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Comments on "A Unified Approach to Extremal Multiplicative Zagreb Indices for Trees, Unicyclic and Bicyclic Graphs"

Jiashun Yang

College of Mathematics and Statistics, South Central University for Nationalities, Wuhan 430074, P. R. China

js.y@163.com

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Abstract. In this comments, an error in K. Xu and H. Hua's article [1] is pointed out. Because of this error, the proof of the Lemma 2.4(2) presented in this article is wrong. In the second part of this comments, the author proposes a correction to this proof.

1 Error in the proof of Lemma 2.4(2)

From the first line and the last in the proof of Lemma 2.4(2) in Page 247, manely

$$\begin{aligned}\Delta_1 &\geq (x+k+l)^{x+k+l} - (x+k)^{x+k} - (y+l)^{y+l},\\ \Delta_2 &\geq (y+k+l)^{y+k+l} - (x+k)^{x+k} - (y+l)^{y+l}.\end{aligned}$$

is incorrect. So the following of the proof is also wrong.

2 Correction to the proof of Lemma 2.4(2)

Lemma 2.1. *If $p \geq 1, x \geq 1$ and $f(x) = \frac{(x+p)^{x+p}}{x^x}$, $f(x)$ is increasing.*

Proof. Note that $\ln f(x) = (x+p)\ln(x+p) - x\ln x$, then $\frac{df(x)}{dx} = f(x)[\ln(x+p) - \ln x] > 0$. The result follows immediately. □

For convenience, let

$$\begin{aligned}N_{G_0}(u) &= \{z_1, z_2, \cdots, z_y\},\\ E(u) &= \{uz_1, uz_2, \cdots, uz_y\},\end{aligned}$$

$$\begin{aligned} N_{G_0}(v) &= \{w_1, w_2, \cdots, w_x\}, \\ E(v) &= \{vw_1, vw_2, \cdots, vw_x\}. \end{aligned}$$

Then we have

$$\begin{aligned} \Pi_2(G') &= \prod_{ef\in E(G')-E(u)-E(v)-\{vu_1,\cdots,vu_k,vv_1,\cdots,vv_l\}} d_{G_0}(e)d_{G_0}(f) \prod_{z\in N_{G_0}(u)} d_{G_0}(z) \\ &\quad \prod_{w\in N_{G_0}(v)} d_{G_0}(w)\cdot y^y(x+k+l)^{x+k+l}, \\ \Pi_2(G'') &= \prod_{ef\in E(G')-E(u)-E(v)-\{vu_1,\cdots,vu_k,vv_1,\cdots,vv_l\}} d_{G_0}(e)d_{G_0}(f) \prod_{z\in N_{G_0}(u)} d_{G_0}(z) \\ &\quad \prod_{w\in N_{G_0}(v)} d_{G_0}(w)\cdot x^x(y+k+l)^{y+k+l}, \\ \Pi_2(G) &= \prod_{ef\in E(G')-E(u)-E(v)-\{vu_1,\cdots,vu_k,vv_1,\cdots,vv_l\}} d_{G_0}(e)d_{G_0}(f) \prod_{z\in N_{G_0}(u)} d_{G_0}(z) \\ &\quad \prod_{w\in N_{G_0}(v)} d_{G_0}(w)\cdot (y+k)^{y+k}(x+l)^{x+l} \end{aligned}$$

Hence

$$\begin{aligned} \Delta_1 = \Pi_2(G') - \Pi_2(G) &\geq y^y(x+k+l)^{x+k+l} - (y+k)^{y+k}(x+l)^{x+l}, \\ \Delta_2 = \Pi_2(G'') - \Pi_2(G) &\geq x^x(y+k+l)^{y+k+l} - (y+k)^{y+k}(x+l)^{x+l}. \end{aligned}$$

Now we prove the Lemma 2.4(2) as following.

(1)If $x \geq y$, then $x + l > y$. By Lemma 2.1, we have

$$\begin{aligned} \Delta_1 = \Pi_2(G') - \Pi_2(G) &\geq y^y(x+k+l)^{x+k+l} - (y+k)^{y+k}(x+l)^{x+l} \\ &= y^y(x+l)^{x+l}\left[\frac{(x+l+k)^{x+l+k}}{(x+l)^{x+l}} - \frac{(y+k)^{y+k}}{y^y}\right] > 0. \end{aligned}$$

(2)If $y \geq x$, then $y + k > x$. Also by Lemma 2.1, we have

$$\begin{aligned} \Delta_2 = \Pi_2(G'') - \Pi_2(G) &\geq x^x(y+k+l)^{y+k+l} - (y+k)^{y+k}(x+l)^{x+l} \\ &= x^x(y+k)^{y+k}\left[\frac{(y+k+l)^{y+k+l}}{(y+k)^{y+k}} - \frac{(x+l)^{x+l}}{x^x}\right] > 0. \end{aligned}$$

So the proof of Lemma 2.4(2) holds.

References

[1] K. Xu, H. Hua, A unified approach to extremal multiplicative Zagreb indices for trees, unicyclic and bicyclic graphs, *MATCH Commun. Math. Comput. Chem.* **68** (2012) 241-256.