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BOOK REVIEW

Topological Index — A New Mathematics Linking Between the Fibonacci Numbers and the Pascal Triangle

by

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Nippon Hyoron Sha, Tokyo, 2012, III+165 pp. ISBN 978-4-535-78666-0, published in Japanese

The term *topological index* contained in the book title "*Topological Index — A New Mathematics Linking Between the Fibonacci Numbers and the Pascal Triangle* (in Japanese)" originates from the author's article published in 1971 (H. Hosoya, *Bull. Chem. Soc. Jpn.*, **44**, 2332–2339 (1971)) and is now referred to under the name *the Hosoya index*. Now that the versatility of the Hosoya index in chemistry has been widely described in reviews and textbooks, the author's stance for publishing this book is to emphasize mathematical aspects of the Hosoya index.

This book consists of seven chapters along with a preface and an afterword. The title of each chapter and the respective section titles are listed as follows (after translation from Japanese to English):

Chapter 1. Basic Series and Polynomials: Fibonacci numbers and Lucas numbers; Pell numbers and Pell-Lucas numbers; de Moivre-Binet formula and the golden ratio; Continued fractions and continuants; Chebyshev polynomials; On-Line Encyclopedia of Integer Sequences (OEIS).

Chapter 2. Graph Theory and Topological Index: Basic graphs; Non-adjacent numbers and topological index; Pascal triangle and Fibonacci numbers; Characteristic polynomials; Z-counting polynomials and the inclusion-exclusion principle; Caterpillar graphs and continued fractions; Molecular graphs and isomers.

Chapter 3. Non-Tree Graphs and Topological Index: Cyclic graphs and Lucas numbers; Characteristic polynomials and matching polynomials for cyclic graphs; graphs and their spectra; Regular polyhedra and their characteristic polynomials; Complete graphs, Hermite polynomials, and Young tableaux; Two-color complete graphs and Laguerre polynomials; Physical meanings of the gnarled Lucas triangle.

Chapter 4. Pell Equation and Topological Index: Continued fractions for square roots; Behavior of the smallest solution of the Pell equation; Structure of the solution of Pellp-1; High-speed solution of the Pell equation; Solutions of the Pell equation other than the smallest one; Caterpillar graphs for general solutions of the Pell equation.

Chapter 5. Diophantine Equation and Topological Index: Euclidean algorithm; Solution of Diophantine indeterminate equation; Cassini equation; New solution of Diophantine indeterminate equation; Proof for Cassini equation.

Chapter 6. Pythagorean Triangles and Topological Index: Pythagorean triangles and their classification; $\Delta 1$ group; Pythagorean triangles with consecutive legs; $\delta 1$ group; Barning-Hall matrices $\mathbf{U}$, $\mathbf{D}$, and $\mathbf{A}$; $\Delta 2$ group; The j/k-th roots of $\mathbf{U}$ and $\mathbf{D}$; Various Pythagorean triangles derived from caterpillars.

Chapter 7. Perspectives of Topological Index: High-speed rational approximation of square roots; Slowly-convergent series; Irreducible Pythagorean triangles for giving rational approximation of square roots; Heronian triangles; Heronian triangles with three sequential numbers.

The afterword (pp. 157–160) contains a short history of the Hosoya index since the author's proposal in 1971. Each chapter as well as the Afterword is accompanied with a selected list of references. The book ends with a well-organized Index (pp. 161–165).

Chapter 1 discusses prerequisite mathematical concepts such as Fibonacci numbers and Lucas numbers. Chapter 2 is devoted to a general introduction of the Hosoya index in combination with the graph theory. Chapters 3–6 are concerned with the relationships between the Hosoya index and respective mathematical concepts. Chapter 7 discusses further aspects of the Hosoya index to be developed in the future.

In summary, "*Topological Index — A New Mathematics Linking Between the Fibonacci Numbers and the Pascal Triangle*" is a valuable book, aimed at both chemists and mathematicians. Throughout all of the chapters, the description style of the author is easy and concise, so that the book is useful both for beginners and experts. Although the book is written in Japanese, it involves many illustrative figures, succinct tables, and meaningful mathematical equations, which would provide even non-Japanese readers with tools for understanding the contents of this book.

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