

The Six (or Seven) Bridges of Kaliningrad: a Personal Eulerian Walk, 2006

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Abstract

The eighteenth-century problem of the Bridges of Königsberg was solved in a memoir dated 1736 and written by the Swiss mathematician Leonhard Euler (1707–1783) soon after he had been appointed to the senior Chair of Mathematics at the St. Petersburg Academy of Sciences. Euler demonstrated that what is now called an Eulerian Walk (that is, a route that traverses all of the bridges once, and once only) was not possible in contemporary Königsberg. Soon after the Conferences of Yalta and Potsdam had assigned the city and its environs to the Soviet Union after World War II, Königsberg came to be known as the city of Kaliningrad (Калининград), capital of the Kaliningrad Oblast, which, since the early 1990s, has found itself as an exclave of the present-day Russian Federation, isolated from mainland Russia by the newly independent republic of Lithuania (and, beyond that, Latvia and Belarus). Furthermore, the Kaliningrad Oblast's only other adjoining neighbour is Poland which, like Lithuania, has been a Member of the European Union since 1 May 2004. This state of affairs thus determines that the Kaliningrad Oblast is, these days, *doubly* anomalous, in that it is not only an *exclave* of the Russian Federation but (simultaneously) it is also a foreign *enclave* within the European Union. It was into this intriguing and unique area that the author recently ventured (in February, 2006) in order to investigate what the current situation is with regard to Eulerian Walks in present-day Kaliningrad, precisely 270 years after Euler considered the problem as it applied to the Königsberg of 1736. This paper evaluates how the disposition of the Bridges has varied over the years. Until 1876, the configuration of the Bridges was precisely what it was in Euler's time, and an Eulerian Walk was thus still not feasible (though it was after the mid-1870s). By the 1930s the original seven Bridges had increased in number to nine or ten, and careful examination shows that an Eulerian Walk was, at that time, also still possible. It is demonstrated that, only a few years ago — at the beginning of the present millennium — an Eulerian Walk was again *not* feasible. In fact, though, thanks to the rebuilding (in 2005) of the (1905) *Kaiserbrücke* — which was reconstructed in order to commemorate the 750th anniversary of the founding of the city in 1255 — an Eulerian Walk *is* once again now possible in present-day Kaliningrad. This claim is demonstrated both by use of the conventional Graph-Theoretical equivalent of Euler's original algorithm and by describing the specific Eulerian Walk that the author (and his travelling companion, Paweł Skrzyński) actually executed in practice, on Sunday, 26 February 2006.

1. Introduction

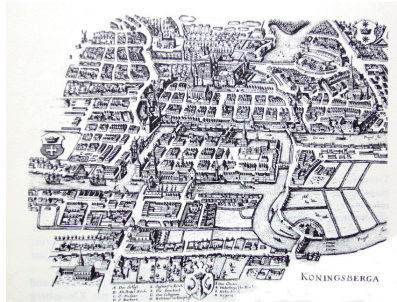


Figure 1. *Eighteenth-Century Königsberg.*

The eighteenth-century problem of the Bridges of Königsberg (Figure 1) was solved in a memoir dated 1736, written by the Swiss mathematician Leonhard Euler (15 April 1707¹ – 18 September 1783²), soon after he had been appointed to the senior Chair of Mathematics at the St. Petersburg Academy of Sciences. Ever since the deliberations and decisions of the Conferences of Yalta and Potsdam after World War II — when it and some of its East-Prussian³ environs

¹ Expressed according to the Gregorian Calendar, already then current in Basle, Euler's place of birth. The 300th anniversary of Euler's birth thus fell on 15 April this year. This article is intended to be a small tribute to Euler on this auspicious occasion.

² Expressed according to the Gregorian Calendar, already then current in Basle and Berlin, places where Euler had worked; however, as Euler died in St. Petersburg his death would have been reported locally on the Julian Calendar as having occurred on 7 September. Some reference-sources and web-sites report the date as such (frequently omitting to point out that this is a Julian date).

³ Other parts of the former East Prussia (some two-thirds of it) were assigned at Yalta and Potsdam to Poland; the *Memelgebiet* (Memelland, the most easterly part of East Prussia that had been retrieved by Germany in 1939) was annexed by Lithuania after 1945. Before the Second World War, the population of East Prussia was 2.4 million, 99 per cent of them ethnic Germans. By 1953, almost no Germans remained. By 1950, 1.35 million expellees lived in the German Federal Republic, with another 48,000 from the *Memelgebiet*. (See W. Benz [Editor], *Die Vertreibung der Deutschen aus dem Osten*, Fischer Taschenbuch, Frankfurt-am-Main, 1995). Others presumably lived in the German Democratic Republic, or emigrated abroad. It is estimated (A. M. de Zayas, *The German Expellees in War and Peace*, Macmillan, New York and London, 1993) that at least 20 per cent of the population perished during the last few months of the war and in the years immediately afterwards. (These data and the references cited were kindly brought to my attention by Dr. Michael Glasby and Professor Ian Whittle.)

were confiscated from Germany and awarded to the Soviet Union by their incorporation into the Soviet Socialist Republic of Russia — Königsberg (Polish: Królewiec, Latin: variously rendered as *Regiomontium* [what Euler called it but there is no real translation of this — it is just a name, on a model of a Latin place-name]), *Regius Mons* or *Mons Regius* [‘Royal Mountain’] and *Regis Mons* [‘King’s Mountain’]) has been known as the city of Kaliningrad (Калининград), capital of the Kaliningrad Oblast, which, since the early 1990s, has found itself in the awkward and inconvenient position of being an exclave of the present-day Russian Federation, isolated from mainland Russia by the newly independent republic of Lithuania (and, beyond that, Latvia and Belarus — see Figure 2). When these republics were part of the Soviet Union, the fact that the Kaliningrad Oblast was isolated from the rest of Russia did not, in practice, much matter: but it does now. Furthermore, the Kaliningrad Oblast’s only other adjoining neighbour is Poland (see Figure 2), which, like Lithuania, has been a Member of the European Union since 1 May 2004. This state of affairs isolates the Oblast even further: for it is, these days, not only an exclave of the Russian Federation, but — since that accession of Poland and Lithuania to the Union only three years ago — it now has, in addition, the peculiar status of (simultaneously) being a foreign enclave within the European Union (see Figure 2).

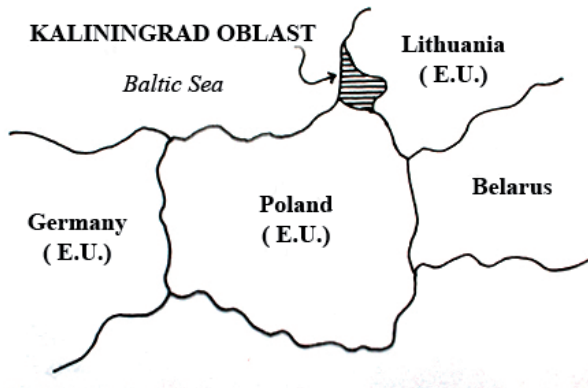


Figure 2. A sketch map showing the location of the Kaliningrad Oblast.

2. The History and Evolution of the Königsberg Bridges Problem

The city is founded on the banks of the River *Pregel* (now, in Russian, *Pregolya* [Преголя]) which flows through the city and surrounds an island (called, in Prussian days, the *Kneiphof*⁴), and then separates into two branches (the *Alter Pregel* [Старая Преголя], to the south, and the *Neuer Pregel* [Новая Преголя], to the north), as shown in Figure 3. The whole complex of the *Kneiphof* (A), and the landmasses B (the *Vorstadt*), C (the *Altstadt*) and D (a large island called the *Lomse Insel*), was spanned by a system of seven bridges, as shown in that Figure. It is frequently claimed — though on ostensibly scant evidence — that it was common folk-lore in the early eighteenth-century that the good burghers of Königsberg used to amuse themselves by trying to devise a

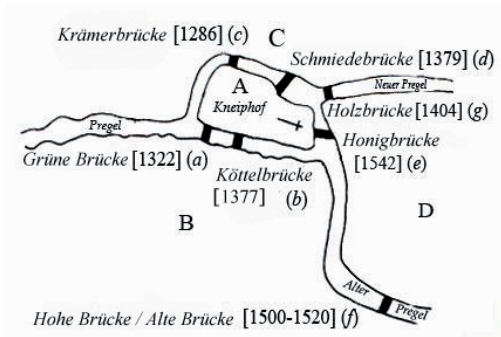


Figure 3. *The Bridges of Königsberg, 1736.*

⁴ The *Kneiphof* (= a ‘pub’, ‘students’ drinking place’, or ‘hotel where students got drunk’), once a bustling little municipality in its own right, is now a verdant park, entirely bereft of buildings save for the Cathedral, currently in course of restoration. It does not appear even to have an official name in Russian: many people still seem to denote it by its former German name, whilst others refer to it as ‘Pregel Island’ or ‘Kant Island’, the latter name arising because the tomb of Königsberg’s most famous son, Immanuel Kant (1724–1804), is in the Cathedral, on the island, and his university (the Albertina University [1544–1944]) was also there. Although the epitome of the former German culture in Königsberg, Kant, being a philosopher whom some orthodox communists thought to have influenced Marx, was, exceptionally, *not* considered *persona non grata* by the Soviets, and so ‘Kant Island’ was probably quite an acceptable name to them, not as redolent of the Prussian past as the Germanic ‘*Kneiphof*’ was. The other (much larger) island, landmass D — the *Lomse Insel* — is now called *Oktyabrskiy* (October Island).

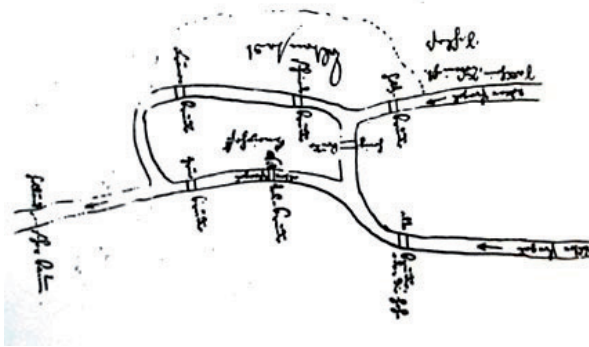


Figure 4. Ehler's sketch, sent to Euler on 9 March 1736.

route around the city that would cross each of the seven bridges once, and once only — what is now referred to as an ‘Eulerian Walk’. Since their attempts had always failed, the belief grew that maybe the feat was impossible. It was Euler⁵ who, in a presentation to the St. Petersburg Academy on 26 August 1735, first treated the question systematically and mathematically. Euler had correspondence on the matter with C. L. G. Ehler, Mayor of Danzig [Polish: Gdańsk]⁵; the sketch map that Ehler drew for Euler is reproduced in Figure 4⁶.

⁵ See H. Sachs, M. Stiebitz, & R. J. Wilson, An historical note: Euler’s Königsberg letters, *Journal of Graph Theory* **1988**, 12, 133–139.

⁶ For some unknown reason, Ehler drew his map with south at the top and, in Figure 4, in order to make north appear at the top, Ehler’s sketch has been rotated 180°. Of course, the consequence of this is that Ehler’s hand-written labellings — in any case, almost illegible — are now upside-down! There are two mysteries about Figure 4 (which is extracted from Sachs, Stiebitz & Wilson [see Note 5]). One minor puzzle is that the sketch was sent by Ehler to Euler (with a letter dated 9 March 1736 — see Note 5) some six or seven months *after* the latter had presented the problem, and its solution, to the St. Petersburg Academy. When writing that letter to Euler, Ehler knew that Euler had solved the problem, and was writing to ask him for the solution. The second — and major — enigma is the appearance of dotted lines across the river, in the south-west corner of the map (as it is displayed in Figure 4), cryptically labelled ‘holländischer Baum’. None of the authors of the paper mentioned in Note 5 could throw light on either of these matters when I raised them at Ilmenau on 27 March 2007, Professor Horst Sachs’s eightieth birthday. This second mystery is perpetuated (by Euler himself) when, in his letter to G. J. Marinoni of 13 March 1736 (which, if Euler, based in St. Petersburg, were using Julian dates, was 24 March on the Gregorian Calendar, and so Euler would have been writing 15 days after Ehler had written his 9 March letter to Euler), Euler sketches a map of contemporary Königsberg and includes a single *line* across the river (definitely *not* indicating a bridge — he clearly and unambiguously depicts bridges by means

Although initially disputing that the task in hand was properly within the realm of Mathematics at all,⁵ Euler showed that the problem was insoluble for the configuration of bridges and landmasses then extant in early eighteenth-century Königsberg, and he generalised his reasoning to any arbitrary arrangement of landmasses and bridges whatsoever. This work's eventual publication

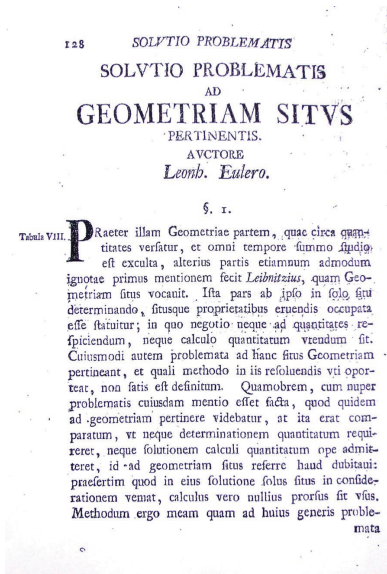
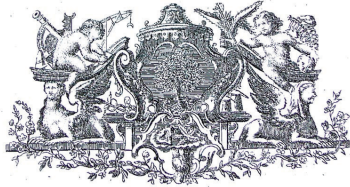


Figure 5. Euler's paper: the first page of the 1736/1741 printing.

COMMENTARII
ACADEMIAE
SCIENTIARVM
IMPERIALIS
PETROPOLITANAE.

TOMVS VIII.
AD ANNUM MDCCLXXXVI.



PETROPOLI,
TYPIS ACADEMIAE MDCCCLXXXVI.

Figure 6. Frontispiece of Volume 8 of the Commentarii, 1736/1741.

of two, parallel, lines with shading between them); this line (a) is in the same position as the *holländischer Baum* of Ehler's map (approximately the location of the present-day railway-bridge), and (b) is somewhat tantalisingly labelled 'h'. Now, whether this is 'h' as an abbreviation for *holländischer Baum* or whether 'h' is used because that is the eighth letter of the alphabet, and was thus the next letter due to be adopted after Euler had previously labelled his seven bridges 'a' to 'g', is again not clear. This sketch by Euler, sent to Marinoni, is illustrated as Figure 3 of the paper by Sachs, Stiebitz & Wilson, cited in Note 5. (See also later, Note 19.)

constituted the first paper⁷ on what later came to be called ‘*Analysis Situs*’. Euler used Leibnitz’s term *Geometria Situs* (‘Geometry of Position’), a

⁷ L. Euler, *Solutio problematis ad geometriam situs pertinentis*, (a) originally published by the St. Petersburg Academy in *Commentarii Academiae Scientiarum Imperialis Petropolitanae* **1736**, 8, 128–140; (b) reprinted in *ibid.*, ed. nova, *Bononiae* **1752** 8, 116–126; (c) also reprinted in *Opera Omnia, Series Prima, Opera Mathematica* (29 vols), Vol. 7, pp. 1–10, *Sub Auspiciis Societatis Scientiarum Naturalium Helveticae*, 1911–1956. English translations of Euler’s paper are to be found in the following three places: (i) J. R. Newman, Leonhard Euler and the Königsberg bridges, *Scientific American* **1953**, 189, 66–70; (ii) J. R. Newman (Editor), *The World of Mathematics*, Vol. 1, Simon & Schuster, New York, 1956, pp. 573–580; (iii) N. L. Biggs, E. K. Lloyd & R. J. Wilson *Graph Theory 1736–1936*, Clarendon Press, Oxford, 1976, Chapter 1, pp. 3–8. Robin Wilson has pointed out that close examination of this paper shows that, despite its well-known 1736 date, it was in fact not actually published until 1741; see the facsimile shown in Figure 2 of R. J. Wilson, An Eulerian trail through Königsberg, *Journal of Graph Theory* **1986**, 10, 265–275. The facsimile of the first page of Euler’s paper illustrated by Wilson is, however, from a later reprint of the paper (probably that in the the *Omnia Opera*, (c), cited above); the first page of the *true* original of Euler’s paper is illustrated in Figure 5, here. Note that, on the frontispiece, illustrated in Figure 6, the actual publication date of 1741 — along with the purported, notional date of 1736 — is explicitly acknowledged. It may be remarked in passing that Euler had no fewer than ten other papers ‘*In Classe Mathematica*’ of this volume (8) of the *Commentarii*, in addition to the one on the Königsberg Bridges Problem. The only other author to publish — in his case, two papers — ‘*In Classe Mathematica*’ of that volume was Daniel Bernoulli; hence, both Euler and Bernoulli were in august company! Also emphatically worth reading is the fascinating, three-page preface to Volume 8 of the *Commentarii*, written on 1 December 1740 (Julian date) by Christian Goldbach (himself, as it happens, a native of Königsberg), directly and effusively praising the infant Tsar Ivan III (usually styled Ivan VI who, by then, had reigned for only six weeks), and, indirectly, through him, his mother, the Regent Anna Leopoldovna, expressing the hope that royal support for the Academy would continue. Goldbach poignantly addresses his dedication to the infant Emperor ‘in the first year of Your age and Reign.’ Declared successor on 5 October — at the age of less than two months — to his great-aunt Empress Anna, the baby Ivan became Emperor on her death only 12 days later, under the regency initially (until he was usurped, just three weeks after that) of a former *éminence grise*, E. J. von Biron, Duke of Courland, and then of Ivan’s mother. (This was a regency which, however, as a result of a *coup d’état* on 6 December 1741 that overthrew the 15-month-old Tsar, was itself destined to last for little more than one further year). It was largely because of all this chaos, and the xenophobic backlash that followed Empress Anna’s policy, during the preceding decade, of promoting Baltic Germans over Russians into prominent positions, that, in 1741 (the year of actual publication of his ‘Königsberg’ paper), Euler accepted Frederick the Great’s invitation to move to the Berlin Academy.

discipline in which⁸ all that matters is how entities are *connected*, as distinct from the more usual geometry in which it is distances ('metric') and angles that are of material importance; later still (thanks to the ability of the British Mathematician J. J. Sylvester to invent names),⁹ the subject became known as 'Graph Theory'. Euler's seminal paper⁷ is thus frequently and generally regarded as having founded the modern subject of Graph Theory⁸ (as well as, incidentally, that of Topology).

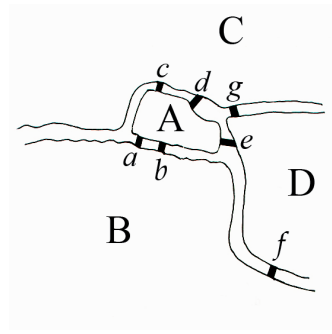


Figure 7. Euler's Königsberg (1736).

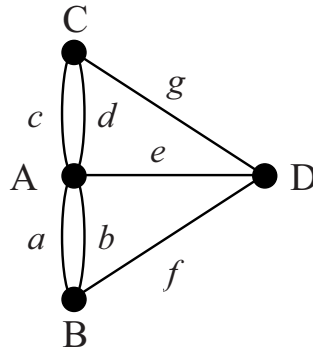


Figure 8. The corresponding graph (W. W. Rouse Ball, 1892⁸).

When expressed in modern terminology — but see the caveat embodied in Note 8 — Euler's procedure is effectively⁸ to replace each landmass by a *point* (*vertex*); the four points/vertices in this example are labelled *A*, *B*, *C*, and *D*. Pairs of those points are joined by a *line* (*edge*) whenever there is a bridge that directly connects the landmasses that those pairs of points represent (see Figures 7 and 8). If more than one bridge connects any two landmasses then a separate line is drawn that connects the corresponding vertices for each and every such

⁸ Robin Wilson has, indeed, also emphasised that, despite common belief and apparently long-established 'folk-lore', Euler himself did *not* use the language of Graph Theory, and that, specifically, the graph of the Königsberg Bridges Problem depicted in Figure 8 was not illustrated in any publication until W. W. Rouse Ball's *Mathematical Recreations and Problems of Past and Present Times*, Macmillan, London, 1892. (See (a) R. J. Wilson, An Eulerian trail through Königsberg, *Journal of Graph Theory* **1986**, *10*, 265–275; (b) B. Hopkins & R. J. Wilson, The truth about Königsberg, *The College Mathematical Journal* [The Mathematical Association of America] **2004**, *35*, 198–207.)

⁹ J. J. Sylvester, Chemistry and algebra, *Nature* **1877–1878**, *17*, 284–284.

bridge. This can be seen from Figures 7 and 8.¹⁰ For example, the *Kneiphof* (A) has two bridges (a and b) connecting it to landmass B , so there are two lines going between the point labelled ‘ A ’ and the point labelled ‘ B ’; the *Kneiphof* likewise has two bridges (c and d) connecting it to landmass C and so there are also two lines going between vertices A and C . Since the *Kneiphof* (A) is, in addition, connected to region D by a single bridge (e), the island landmass A (the *Kneiphof*) has a total of five bridges emanating from it. The *degree* of vertex A is thus five. If the network diagram (*graph*) is completed in this way for all four vertices we have the situation as depicted in Figure 8, in which vertex A has degree 5, and vertices B , C and D all have degree 3. We shall subsequently abbreviate information like this into the shorthand form: ‘ $A(5), B(3), C(3), D(3)$ ’. When expressed in this modern graph-theoretical terminology, Euler’s conclusions for a general network of this type — describing *any* arbitrary system of islands, landmasses and connecting bridges whatsoever — were:

- (a) *If more than two vertices are of odd degree*, then an Eulerian Walk is *not* possible.
- (b) *If exactly two vertices are of odd degree, and the rest — no matter how many of them there are — are of even degree*, then an Eulerian Walk is always possible, if it starts in (either) one of the two landmasses represented by the vertices of odd degree, and ends in the other.
- (c) *If all vertices are of even degree*, then the required Eulerian Walk may be accomplished starting from any of the landmasses and ending at any landmass. (In this case, an Eulerian *Circuit* — that is, a closed Eulerian Walk that starts and ends at the same vertex/landmass — is always possible).

Euler did not actually trouble to prove the sufficiency of statement (c), above; he regarded it as self-evident, referring to it (when translated from the Latin) as ‘an easy task . . . after a little thought.’ His intuition was in fact correct, but the equivalent of statement (c) was not formally and rigorously proved for some 130 years, when it was elegantly and efficiently disposed of in (remarkably) a posthumous paper by Carl Hierholzer (1840–1871), published in

¹⁰ This account follows Euler’s practice of using upper-case letters ($A - D$) for the landmasses, and lower-case letters for the bridges ($a - g$, initially; later, for two *post-eighteenth-century* bridges, the letters h and i will be used, and a twentieth-century motorway-bridge [known as the *Estacada* Bridge] will subsequently be denoted by the ‘compound’ symbol ‘ (a,c) ’).

the 1873 *Mathematische Annalen*.¹¹ Hierholzer was, in fact, not actually aware of Euler's eighteenth-century work, and, accordingly, he had no option but to devise it all again, independently and *ab initio*.^{11, 12} This posthumous paper was written up by Hierholzer's contemporaries (his colleague C. Wiener, helped by the algebraist J. Lüroth^{8,11}) after the *Privatdozent's* untimely and premature demise on 13 September 1871.¹¹ One hundred and fifteen years later — in order to commemorate the 250th anniversary of Euler's 1736 paper — Fowler offered a new sufficiency proof¹³ that proceeded by induction on the number of edges. Fowler also remarked¹³ that essentially the same proof can be given by using induction on the number of vertices.

Because the 1736 Königsberg graph has all four of the vertices *A*, *B*, *C* and *D* of odd degree (see Figure 8), it may properly be deduced, as, indeed, Euler himself definitively concluded (from his equivalent of Rule (a), above — but expressed in his own [different, *not* graph-theoretical] terminology), that an Eulerian Walk was *not* possible in the Königsberg of Euler's era.

¹¹ C. Hierholzer, *Über die Möglichkeit, einen Linienzug ohne Wiederholung und ohne Unterbrechung zu umfahren*, *Mathematische Annalen* **1873**, 6, 30–32. An English translation of this paper is to be found in N. L. Biggs, E. K. Lloyd & R. J. Wilson, *Graph Theory 1736–1936*, Clarendon Press, Oxford, 1976, Chapter 1, pp. 11–12. For some helpful insight into Hierholzer's proof, see also J. H. Barnett, Early writings on graph theory: Euler circuits and the Königsberg bridge problem, in J. Lodder, J. Barnett, G. Bezhanishvili, D. Pengelley, D. Ranjan, & H. Leung, *Historical Projects in Discrete Mathematics and Computer Science*, in B. Hopkins (Editor) *Discrete Mathematics Resource Guide*, The Mathematical Association of America, Washington D.C., *in press*. The book just cited is currently available on the following internet web-site at the New Mexico State University at Las Cruces: www.math.nmsu.edu/hist_projects (note the 'underscore' separating 't' from 'p' in this web-site address); the article by J. H. Barnett referred to is on pp. 74–86 (with the references on pp. 104–107) of this web-site.

¹² Sporadic attention was, however, paid to Euler's solution to this problem, in the succeeding years: more than a century later, for example, in the 1851 *Nouvelles Annales de Mathématiques* (**1851**, 10, 106–119), E. Coupy published a French translation of Euler's paper, entitled: *Solution d'un problème appartenant à la géométrie de situation, par Euler*, and he applied Euler's algorithm to the analogous problem of the Bridges of Paris, connecting the *Rive Droite*, the *Rive Gauche* and the islands of the River *Seine*. Three-quarters of a century after that, Euler's paper⁷ was also translated into German, in: A. Speiser, *Klassische Stücke der Mathematik*, Orell Füssli, Zürich & Leipzig, 1925, pp. 127–138.

¹³ P. A. Fowler, The Königsberg Bridges — 250 years on, *American Mathematical Monthly* **1988**, 95, 42–43.

3. Königsberg Since Euler's Time

A glance at the map of Königsberg that is included in the 1870 *Baedeker* travellers' handbook¹⁴ confirms that, at the moment when Bismarck formed the German Empire (the 'Second Reich'), the configuration of the bridges was precisely the same as it had been some 130 years earlier, in Euler's time. However, in 1876 a further bridge was reported,¹⁵ joining landmasses *B* and *C*. This was in fact a railway bridge, though it could apparently also accommodate pedestrians.¹⁵ At that stage, therefore, there was a total of eight bridges. This consideration then caused the degrees of the four vertices to be altered to the following: $A(5)$, $B(4)$, $C(4)$, $D(3)$; and so an Eulerian Walk was then at last theoretically possible (starting at the *Kneiphof* (*A*) and ending on region *D*, or vice-versa).^{15,16} In 1905, landmasses *B* and *D* were connected via the *Kaiserbrücke*, thereby enabling a Walk between *A* and *B*, or vice versa. By the time that Königsberg was in the *Third* Reich, circumstances concerning the bridges had changed yet again. Precisely 200 years after Euler considered the problem of the city's seven eighteenth-century bridges, the 1936 *Baedeker*¹⁷ contained a map that indicated the presence of *ten* bridges, and the landmass vertex-degrees were then $A(5)$, $B(6)$, $C(5)$, $D(4)$. Therefore, an Eulerian Walk was apparently possible in the immediate pre-war period, too, beginning at the *Kneiphof* (*A*) and ending in landmass *C* (or vice-versa): this statement does, though, assume that what was manifestly the railway bridge (the *Reichsbahnbrücke*) was passable by pedestrians as well (as is the case with the

¹⁴ *Baedeker's The Rhine and Northern Germany. Handbook for Travellers*, 4th Edition, Karl Baedeker, Coblenz, 1870, map between p. 338 and p. 339.

¹⁵ L. Saalschütz [untitled] *Schriften der Physikalisch-Ökonomischen Gesellschaft zu Königsberg in Preußen* **1876**, 16, 23–24. Robin Wilson has reported (R. J. Wilson, An Eulerian trail through Königsberg, *Journal of Graph Theory* **1986**, 10, 265–275) that Saalschütz even went so far as to enumerate no fewer than 48 different possible Eulerian Walks, starting in the *Kneiphof* (*A*) and ending in the *Lomse Insel* (*D*).

¹⁶ It is perhaps stretching things somewhat, but Robin Wilson (R. J. Wilson, An Eulerian trail through Königsberg, *Journal of Graph Theory* **1986**, 10, 265–275) has drawn attention to the fact that it has been observed (R. Cooke, Letter to the Editor, *American Mathematical Monthly* **1984**, 91, 662–664) that a map of Königsberg *ca.*1800 indicates the existence of a ferry service that joined landmasses *A* and *C*. If this ferry service were counted as effectively being a 'bridge', this would have then made the vertex degrees $A(6)$, $B(3)$, $C(4)$, $D(3)$, thereby theoretically enabling an Eulerian 'Walk' that went from *B* to *D* (or vice-versa).

¹⁷ *Baedeker's Germany. A Handbook for Railway Travellers and Motorists*, Karl Baedeker, Leipzig, 1936, the map between p. 124 and p. 125.

present-day railway-bridge — please see later.¹⁸) If, however, that assumption is not true — and it cannot be ascertained, from the 1936 *Baedeker* map, whether it is true or not — then the degrees for bridges accessible to pedestrians in 1936 would have been $A(5)$, $B(5)$, $C(4)$, $D(4)$. An Eulerian Walk would thus also then still have been possible, but now from the *Kneiphof* (A) to landmass B (or vice-versa). Either way, therefore, whether or not the railway bridge depicted in the 1936 map of the city was traversable by pedestrians, an Eulerian Walk *was* still possible there in the mid 1930s. Surprisingly, however, the map in the 1938 German-language edition of *Baedeker*,¹⁹ published only two years later, seems to indicate the *absence* of one bridge (to the east of the *Reichsbahnbrücke* and connecting landmasses B and C) whose presence was very clearly indicated by the map published with the 1936 English-language edition.¹⁷ Accordingly, the landmass vertex-degrees in 1938 would appear to have been $A(5)$, $B(5)$, $C(4)$, $D(4)$ if the *Reichsbahnbrücke* were traversable by pedestrians, and to have been $A(5)$, $B(4)$, $C(3)$, $D(4)$ if it were not. Again, because precisely two landmass-vertices are of odd degree, it is immediately seen that, in 1938, an Eulerian Walk *was* possible, either by a route from A to B (or vice versa) or — if the *Reichsbahnbrücke* had been inaccessible to those on foot — by a route from A to

¹⁸ According to an article entitled *The Bridges and 'the Pregel's Odour'* by Ivan Chechot of St. Petersburg, which is to be found on the Kaliningrad internet web-site <http://ncca-kaliningrad.ru/art-guide/?by=p&lang=eng&au=022chechot>, it seems that the present-day *Reichsbahnbrücke* is essentially the same as the one that stood during Third-Reich times, as Chechot reports that it was constructed in the period 1913–1926, the long building-time being attributed to a delay due to World War I. Chechot specifically confirms that cars and trams could pass on the lower tier of the bridge, and railway traffic on the upper tier. He claims, too, that there was also at one time a device that enabled the middle part of the bridge to be turned in order to allow ships to pass. This facility seems, though, now to have disappeared and Chechot states that the bridge is, these days, apparently static. However, in the web-site <http://commons.wikimedia.org/wiki/Image:Most.jpg> this claim has been contradicted: it is stated there that the ‘... middle section of the bridge can be moved vertically to let ships pass’. Be that as it may, it is, however, almost certain that, as now, the *Reichsbahnbrücke* would have been available to any pedestrian who wished to undertake an Eulerian Walk in the mid-to-late 1930s.

¹⁹ *Baedekers Autoführer Band 1 Deutsches Reich (Ohne das Land Österreich)* Karl Baedeker, Leipzig, 1938, the map between p. 546 and p. 547. In connection with the mystery described in Note 6, it may be mentioned in passing that the maps included in the 1936 and 1938 *Baedekers*, referred to in Note 17 and (above) in the present Note, record the existence of a railway station called *Bhf.* (1936) or *Hst.* (1938) *Holländerbaum* and a street called *Holländer Baum Straße* (1936) or *Holländerbaum Straße* (1938), very close to the present-day railway-bridge and also near to where Ehler tantalisingly wrote ‘*holländischer Baum*’ on his sketch-map (Figure 4), drawn⁶ for the benefit of Euler. (Note: these 1930s maps say ‘*Holländer*’ NOT ‘*holländischer*’, as written by Ehler on his 1736 sketch [Figure 4].)

C (or vice versa). The numerous changes in the four landmass vertex-degrees during the two centuries after Euler are conveniently summarised in Figure 9.

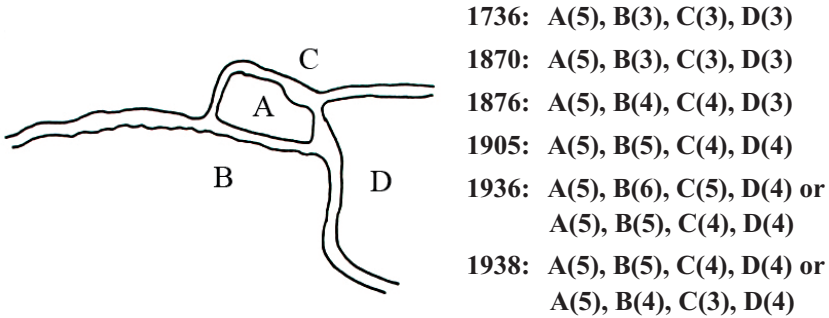


Figure 9. Landmass vertex-degrees during the course of the two centuries after Euler's investigation of 1736.

In the period 1944–1945, Königsberg suffered terrible damage in Allied bombings²⁰ and, having been assigned at Yalta and Potsdam to the Soviet

²⁰ During the war-time storming of Königsberg, damage was sustained by the *Holzbrücke* (Timber Bridge, bridge *g* of Figure 3, built in 1404, replaced in metal in 1904), the *Schmiedebrücke* (Blacksmith's bridge, *d*, built in 1379 [or, possibly, 1397], replaced [in wood] in 1787, rebuilt in metal in 1896), the *Köttelbrücke* (Offal Bridge, bridge *b*, built in 1377, replaced, in steel, in 1886) and the *Kaiserbrücke* (Emperor Bridge, connecting landmasses *B* (the *Vorstadt*) and *D* (the *Lomse Insel*) of Figure 3, built in 1905 — see later, as bridge *i* of Figure 14). In the Soviet period, during the construction, in 1972, of the *Estacada* motorway-bridge, carrying part of the *Leninsky Prospekt*, the *Krämerbrücke* (Merchants' Bridge, bridge *c*, originally built in 1286, rebuilt [in wood] in 1787, replaced in metal in 1900), the *Grüne Brücke* (Green Bridge, bridge *a*, first built in 1322, burned down in 1582, rebuilt [again in wood] by 1590, and replaced in steel in 1907), the *Köttelbrücke* and the *Schmiedebrücke* were all demolished. The *Holzbrücke* had an articulated section that was capable of being raised in order to let ships pass; this was replaced by the corresponding section of the (demolished) *Köttelbrücke*. In the mid-1980s, the *Kaiserbrücke* (1905) was also finally taken down (but see later). The *Hohe Brücke* (High Bridge, bridge *f*, also known as the *Alte Brücke* [Old Bridge]) was built between 1500 and 1520 and renovated in 1882 (when its wooden parts were replaced by steel) and was taken down in 1937 to be replaced (in 1939) by a new bridge, built just to the east of it. This latter bridge is still intact, together with a few remains of the earlier bridge — the bases of support-pillars are evident in the water, and at least the external part of the turreted bridge-keeper's house appears to be well preserved. Also still intact, and connecting the *Kneiphof* (*A*) with the *Lomse Insel* (*D*), is the *Honigbrücke* (Honey Bridge, bridge *e* of Figures 3 and 7), originally constructed in wood in 1542 and replaced in metal in the period 1879–1882. (The information stated in this footnote has

Union, it was absorbed into the Soviet Socialist Republic of Russia.²¹ In the summer of 1946, it was renamed Kaliningrad (Калининград), after the then very recently deceased Mikhail Ivanovich Kalinin (1875–1946), formally the Soviet Union’s Head of State — somewhat grandly called, after 1938, ‘The President of the Presidium of the Supreme Soviet’. Remarkably, for someone whose name is so little known outside his own country, Kalinin held this office of Head of State from the Soviet Union’s inception, in 1919, until his death, in June 1946. The city was re-named after him just one month later. During the succeeding 45 years, Kaliningrad became a closed city, a military and naval centre, access to which was largely denied to outsiders, and especially to those from the West, beyond what came to be known as the ‘Iron Curtain’.

With the break-up of the Soviet Union, however, in the early 1990s, and the general easing of tensions within most of Europe, Kaliningrad — though now a city dominated by 1950s Soviet apartment-blocks and thus, architecturally, a mere shadow of its former, pre-war self — once again began to become accessible to the wider world. Early last year, I was, therefore, curious to know what the current situation was with regard to ‘Eulerian Walks’ in the Year of Grace 2006, precisely 270 years after Euler. I first consulted the computer software ‘Google Earth’, to see if the current bridge-configuration could be established that way. However, this is a part of the world where the Google-Earth satellite-photographs are not of very high quality or resolution. My next ploy, therefore, was to down-load from the internet some road maps of Kaliningrad, from the web-site <http://guide.kaliningrad.net>. From these maps I deduced that two of the former bridges (*b* and *d* of Figure 3, the *Köttelbrücke* [Offal Bridge, built in 1377, renovated (in metal) 1886] and the *Schmiedebrücke* [Blacksmith’s Bridge, built in 1379 (or, possibly, 1397), rebuilt (in wood) in

largely been gleaned from a postcard compilation issued at Kaliningrad in 2005 to commemorate the 750th anniversary of the city’s foundation, [1255–2005]: Мосты Кёнигсберга [*The Bridges of Königsberg*], Album IX, 12 postcards, *Reklamno-informatsionnyy Kholding “39 Region”*, Kaliningrad, 2005. [Web-site: www.39.ru; e-mail: koenigsberg@39.ru].

²¹ The original Soviet intention had apparently been to incorporate the Kaliningrad Oblast into the Soviet Socialist Republic of Lithuania — which would have made sense because of previous historical connections between the two and, more obviously, because, physically, the Oblast is immediately *adjacent* to Lithuania. However, such was the ruined state of the area after World War II that the Soviet Socialist Republic of Lithuania apparently did not want it. The Kaliningrad Oblast was, accordingly, absorbed into the Russian Soviet Socialist Republic, as an exclave of that Republic.

1787, and renovated (in metal) in 1896], respectively) connecting the *Kneiphof* to the landmasses *B* and *C*, respectively, seem now no longer to exist, and that there are also numerous other changes that have occurred since the 1930s. Depending on the outcome of the question (crucial *now* — though, as we have seen, not so in 1936 and 1938) as to whether or not what was evidently a railway bridge is passable by pedestrians,²² it did look as though an Eulerian Walk *was* possible, as the vertex degrees seemed to be as follows: *A*(3), *B*(4), *C*(4), *D*(5). It looked, therefore, as though there were eight bridges, over which an Eulerian Walk did appear to be possible, from the *Kneiphof* to landmass *D* (or vice-versa). It was with the purpose of executing such an Eulerian Walk that I and my redoubtable Polish travelling companion, Paweł Skrzyński, set off from Gdańsk in the very early morning of Saturday, 25 February 2006, on the 130 km journey that would take us across the north-eastern border of the European Union to the capital city of the Kaliningrad Oblast, *exclave* of the Russian Federation and *enclave* of the European Union — truly a modern-day political anomaly.

4. In Present-Day Kaliningrad, 25/26 February 2006

We travelled on the Polish coach-service, departing Gdańsk at 7.00 a.m. and arriving in Kaliningrad just under five hours later. The border checks on both sides were thorough, though not oppressive. We registered at the hotel, the Hotel Kaliningrad on *Leninsky Prospekt*, having walked to it from the coach station. Until dark, we made an informal, haphazard and non-systematic inspection of some of the bridges. The railway bridge was rather too far away for us to investigate at that preliminary stage but we went to look at some of the others. The first thing we noted was that the two bridges labelled by Euler (in Figure 3) as *a* and *c* (which are, respectively, the former *Grüne Brücke* [Green Bridge, built in 1322, re-built in 1590, and renovated in 1907] — over which Kant used to walk each day to work — and the *Krämerbrücke* [Merchants' Bridge, built in 1286, reconstructed in 1787, and renovated in 1900]) have been replaced — this was done in 1972 — by a somewhat un-aesthetic motorway road-bridge, called the *Estacada* Bridge, carrying an elevated part of the *Leninsky Prospekt* (over

²² As mentioned earlier, in connection with the maps available in the 1936 and 1938 *Baedekers*,^{17,19} this was not determinable solely from an examination of the modern road-map. The web-site <http://commons.wikimedia.org/wiki/Image:Most.jpg> did, though, display a photograph by Volkov Vitaly of the present-day railway-bridge and further claimed this railway bridge to be a two-tier one, capable of carrying pedestrians (as well as, incidentally, road vehicles) on its lower level, in addition to railway traffic on its upper level — a claim that we subsequently verified personally, in Kaliningrad itself, by direct inspection, ‘on the ground’.

which bridge pedestrians can, however, also pass). This actually goes *above and over* the *Kneiphof* island and can thus be considered not as two bridges connecting landmasses *B* and *C* to the island *A* but as a single bridge that connects *B* directly to *C*, without passing through the *Kneiphof* (*A*) at all. On the other hand, there *is* provision for the pedestrian to pass down some steps in the middle of the bridge, and thereby visit the *Kneiphof* (and, if he so desires, to depart from the *Kneiphof*, by ascending once again these steps in the middle of this bridge directly connecting *B* to *C*, and thus to emerge again, at will, onto either landmass *B* or landmass *C*). Hence, it would also still be legitimate to regard this elevated motorway-bridge as *two* bridges, both incident on the *Kneiphof*. As will be seen later, this choice is *not* material to whether or not an Eulerian Walk can be performed, and, in fact — as will be seen — whichever interpretation is adopted, an Eulerian Walk *is* still possible — at least, it was in February, 2006.

However, a stark (and, for our plans, potentially devastating) discovery that we made on this preliminary ‘recce’ on the first day was that two of the bridges — connecting landmass *B* to landmass *D*, and *D* to *C*, to the east of, respectively, bridges *f* and *g* (on Euler’s labelling, in Figures 3 and 7) — that were shown on the road-map that I had down-loaded from the internet whilst still in England were, on inspection, seen to be bridges *whose construction had evidently been abandoned* — they just *ended in mid-air*; (see Figure 10 which shows the author standing by one of these extraordinary constructions). There had evidently been



Figure 10. *The author standing beside one of the ‘phantom’ bridges.*

an intention to connect the region between the two bridges — and, beyond them, in each direction on either side — by means of an elevated motorway, but this work remains uncompleted. These ‘phantoms’ (Figure 11) could not, therefore,

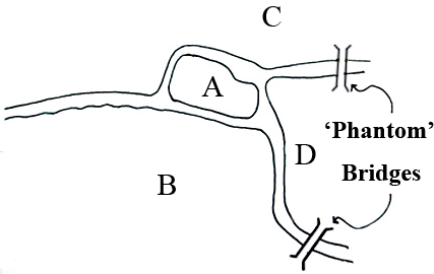


Figure 11.

Location of the 'phantom' motorway-bridges.

properly be considered as genuine bridges for, ending in 'mid-air', as they did, they were equally inaccessible to both pedestrians and road vehicles (see Figure 10). This unexpected and unwelcome finding, forcing us to eliminate two 'bridges' from our consideration, fundamentally sabotaged my plan, for the vertex degrees now changed from what I had thought them to be whilst we were in England ($A(3), B(4), C(4), D(5)$) to $A(3), B(3), C(3), D(3)$; or, if the former *Grüne Brücke* (bridge *a*) and the former *Krämerbrücke* (bridge *c*) are considered to be just one bridge (the *Estacada Bridge*, denoted by the symbol '(*a,c*)' — now connecting *B* and *C* directly with each other without involving *A* at all — the vertex degrees would be $A(1), B(3), C(3), D(3)$. Either way, we would be back to the eighteenth-century situation of having all four vertices of odd degree, and thus an Eulerian Walk would not, after all, have been possible.



Figure 12.

Distant view (looking north) of the newly reconstructed (2005) Kaiserbrücke (1905).

However, in a development reminiscent of the change of situation in 1876, when Saalschütz¹⁵ reported the construction of another bridge (i.e., the

Reichsbahnbrücke) that made the Walk possible, we discovered that, *although not on the map obtained from the internet,*²³ *a new, and charming, footbridge had been constructed only the previous year: 2005 was the 750th anniversary of the founding of the city*²⁴ and, as part of the commemoration of this, a new



Figure 13.

Art-nouveau wrought-iron and decorative lamps of the newly reconstructed Kaiserbrücke (2005), joining the Vorstadt (B) to the Lomse Insel (D).

footbridge was constructed at the crossing-point between landmasses *B* and *D* at the place where the former *Kaiserbrücke* (Emperor Bridge, originally built precisely 100 years earlier, in 1905) used to be. Furthermore, this new bridge is the only recently built one that has been constructed with any attempt to acknowledge the visual aesthetic appeal of the pre-war bridges. It even has *art-nouveau* ornamental lamp-posts and wrought-iron work, and a ‘bridge-keeper’s house’ in the turreted design similar to that of the 1905 *Kaiserbrücke*; (see Figures 12 and 13). Whatever its architectural appeal, however, the main delight for me in finding out about the existence of this new ‘*Kaiserbrücke*’ was that it saved the day as far as being able to carry out an Eulerian Walk was concerned: for, the vertex degrees were now finally (and, by personal inspection, *definitively*) established as being *A*(3), *B*(4), *C*(3), *D*(4) (or — on the other interpretation about the modern equivalent of Euler’s bridges *a* and *c* in Figures

²³ Neither was it shown on a large, folding city-map that we bought in Kaliningrad; however, closer inspection revealed that it *was* on a miniature tourist-map, given to us by the hotel reception.

²⁴ Founded in 1255 by the Teutonic Knights and named ‘Königsberg’ in honour of King Přemysl Otakar II of Bohemia.

3 and 7 that regards them as constituting a single bridge [the *Estacada* Bridge, which is denoted by the ‘combined’ symbol ‘(a,c)’] going directly from *B* to *C* without involving *A* in any way — $A(1), B(4), C(3), D(4)$. An Eulerian Walk is, therefore, now possible. It must go from landmass *C* (where the hotel at which we stayed, Hotel Kaliningrad, is situated) to the *Kneiphof* (*A*) (or vice-versa); see Figure 14, which definitively depicts the present-day configuration of ‘The Seven (or Six) Bridges of Kaliningrad’.

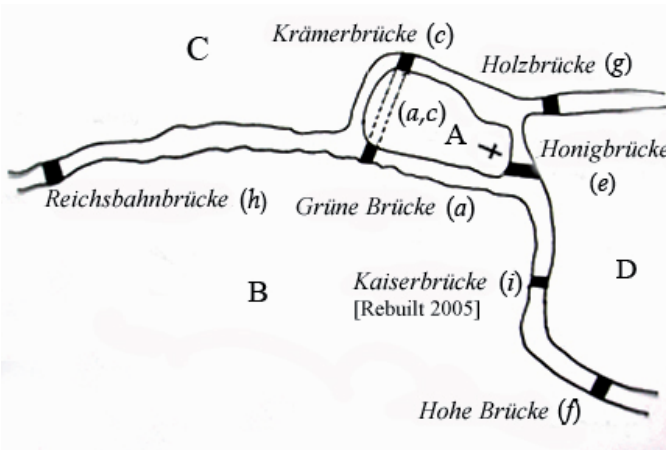


Figure 14. The definitive present-day configuration of ‘The Seven (or Six) Bridges of Kaliningrad’ (as of 26 February 2006).

5. The Eulerian Walk in Present-Day Kaliningrad (26 February 2006)

Comforted by this finding, we made our way back to the Hotel Kaliningrad and arranged the sequence of landmasses and bridges that we would traverse in our execution of the Eulerian Walk, to be accomplished the following day. The sketch maps are shown in Figures 15 and 17, and the corresponding graphs are illustrated in Figures 16 and 18, respectively. The Walk can be done in more than one way but the actual route(s) that we took, from landmass *C* to the *Kneiphof* (*A*), across the *seven* (or, on the other interpretation of the elevated *Estacada* motorway-bridge [carrying part of the *Leninsky Prospekt*], (a,c), above the *Kneiphof*, the *six*) Bridges of Kaliningrad, were as follows:

(1) THE 'SEVEN-BRIDGE' EULERIAN WALK

(FIGURE 15, THE ASSOCIATED GRAPH FOR WHICH IS SHOWN IN FIGURE 16):

$C \rightarrow (\text{bridge } h) \rightarrow B \rightarrow (\text{bridge } a) \rightarrow A \rightarrow (\text{bridge } c) \rightarrow C \rightarrow (\text{bridge } g) \rightarrow D$
 $\rightarrow (\text{bridge } f) \rightarrow B \rightarrow (\text{bridge } i) \rightarrow D \rightarrow (\text{bridge } e) \rightarrow A$

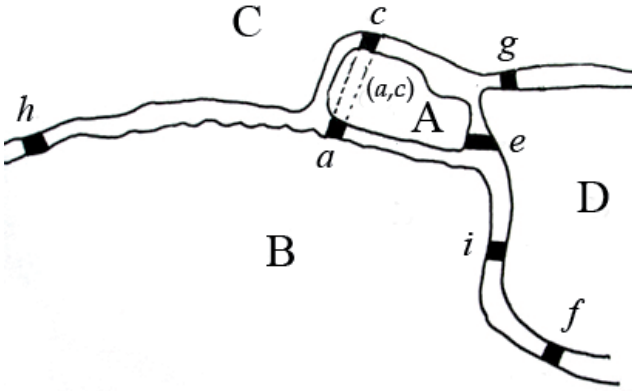


Figure 15. *The Seven Bridges of modern-day Kaliningrad.*

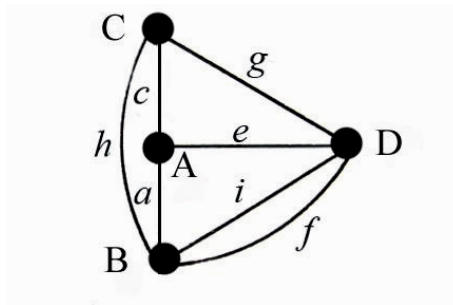


Figure 16. *The Graph of the Seven Bridges of modern-day Kaliningrad.*

(2) THE 'SIX-BRIDGE' EULERIAN WALK

(FIGURE 17, THE ASSOCIATED GRAPH FOR WHICH IS SHOWN IN FIGURE 18):

$C \rightarrow (\text{bridge } h) \rightarrow B \rightarrow (\text{'combined' bridge, ('a,c')}) \rightarrow C \rightarrow (\text{bridge } g) \rightarrow D$
 $\rightarrow (\text{bridge } f) \rightarrow B \rightarrow (\text{bridge } i) \rightarrow D \rightarrow (\text{bridge } e) \rightarrow A$

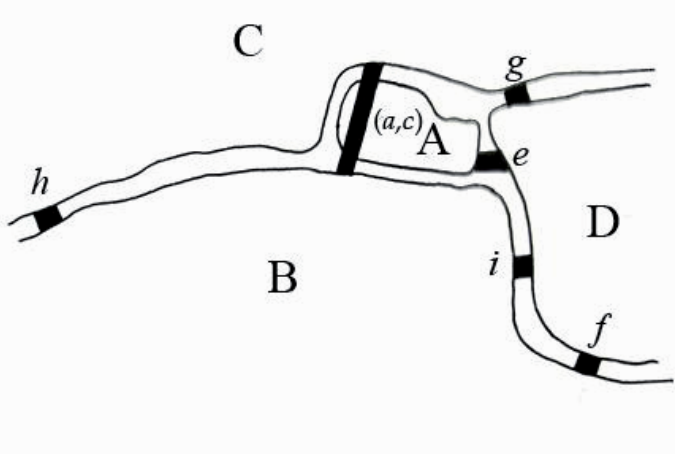


Figure 17. The Six Bridges of modern-day Kaliningrad.

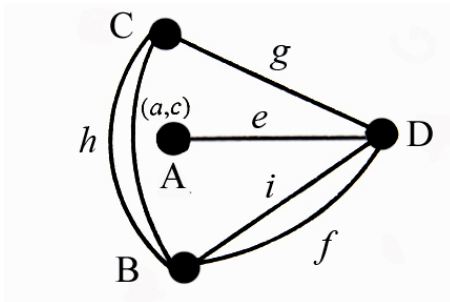


Figure 18. The Graph of the Six Bridges of modern-day Kaliningrad.

Dining at the hotel the evening before the Walk we reflected on how, in Kaliningrad, there is effectively nothing left from the old Königsberg. The *Altstadt* (on the north bank, landmass C) has been virtually razed to the ground and replaced by (among other things) part of the *Leninsky Prospekt* and Hotel Kaliningrad. Some years ago, the remains of the *Schloss* (also on, and very near, the north bank of the *Neuer Pregel*, in region C) were demolished²⁵ and replaced by a stark (though, actually, not entirely unattractive) tower-block called, appropriately, ‘The House of Soviets’. The *Kneiphof* used to be a thriving (but extremely small) independent entity before Frederick William I incorporated it, along with nearby *Löbenicht* (just north-east of the *Altstadt* [landmass C]), into the city of Königsberg in 1724; now it is *just* a park, the only building presently on it being the shell of the (picturesque, and partially restored) Cathedral (marked with a Christian Cross in Figures 3 and 14). Other than that, there is (as earlier mentioned in detail) a somewhat un-aesthetic motorway-bridge, (*a,c*), the *Estacada* Bridge, carrying part of the *Leninsky Prospekt*, that runs straight across the *Kneiphof* at an elevated level (with, as already explained, steps down in the middle so that the pedestrian can descend from it to the island), and quite an elegant little bridge — the *Honigbrücke* (Honey Bridge, (*e*), built in 1542, renovated 1882) — on the eastern end of the *Kneiphof*, just by the Cathedral, connecting that island to the other island-region, D. The elevated *Estacada* Bridge, (*a,c*), runs over (and at some distance above) where the *Kneiphof Langgasse* used to be. The (Albertina) University of Königsberg was founded in 1544 by Duke Albrecht I of Prussia (the last *Hochmeister* [Grand Master] of the Teutonic Knights, who reigned 1525–1568) — acting, though, under the suzerainty of King Sigismund II of Poland. Its rolls were initially filled from the Polish-Lithuanian Commonwealth and from Prussia but, after the Thirty Years’ War (1618–1648), the Albertina University — affiliated, as it was, with the Protestant Christian faith — attracted applicants from all over the German-speaking world. It had a distinguished history of precisely 400 years, until it was obliterated in 1944. Quoted below is the entry in the Wikipedia ‘on-line encyclopaedia’²⁶ concerning Königsberg University:

²⁵ The *Schloss* was thought, in Soviet times, to be too representative of Prussian imperialism. Now, however, there are (somewhat ambitious and possibly unrealistic) plans to re-build it.

²⁶ Wikipedia internet web-site on Kaliningrad: <http://en.wikipedia.org/wiki/Kaliningrad>.

‘Königsberg became a centre of education when the Albertina University was founded by Albert of Prussia in 1544. The university was situated opposite the north and east side of the Königsberg Cathedral. In 1560 Albert’s sovereign, Polish king Sigismund II of Poland equalled (*sic*) the university in law with the University of Kraków. In 1900 it contained the Municipal Library. In 1862 a new university, in the Renaissance style, was completed. The facade was adorned by an equestrian figure in relief of Albert of Prussia, the founder. Below it were niches containing statues of Martin Luther and Philipp Melanchton.²⁷ Inside was a handsome staircase, borne by marble columns. The Senate Hall contained a portrait of Emperor Frederick III and a bust of Immanuel Kant by Friedrich Hagemann. The adjacent hall (‘Aula’) was adorned with frescoes painted in 1870. The university library was situated in *Dritte Fliess-Straße* and contained over 230,000 volumes. There were 900 students in 1900.’

When one reads this heart-breaking entry, it is almost as if Oxford, Cambridge, Heidelberg or Padua, had been wiped from the face of the earth. Immanuel Kant was educated at the Albertina University, and served on its faculty for almost fifty years. Königsberg was also the *alma mater* of Christian Goldbach (who, as has already been noted,⁷ in addition to being the originator of what is now known as the Goldbach Conjecture, also — in his capacity as Secretary of the St. Petersburg Academy — wrote the preface to the volume [No. 8] of the *Commentarii* in which Euler published his 1736 paper [see Note 7]). The university’s nineteenth-century contributions to European mathematical and scientific culture were remarkable. In the early part of that century, Carl Jacobi was an undergraduate and, subsequently, a faculty member who, with contemporary colleagues — the physicist Franz Neumann and the astronomer Friedrich Bessel — introduced²⁸ a ‘. . . new research-oriented attitude in university instruction. The triad of Bessel, Jacobi and Franz Neumann thus became the nucleus of a revival of mathematics at German universities.’²⁸ These in turn influenced their pupils Carl Borchardt and the young Gustav Kirchhoff,

²⁷ Usually, in English, spelt ‘Philip’ and (in any language) ‘Melanchthon’.

²⁸ C. J. Scriba, in *Dictionary of Scientific Biography*, American Council of Learned Societies, New York, 1970–1990, quoted in J. J. J. O’Connor & E. F. Robertson, Mac Biography of Carl Gustav Jacob Jacobi on the internet web-site <http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Jacobi.html> (University of St. Andrews).

the latter of whom published two^{29,30} classic papers from the Albertina University — one²⁹ (enunciating what have long since become known as Kirchhoff's First & Second Laws of electrical networks) in 1845, when its author was still an undergraduate, and the other,³⁰ two years later, a momentous sequel, still of immense importance in the theory of graphs and in the context of determining the independence of a set of n linear equations in n unknowns. Kirchhoff's presence as a self-styled *Studiosus*²⁹ in the University was soon followed by fellow physicist Hermann von Helmholtz as a faculty member. In the 1880s, Carl von Lindemann (who, in 1882, whilst at Königsberg, proved that π is transcendental), was supervising A. J. W. Sommerfeld in his research and teaching the undergraduate Adolf Hurwitz who, when he in turn joined the Albertina faculty, himself befriended and nurtured Hermann Minkowski (who later developed a new view of Space and Time) and David Hilbert, the latter *alumnus* also subsequently — at the start of a distinguished career — going on to join the teaching faculty at Königsberg.

The University used to be in an extremely small area to the north and east of the Cathedral, on the *Kneiphof*, between the Cathedral and the start of the *Neuer Pregel*. Now it is just an empty space. On seeing it (see Figure 19) I was put in mind of the sonnet *Ozymandias* (1817), by the early nineteenth-century English poet P. B. Shelley, about 'a traveller in an antique land' coming across a crumbling statue of an ancient ruler:

*'Nothing beside remains. Round the decay
Of that colossal wreck, boundless and bare,
The lone and level sands stretch far away.'*

²⁹ *Studiosus* Kirchhoff, *Ueber den Durchgang eines elektrischen Stromes durch eine Ebene, insbesondere durch eine kreisförmige*, *Annalen der Physik u. Chemie* ('*Poggendorf's Annalen*') **1845**, 64, 497–514. In the by-line to this paper, Kirchhoff uses his undergraduate denomination, i.e., *Studiosus*, in preference to the initial letter of his Christian name and — in what is presumably a reference to the research seminars initiated by Bessel, Jacobi and Neumann — Kirchhoff describes his affiliation as '*Mitglied des physikalischen Seminars zu Königsberg.*'

³⁰ G. Kirchhoff, *Ueber die Auflösung der Gleichungen, auf welche man bei der Untersuchung der linearen Vertheilung galvanischer Ströme geführt wird*, *Annalen der Physik u. Chemie* ('*Poggendorf's Annalen*') **1847**, 72, 497–508.



Figure 19. *The author standing beside the forlorn statue of Duke Albrecht I of Prussia (reigned 1525–1568), Founder, in 1544, of the Albertina University.*

Somewhat pathetically, there is a statue of the august (sixteenth-century) Founder of the University (Duke Albrecht I of Prussia), forlornly surveying this empty space — see Figure 19 — which once housed a distinguished university that operated for four centuries until the very year (1944) in which the present author was born³¹. . . This is truly a case of ‘*Sic transit gloria mundi*’!

³¹ There is now a Russian (formerly, Soviet) University of Kaliningrad (founded 1967), situated in landmass *C*, the former *Altstadt*, which is entirely unconnected with the Albertina University of Königsberg (1544–1944). Even the Albertina University, however, so terminally physically devastated in 1944, seems to have survived, at least in memory, and perhaps in spirit: in note 91, on page 702 of C. Clarke, *Iron Kingdom: the Rise and Downfall of Prussia, 1600–1947*, Allen Lane, London, 2006, there is a reference to *Jahrbuch der Albertus-Universität zu Königsberg/Preußen*, Vol 6, which, remarkably, is dated 1955. I am indebted to Mr. C. W. Haigh for very kindly drawing my attention to this reference, and to Mr. John Pidoux for further information/speculation on the matter.

Starting our Eulerian Walk proper, we followed the route that was outlined earlier in this account. We left landmass *C* (Figure 14) by the most distant of the bridges with which we had to deal, and the only one of which we had not made a preliminary close inspection the day before — the two-tiered railway bridge (bridge *h*, in Figure 14), which, we verified, was indeed a bridge passable by pedestrians (as well as road vehicles) on its lower level, and by railway traffic on its upper level. We started to cross this just before noon. We then went along bridges *a* and *c* (or, if preferred, the [single] bridge, the *Estacada* Bridge, above the *Kneiphof* — that we are denoting by the symbol (a,c) — directly connecting landmasses *B* and *C* without making any contact whatsoever with the *Kneiphof*, *en route*) to landmass *C* (notionally touching base with the *Kneiphof* via the staircases down to the island, half way through, if it had been desired to do the seven-bridge [Figures 15 and 16], rather than the six-bridge [Figures 17 and 18], Kaliningrad Eulerian Walk). We then crossed to landmass *D* via bridge *g* (the former *Holzbrücke* [Timber Bridge, built in 1404 and renovated in 1904], one of only two bridges still to carry trams across it³²) and then, ignoring for the moment bridges *e* and *i* of Figure 14, we made for, and crossed back to landmass *B* by means of, bridge *f* (the still-extant *Hohe Brücke* or *Alte Brücke* [High Bridge or Old Bridge, built between 1500 and 1520, renovated in 1882–1883, and rebuilt in the period 1937–1939], the only bridge with remaining traces of the original bridge-supports, in the river adjacent to it, and with the former bridge-keeper’s house on the bank still intact and also the second of the only two bridges to bear tramway traffic³²). From there, we walked to, and crossed, the aesthetic and newly built (2005) replacement for the 1905 *Kaiserbrücke* (bridge *i*) back to land-mass *D*, and finally completed the Eulerian Walk by crossing (in some triumph, as may be imagined) the charming *Honigbrücke* (Honey Bridge, built [in wood] in 1542, and renovated [in metal] in the period 1879–1882) (bridge *e*), leading to the south-east side of the Cathedral, on the *Kneiphof* (see Figure 20). The whole Eulerian Walk had taken almost exactly an hour. It has to be borne in mind that this Walk was done in February; snow was everywhere on the ground and many parts of the route were quite treacherous underfoot. Care had, therefore, to be taken. In spring or summer, the Walk could probably be accomplished very much more rapidly.

³² However, my colleague Mr. John Pidoux, who has visited Kaliningrad more recently, informs me that trams no longer run over these bridges.



Figure 20.

The author standing on the final bridge of the Walk, the Honigbrücke (Bridge e [of Figure 14]), connecting the Lomse Insel (D) with the Kneiphof (A); the fourteenth-century Cathedral — the only building now extant on the Kneiphof — is in the background.

After that, with our ‘mission’ achieved, as we made our way to the railway station for the 18.48 from Kaliningrad to Gdańsk and Gdynia (Poland), I could not help dwelling on the stark fact that, 100 years ago, the railway journey that we were about to embark upon would have been an entirely internal one, within Germany, between two of the country’s administrative units (Königsberg, *Ostpreußen* and Danzig, *Westpreußen*) that actually adjoined one another: now, by the quirks of war, politics and history, it is not only an international journey but it crosses the all-important border that divides the European Union from the rest of the continent. . .

6. Postscript

My fellow traveller, Paweł Skrzyński, and I are not the only recent mathematical tourists that Kaliningrad has received from what used to be called ‘the West’. When I presented the material reported in this paper at a conference in Ilmenau to celebrate the eightieth birthday of Professor Horst Sachs on 27 March 2007, Professor H.-D. Gronau (Rostock) very kindly drew my attention to a most entertaining article published seven years ago by P. Taylor,³³ describing a *fin-de-siècle* visit that that author made to Kaliningrad (approaching it from Lithuania) in the year 2000. Taylor also couched his account in the form of a personal, touristic diary. However, that author and his Lithuanian party did

³³ P. Taylor, What ever happened to those bridges? *Mathematics Competition (Journal of the World Federation of National Mathematics Competitions)* **2000**, 13, 87–97. This article is also available on the following web-site of the Australian Mathematics Trust: <http://www.amt.edu.au/koenigs.htm>.

not attempt to execute an Eulerian Walk. This is just as well for, of course — as previously argued (§ 4) — without the saving grace of the rebuilt *Kaiserbrücke* (2005), such a Walk would not have been possible at the time of Taylor’s (albeit recent) visit, which took place in the very last year (as it should be reckoned) of the old millennium, five years before the *Kaiserbrücke* was reconstructed.

7. Acknowledgements

I should like to thank several colleagues and friends for their help in supplying references for this article, which is intended to be some small tribute to Leonhard Euler on the 300th anniversary of his birth. I am grateful: to Mr. Hubert Pragnell, Mr. Robert Mathews and Dr. Laurence Boyle for putting their wonderful collections of *Baedeker* travel-guides at my disposal; to Mr. Claude Haigh for bringing to my attention the *Jahrbuch* reference in Note 31, and to Mr. John Pidoux for expanding on it; to Dr. Michael Glasby and Professor Ian Whittle for telling me about the information in Note 3; and to Professor Hans-Dietrich Gronau for his kindness in making available a copy of the articles cited in Note 33 and the *Note Added in Proof*. I have had much help with translations, from: Mr. Valeri Avdeenko and Mr. Paul Pollak (Russian); Mr. Pollak, Mr. Mathews, Mr. Martin Miles, and Mrs. Dorothea Pragnell (German); Mr. Pollak and Dr. Glasby (Latin); and Mr. Paweł Skrzyński (Polish). I am grateful, too, that, in addition, Mr. Pollak, Mr. Haigh and Mr. Mathews have willingly given their services as highly meticulous and eagle-eyed proof-readers. Concerning ‘production’, Mr. Dharmendra Vegad, Mr. Mathews and Mr. Skrzyński have afforded invaluable technical help with the artwork and typesetting. Over a period of nearly 40 years I have from time to time enjoyably discussed the Königsberg Bridges Problem with Professor Robin Wilson and Professor Horst Sachs. My general mathematical mentor in all areas, and on Euler’s work concerning this problem in particular, has, however, been Mr. Paul Pollak, to whom I am, as ever, exceedingly grateful for the benefit of his erudition, so freely and so continuously made available. My final thanks are reserved for Mr. Paweł Skrzyński, who had the courage, patience and resourcefulness to accompany, support and guide me on the ‘voyage of discovery’ here described.

Note Added in Proof. Since submission of this manuscript a very similar, independent, and parallel study has been published: I. Gribkovskaia, Ø. Halskau & G. Laporte, The bridges of Königsberg — a historical perspective, *Networks* **2007**, *49*, 199–203. These authors have, however, chosen to exclude the *Reichsbahnbrücke* (bridge *h* of Figure 14) from consideration (even though pedestrians can traverse it). They do not consider Eulerian Walks but they do explicitly conclude that an Eulerian *Circuit* is *not* possible in present-day Kaliningrad.