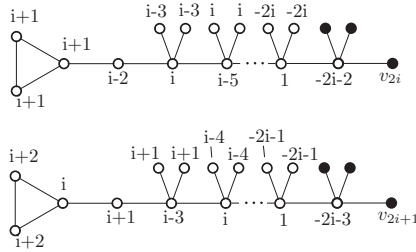


Figure 3: The contributions of the vertices to the difference for every black vertex in graphs  $G_{2i}$  and  $G_{2i+1}$ .

instance, in  $G_1$  the vertex of degree 4 has distance 1 to a black vertex in  $G_1$  and 4 to a black vertex in  $\text{con}(G_1)$ , so its contribution to  $\Delta_1 - \Delta_0$  is  $-3$  for every black vertex.

Denote by  $\delta_k$  the sum of the contributions of all white vertices to a black vertex and let  $\Delta'_k = \Delta_k - \Delta_{k-1}$ . Then,  $\Delta'_k = 3\delta_k + 3$ , since there are three black vertices in  $G_k$  and each pair of them contributes additional 1 to  $\Delta'_k$ .

In what follows, we compute  $\delta_k$ . First, we add the contributions of the vertices of the triangle and the contributions of the vertices of degree at least 2 at even distance from a black vertex. Then, we add the contributions of the remaining vertices of

degree at least 2 and finally the contributions of the pendent vertices. Hence,

$$\begin{aligned} \delta_{2i} &= 3(i+1) + \sum_{j=1}^i j + \sum_{j=0}^i (1+2j - (2(i+1) + 1 - j)) \\ &+ 2 \left( \sum_{j=2}^i j + \sum_{j=1}^i (2j+1 - (2i+4-j)) \right) = -i - 1 \end{aligned}$$

and

$$\begin{aligned} \delta_{2i+1} &= 3i+4 + \sum_{j=1}^{i+1} j + \sum_{j=0}^i (1+2j - (2i+4-j)) \\ &+ 2 \left( \sum_{j=2}^{i+1} j + \sum_{j=1}^i (2j+1 - (2i+5-j)) \right) = -i + 2 . \end{aligned}$$

Therefore  $\Delta'_{2i} = 3\delta_{2i} + 3 = -3i$  and  $\Delta'_{2i+1} = 3\delta_{2i+1} + 3 = -3i + 9$ . Thus we infer

$$\Delta_{2i} = 2 + \sum_{j=0}^{i-1} (-3j+9) + \sum_{j=1}^i (-3j) = -3i^2 + 9i + 2$$

and

$$\Delta_{2i+1} = 2 + \sum_{j=0}^i (-3j+9) + \sum_{j=1}^i (-3j) = -3i^2 + 6i + 11 .$$

□

For non-chemical graphs we can prove even more. In particular, for every integer  $\Delta$ , we show that there are infinitely many graphs  $G$  such that  $W(G) - W(\text{con}(G)) = \Delta$ .

Denote by  $G$  the graph consisting of one triangle to which a path of length 4 is attached (by one of its endvertices). Denote the three vertices of degree 2 of the attached path by  $a$ ,  $b$ , and  $c$ , respectively. Now attach to  $a$ ,  $b$ , and  $c$  exactly  $q$ ,  $r$ , and  $s$  pendent edges, respectively, and denote the resulting graph by  $G_{q,r,s}$ .

Let  $\Delta_{q,r,s} = W(G_{q,r,s}) - W(\text{con}(G_{q,r,s}))$ . In the following theorem we show that for every integer  $\Delta$  there is infinitely many graphs  $G_{q,r,s}$  such that  $\Delta_{q,r,s} = \Delta$ .

**Theorem 11.** *For every integer  $\Delta$ , there are infinitely many graphs  $G_{q,r,s}$  such that  $\Delta_{q,r,s} = \Delta$ . In particular, for every  $\ell = 3z - \Delta$  such that  $s = 4\ell$ ,  $r = 4\ell^2 - 2 - (5\ell + \Delta)/3$ , and  $q = r - s + 2\ell$  are all positive, we have  $\Delta_{q,r,s} = \Delta$ .*



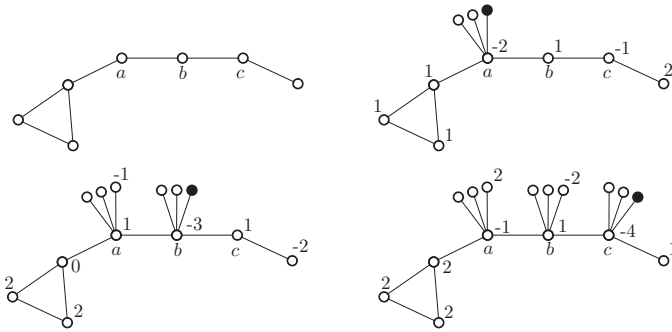


Figure 4: The graphs  $G_{0,0,0}$ ,  $G_{3,0,0}$ ,  $G_{3,3,0}$ , and  $G_{3,3,3}$ .

*Proof.* It is easy to compute that  $W(G_{0,0,0}) = 51$  and  $W(\text{con}(G_{0,0,0})) = 45$ , and so  $\Delta_{0,0,0} = 6$ . Now we construct  $G_{q,r,s}$  from  $G_{0,0,0}$  by attaching  $q$  pendent vertices to  $a$ ,  $r$  pendent vertices to  $b$ , and finally  $s$  pendent vertices to  $c$ .

Analogously as in the previous proof, we compute the contribution of the vertices to  $\Delta_{q,r,s}$  for every added vertex, in particular, we compute it for the vertices of the same type, e.g. all pendent vertices at vertex  $a$  are of the same type, and then multiply the values with the number of the vertices of that type. The contributions (for each of the black vertices) are depicted in Fig. 4.

Since the sums of contributions when attaching pendent vertices to  $a$ ,  $b$ , and  $c$  are  $3$ ,  $1 - q$ , and  $3 + 2q - 2r$ , respectively, and since each pair of attached vertices contributes by  $1$  (observe that the distance between them is  $2$ ), we have that

$$\begin{aligned} \Delta_{q,r,s} &= 6 + 3q + \binom{q}{2} + (1-q)r + \binom{r}{2} + (3+2q-2r)s + \binom{s}{2} \\ &= \frac{1}{2} (q^2 + 5q + r^2 + r + s^2 + 5s - 2qr + 4qs - 4rs + 12). \end{aligned}$$

Our aim is to find infinitely many solutions of  $\Delta_{q,r,s} = \Delta$ , i.e.,

$$q^2 + 5q + r^2 + r + s^2 + 5s - 2qr + 4qs - 4rs + 12 - 2\Delta = 0. \quad (4)$$

After multiplying by  $4$ , Eq. (4) can be rewritten in the form

$$(2q - 2r + 4s + 5)^2 - (2s + 5)^2 = 8s^2 - 24r - 48 + 8\Delta$$

that is  $x^2 - y^2 = a$ . We choose the solution  $x + y = (8s^2 - 24r - 48 + 8\Delta)/2k$  and  $x - y = 2k$ . Obviously, for such a choice it holds that  $2k|(8s^2 - 24r - 48 + 8\Delta)$ . Then

$$x = \frac{2s^2 - 6r - 12 + 2\Delta + k^2}{k} \quad \text{and} \quad y = \frac{2s^2 - 6r - 12 + 2\Delta - k^2}{k}.$$

By(5)  $y = 2s + 5$ , so we obtain

$$r = \frac{2s^2 - 12 + 2\Delta - k^2 - 2sk - 5k}{6} \tag{5}$$

and since  $x = 2q - 2r + 4s + 5$ , we have that  $\frac{2s^2 - 6r - 12 + 2\Delta + k^2}{k} = 2q - 2r + 4s + 5$ . After a substitution for  $r$  from Eq. (5), we obtain  $\frac{2k^2 + 2sk + 5k}{k} = 2q - 2r + 4s + 5$ , and so  $q = r - s + k$ .

Now we return our attention back to  $r$ . Let  $k$  be even, say  $k = 2\ell$ . Then

$$r = \frac{2s^2 - 12 + 2\Delta - 4\ell^2 - 4s\ell - 10\ell}{6} = \frac{(s - \ell)^2 - 3\ell^2 - 5\ell + \Delta - 6}{3}.$$

Choosing  $s = 4\ell$ , we infer that  $r = 3\ell^2 - \ell^2 - 2 - \frac{5\ell - \Delta}{3}$ . Hence, by setting  $\ell = 3z - \Delta$ , then  $q$ ,  $r$ , and  $s$  are integers. Obviously, if  $z$  is big enough, then  $q$ ,  $r$ , and  $s$  are non-negative. □

## 4 Discussion

Throughout the article we have discussed the Wiener index of common neighborhood graphs. We have shown that the diameter of the common neighborhood graph is at most the diameter of the original graph. On the other hand, the difference between the Wiener indices of these two graphs may be arbitrarily large. In fact we have shown that the latter is true even in the case of chemical graphs. Moreover, for any integer  $\Delta$  there is an infinite family of graphs such that  $W(\text{con}(G)) - W(G) = \Delta$  for every graph of the family. We believe that the following conjecture is true.

**Conjecture 12.** *There is an absolute constant  $C$  such that for every graph  $G$  it holds that*

$$W(\text{con}(G)) \leq C \cdot W(G).$$

Nevertheless, it is not completely clear how the difference grows in terms of the number of the vertices in the graphs. Let  $G^*$  be a graph on  $n$  vertices constructed from a path of length  $n/3$  that has attached roughly  $n/3$  on the first and on the second vertex and one triangle on the last vertex. We have observed that the difference  $W(\text{con}(G^*)) - W(G^*) \in \Theta(n^3)$ , more precisely the leading coefficient is  $n^3/27$ . Regarding this, it could be interesting to determine the extremal graphs for which the considered difference is maximal.

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