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METEOR – application of a decision support tool based on discrete mathematics

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Abstract

In the present paper we introduce an improved version of the decision support system METEOR (Method of evaluation by order theory), first presented by Pudenz & Brüggemann (2002). METEOR is applied to evaluate the effects of nine water management strategies on the complex surface water system in the cities of Berlin and Potsdam. The METEOR approach is based on partial order theory, in particular on the HDT (Hasse Diagram Technique) approach. In contrast to evaluation tools such as PROMETHEE, NAIADE or AHP (Analytic Hierarchy Process) indicator aggregation is not performed for all indicators at once, but indicators are aggregated by a systematic, stepwise procedure. Effects of step-by-step aggregation of indicators can be analysed by concepts such as the number of comparabilities and by jump and bump relations. METEOR combines the advantage of real decision support, identifying one best solution, with participation and transparency.

1. Introduction

Decisions concerning management of surface waters need to be supported by information about potential chemical pollution. Especially in cities, a spatial and temporal exposure pattern of various substances is to be expected for both inorganic and organic toxicants as well as nutrients and heavy metals. To evaluate the chemical pollution of surface waters, many methodological approaches are available, requiring in principle the same working steps (Klauer et al. 2001):

1. The definition of options, in our case water management strategies, which are to be evaluated.
2. The generation of a set of indicators, appropriate for evaluation of options with respect to a certain goal, such as environmental hazards.
3. Estimation of the effects of the options.
4. Evaluation of the options, for example by powerful algorithms supporting the process of decision making such as PROMETHEE (Brans & Vincke 1985), AHP (Analytic Hierarchy Process, Saaty 1994), MAUT (Schneeweiss 1991), ELECTRE (Roy 1990) or NAIADE (Mararazzo & Munda, 2001).

The fourth step, the algorithmic aspect of evaluation, is often almost disregarded in real decisions, yet can be considered to be just as important as the first three steps: The chosen evaluation approach will strongly influence the evaluation procedure with respect to the evaluation result, the participation of stakeholders and the transparency of the result. Decisions about complex problems such as water management will typically include conflicting indicators. For example indicators used to evaluate the effects of nutrient emission of waste water treatment plants will not coincide with those indicating costs. In most cases a high purification standard will increase operation expenses. To solve such conflicts, the most commonly used approaches within decision support systems (DSS) listed above, include a methodological step of indicator aggregation. The benefit of the aggregation step is that a linear ranking of the options can be obtained, identifying one best solution. Aggregation also enables the weighting of indicators, which is an important tool for including the stakeholders preferences in the decision process. Aggregation of indicators, however implies a compensation among them: a bad evaluation in one or more indicator(s) can be compensated for by a good evaluation in other indicators. As indicators can represent different aspects such as ecology and economy, compensation can be considered as a comparison of "cheese to chalk". Furthermore, the evaluation result becomes difficult to interpret, because the influence of the ranking parameters become almost non-transparent. For these reasons, researchers and stakeholders complain about the "weighting camouflage" in decision support (Strassert 1995).

To avoid the disadvantage of low transparency induced by complicated DSS algorithms, indicator weighing and preference functions, alternative approaches such as Hasse Diagram Technique (HDT) can be used. The HDT is based on very simple elements of partial order theory and can be used to analyze the structure of multivariate data-sets, whenever a number of options can be characterized by multiple attributes (indicators). Table 1 shows an example of a data matrix, in which 5 options (a, b, c, d, e) are characterized by two indicators (I1 and I2). The respective evaluation result is visualized by a Hasse Diagram (Figure 1). As a methodological precondition of HDT, all indicators need to be orientated consistently in such a way that, for example, small numbers always indicate a good rating. Options are sorted on the basis of a simple ≤-comparison, separately, however simultaneously for each indicator. HDT provides several tools for convenient and detailed data analysis such as the concept of antagonistic indicators (Simon et al. 2004), which support to maintain the transparency of the evaluation result. There are, however, also disadvantages: participation, e.g. by indicator weighting, is not included in the evaluation process, and approaches such as HDT do not directly result in an unique decision. As no compensation among indicators is carried out, conflicting evaluations of indicators cannot be methodologically removed. Consequently multiple favourable options can be identified as incomparable winner solutions. In our example there are two incomparable winner options, namely a and c (Figure 1). For this reason, HDT, may be considered as an evaluation tool rather than a decision support system (Wiegleb 1997).

Table 1: Data matrix of the Hasse Diagram of Figure 1

Indicators / Options	I1	I2
a	10	0
b	20	20
c	5	60
d	12	70
e	40	40

METEOR (Method of evaluation by order theory), attempts to resolve the dilemma among obtaining a clear decision (one best solution), maintaining trans-

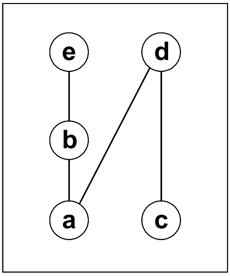

Figure 1: Hasse Diagram of the data matrix of Table 1

parency and allowing participation. Contrary to all DSS approaches we know thus far, METEOR allows a systematic step-by-step aggregation of indicators, including their weighting. This procedure is equivalent to the implementation of a "fitness" function, although it is not necessarily carried out for all indicators at once. The option of step-by-step aggregation of indicators provides the freedom to thoroughly analyse the effects of indicator weights and compensation. Furthermore, preferences (indicator weights) which are most sensitive to the evaluation result can be easily identified. The application of the METEOR approach is exemplified by the evaluation of the effect of nine water management strategies on the chemical pollution of the surface water system of the adjacent cities of Berlin and Potsdam, with an emphasis on nutrients.

2. Material and methods

2.1 Study site, water management strategies and indicators

The study site is the complex system of surface waters in the adjacent cities of Berlin and Potsdam (Figure 2). The main rivers are Havel, Spree and Dahme. Additionally there are tributaries and canals (for a more detailed description, see Simon et al. 2004). To be able to detect spatial effects of the different water management strategies, the surface waters are divided into 14 river sections, each of which is evaluated separately by the same set of indicators. Altogether nine water management strategies, also called scenarios, are evaluated (Table 2). Each scenario consists of three modules (A), (B) and (C). In module (A) "hydrological boundary conditions" measures concerns the quality and the amount of water entering the study site are defined. In module (B) "waste water treatment plants" (wwtp) measures concerns the purification standard of the waste water treatment plants and the spatial distribution of the waste water are described, including the closing of certain wwtp’s. In module (C) "rainwater treatment" measures to manage storm water events are included.

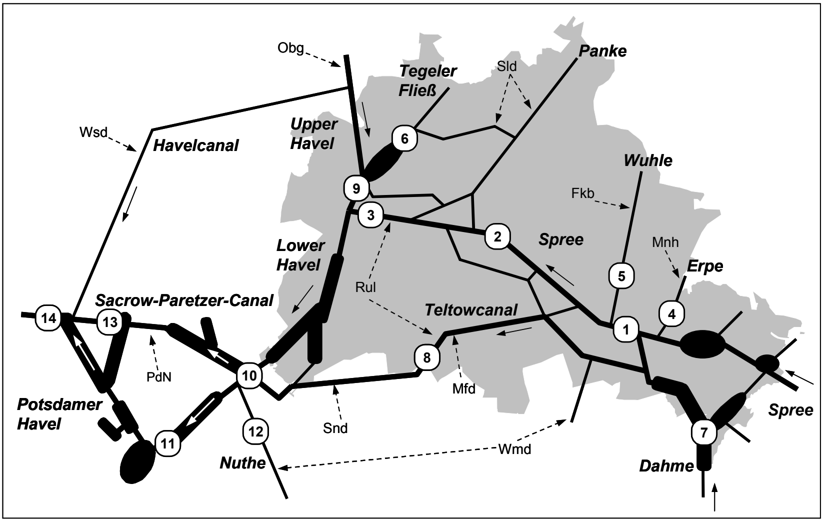

Figure 2: Schematic diagram of the surface water system of Berlin and Potsdam

River sections: (1) Spree Köpenick (including Dahme), (2) Spree Mühlendamm, (3) Spree Sophienwerder, (4) Erpe (Neuenhagener Mühlenfließ), (5) Wuhle, (6) Inflow to lake Tegeler See, (7) Dahme Schmöckwitz, (8) Teltowkanal, (9) Upper Havel, (10) Lower Havel, (11) Havel Caputh, (12) Nuthe Babelsberg, (13) Sacrow-Paretzer-Kanal, (14) Havel Ketzin.
Waste water treatment plants: Fkb=Falkenberg, Mfd=Marienfelde, Mnh=Münchehofe, Obg=Oranienburg, PdN=Potsdam Nord, Rul=Ruhleben, Sld=Schönerlinde, Snd=Stahnsdorf, Wmd=Waßmannsdorf, Wsd=Wansdorf,.
Dashed lines show wastewater pipe lines. Shaded area = city of Berlin.

The evaluation of the chemical pollution of surface waters is exemplified by three indicators (Table 3). The nutrients phosphorus and nitrogen, both identified to determine the eutrophication level. Their values are quantitatively modelled by MONERIS (MOdelling Nutrient Emissions in RIver Systems) (Behrendt et al. 1999, 2000). In addition, contamination by chemicals such as polycyclic aromatic hydrocarbons or heavy metals is unspecifically indicated by the indicator "short-term chemical pollution". The indicator qualitatively describes the impact of storm water events on the surface waters. To finally complete the ecological aspect of the evaluation, the quantitative indicator "reduction of the discharge" is added to the set of indicators. The definition of a significant discharge reduction is based on statistics (Simon et al. 2004). The evaluation of the effects of the nine scenarios on each of the 14 river sections by a set of four indicators leads to a information base *IB* of 14*4 = 56 elements. The resulting data matrix consists of 9 * 14 * 4 = 504 entries, which is the basic evaluation matrix (see appendix).

Table 2: Water management strategies

Abbreviations of waste water treatment plant names are given in Figure 1.

Abbre-viations of Scenarios	**Measures of module (A): hydrological boundary conditions**	**Measures of module (B): waste water treatment**		**Measures of module (C): entry of storm water**
		purification technique	closing of waste water treatment plants	
1a	current state (average of the years 1993-1997)			
1	reduced amount of water			
2		technical upgrade	Fkb, Mfd, Obg	
3	reduced amount of water and lower nutrient concentrations	advanced waste water treatment (micro-filtration)		emission 50% reduced
4			Mfd, Odg	
5			Fkb, Mfd, Obg	
6i		alternative sanitary technique		
6ii			Mfd, Obg, Mnh, Snd	
6iii			Mfd, Obg, Mnh, Sld	

2.2 The METEOR approach

The METEOR approach was first introduced by Pudenz & Brüggemann (2002) and is improved in this paper. The METEOR approach is based on Hasse Diagram Technique (HDT), which belongs to partial order theory. Within the HDT algorithm indicators are not aggregated and consequentely there is no compensation among them. For a detailed description of the HDT approach see for example Brüggemann et al. (2001). We introduce *E* as the ground set of options (scenarios), and *IB* as set of indicators. Then the specific partial order used in

HDT can be symbolised by (E, IB). The order relation $x \leq y$, meaning that scenario x is better than scenario y, corresponds to the product order, defined by the indicators of IB: Let $q_i \in IB$ be the i[th] indicator, and $q_i(x)$ the value of q_i for scenario x. Then $x \leq y \Leftrightarrow q_i(x) \leq q_i(y)\ \forall\ q_i \in IB$. At least in one case, a < - relation holds. Often the notation (E/R, IB) is used to indicate that the ground set of the partial order is the quotient set; the equivalence relation being the simultaneous equality of indicator values. If not explicitly stated, we consider our ground set as a quotient set. In order to avoid excessive technical notation, we continue to write simply E.

Table 3: Set of indicators

Abbreviations of indicators	**Indicator**	**Description**
P	Phosphorus	Difference of total phosphorus concentration from target concentration
N	Nitrogen	Concentration of total nitrogen
S	Short-term chemical pollution	Short-term chemical pollution of river sections by storm water events
Q	Discharge reduction	Reduction of the discharge in a river section

The evaluation result, a partial order, is visualized in a Hasse Diagram (HD). The vertical arrangement of options which are connected by lines, represents their relative evaluation from "good" to "bad". Note that non-connected scenarios (often more or less in a horizontal arrangement), exhibit conflicts among indicators. There are at least two indicators (one pair) where one indicator is evaluated better in one scenario and worse in the other scenario and the other way round. The incomparabilities between any of two options can be analysed by the HDT-originated concept of antagonistic indicators (Simon et al. 2004): Let E be the ground set, i.e. here the set of scenarios, introduce $E_i, E_j \subset E$, $E_i \cap E_j = \phi$. Then the smallest subset $IB^* \subset IB$ is searched for which the following is valid: $\forall x \in E_i$, $\forall y \in E_j$ in $(E_i \cup E_j, IB^*)$, $x \parallel y$. The indicators belonging to IB^* are called antagonistic (with respect to E_i, E_j). The terms (E_i, IB), (E_j, IB) are also called "hierarchies"; interpreted as undirected graphs, the poset ($E_i \cup E_j$, IB^*) has at least two components.

Even though compensation of indicators by a fitness function is not implemented in the basic HDT approach, an analogous methodological step can easily be realised: any of two original indicators can be aggregated to generate a new "aggregated indicator", for example by their weighted sum. This aggregation is equivalent with an order preserving map ℓ : The original poset $(E, IB) \xrightarrow{\ell} (E, IB')$ is the resulting information base as follows ω {q_1, ..., q_m} = IB and $q_{im} = w_i q_i + w_m q_m$ then $IB' = (IB \setminus \{q_i, q_m\} \cup \{q_{im}\})$. Note that indicator values have to be normalised beforehand in order to make them numerically comparable. The step-by-step aggregation can be carried out systematically until one single "aggregated meta indicator" is obtained, which is a weighted sum of all original indicators. This stepwise aggregation procedure we introduce as the METEOR approach. The effect of every single aggregation step can be visualised in a HD, and can be detected by analysing the structural changes among the sequence of HD's, i.e. by comparing the number of comparabilities (see section 2.5). Step-by-step aggregation of indicators will change the structure of the HD's towards a linear ranking. METEOR can thus be seen as a sequence of order preserving maps $\ell_{r,s}$, where a sequence of original and aggregated indicators leads to a sequence of information bases $IB^{(i)}$, $IB^{(0)} \equiv IB$ such that

$$(E, IB^{(0)}) \xrightarrow{\ell_{01}} (E, IB^{(1)}) \xrightarrow{\ell_{12}} \ldots\ldots \xrightarrow{\ell_{\zeta-1,\zeta}} (E, IB^{(\zeta)})$$

is valid and the final partially ordered set $(E, IB^{(\zeta)})$ represents a linear order.

2.3 Aggregation of indicators

The step-by-step aggregation can be performed in many different ways. The indicator weights were obtained according to the principles of the AHP weighting scheme (Saaty 1994). In a systematic pair-wise comparison of all indicators relative preferences are defined. After being checked for consistency, the relative preferences are transformed to absolute weights, summing up to 1. As an example for step-by-step aggregation of indicators the following sequence can be defined:

START: $S_1 = q_i$, $q_i \in IB$, select one attribute

$Q_1 = \{ q_i \}$

$\mathcal{W} = \{w_1, w_2, ... w_{(IB)}\}$, $\sum w_i = 1$, the set of weights as described above.

STEPS: $S_j = S_j \oplus q_k$, $q_k \in IB \setminus Q_{j-1}$

$\oplus$: weighted combination, i.e. $S_j + (w_k)q_k$, w_k taken from the set $\mathcal{W}$.

$Q_j = Q_{j-1} \cup \{q_k\}$

STOP: (1) j = card *IB*, or

(2) (E, Q_j), $j \leq$ card *IB*, is already a linear order.

(3) (optional) an aggregation by $\oplus$ would include an unacceptable mismatch of indicators of different scaling levels

For example: $IB = \{q_1, q_2, q_3\}$

START: $S_1 = q_1$

$Q_1 = \{q_1\}$

STEP: $S_2 = w_1q_1 + w_2q_2$

$Q_2 = \{q_1, q_2\}$

STOP: $S_3 = S_2 \oplus q_3 = S_2 + (w_3)q_3$

$S_3 = w_1q_1 + w_2q_2 + w_3q_3$.

Graphically this step-by-step-procedure can be displayed as shown in Figure 3. Clearly any other fusion scheme can be thought of, even a dichotomic combination can be performed, see for example Voigt et al. (2005, this issue).

In our example m-1 aggregation steps are possible, with m being the number of indicators. Namely there are 4 original indicators multiplied by 14 river sections minus 1 equals 55. Here we used a two-fold aggregation scheme of the indicators, including a spatial and a thematic aspect: Concerning the spatial aspect of the 14 river sections, two aggregation steps are performed. In the first step of spatial aggregation, the 14 river sections are aggregated into three groups of rivers: The first group, abbreviated "MS", contains all sections of the main rivers Spree, Dahme and Havel (Figure 2: no. 1, 2, 3, 7, 9, 10, 11, 13, 14). The second river group, abbreviated "Tr", contains all tributaries (Figure 2, no. 4, 5, 6, 12), and the third group, abbreviated "Ca" contains the canal (Figure 2, no. 8). In the second step of aggregation, the three

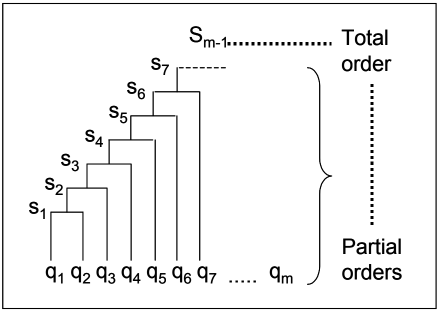

Figure 3: Step-by-step aggregation of indicators

river groups (MS, Tr, Ca) are unified to one section, representing the whole system of surface water. Note that the indicators referencing a river section are combined to indicators referencing now a group of river sections. Concerning the thematic aspect, the original indicators representing phosphorus (P), nitrogen (N), short term pollution (S) and the discharge reduction (Q) are aggregated one-by-one in three steps. Beginning with the four original indicators P, S, Q, N, followed by three indicators, the aggregated PS indicator and the original Q and N indicators. Followed by two indicators, the aggregated PSQ indicator and the original N indicator, and finally by one PSQN aggregated indicator. Note, that spatial and thematic aggregation can also be performed in arbitrary order. The two aspects of indicator aggregation were combined systematically. This leads to a matrix $\mathcal{V}$ of 3*4 evaluation results, which can be represented by a 3*4 matrix of HD's. Figure 4 shows the scheme of the METEOR evaluation matrix $\mathcal{V}$. The rows of $\mathcal{V}$ are the evaluation results, referring to the three different states of spatial aggregation (14 river sections, 3 river groups, 1 river system). The columns of $\mathcal{V}$ are the evaluation results according to the four states of thematic aggregation of the P, S, Q and N indicators. Any entry of $\mathcal{V}$ describes a specific aggregation. In $\mathcal{V}_{1,2}$ for example exclusively thematic aggregation of the original indicators P and S to the aggregated PS indicator is performed. The resulting three indicators, PS, Q and N are applied to all 14 river sections. The $\mathcal{V}_{2,2}$ entry includes both, spatial and thematic aggregation. The three indicators PS, Q and N are now applied to the three river groups MS, Tr and Ca.

2.4 Weighting of indicators

According to the aggregation scheme, the weighting of indicators is considered under two aspects: (1) spatially because the 14 river sections might be considered to be of difference importance, and (2) thematically because the four indicators P, S, Q and N can be given different preferences. In our study indicator weights were defined by the members of the project group. The two-fold weighting scheme of spatial and thematic indicators facilitates the definition of preferences according the principles of the AHP approach (see section 2.3), as only small number of pairs of indicators has to be compared. For the spatial dimension the following relative weights are defined: MS=1.5*Tr and MS=3*Ca. Consequently Tr=2*Ca. The normalized weight are: MS=0.5; Tr=0.33 and Ca=0.17. For the thematic dimension the following relative weights are defined: P=7*N; P=2*Q and P=1.5*S. Consequently Q=3.5*N and S=4.6*N. The normalized weights are: P=0.43; S=0.29; Q=0.22 and N=0.06. Both sets of indicator weights are applied to the two fold aggregation scheme in a way, that the relative preferences of the thematic indicators are maintained within the weighting scheme of the spatial level, and vice versa.

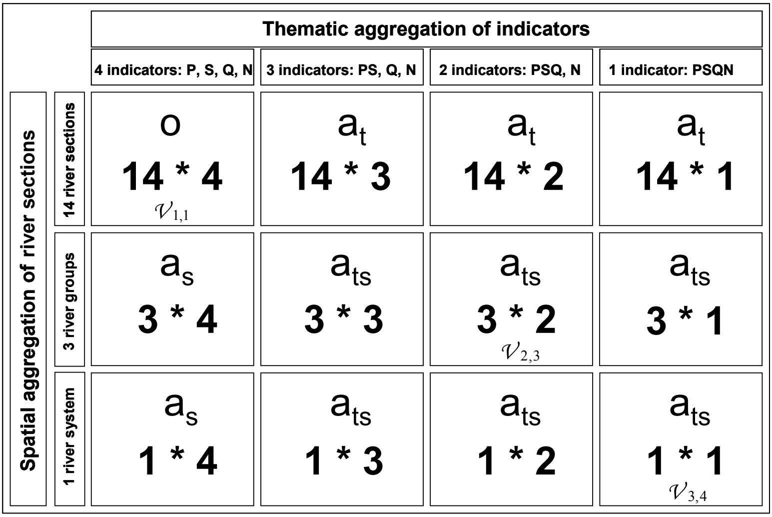

Figure 3: Scheme of the METEOR 3*4 evaluation matrix

The evaluation matrix $\mathcal{V}$ shows the nature of each data set: the original (o), and aggregated spatially (a_s) and thematically (a_t), and the number of indicators.

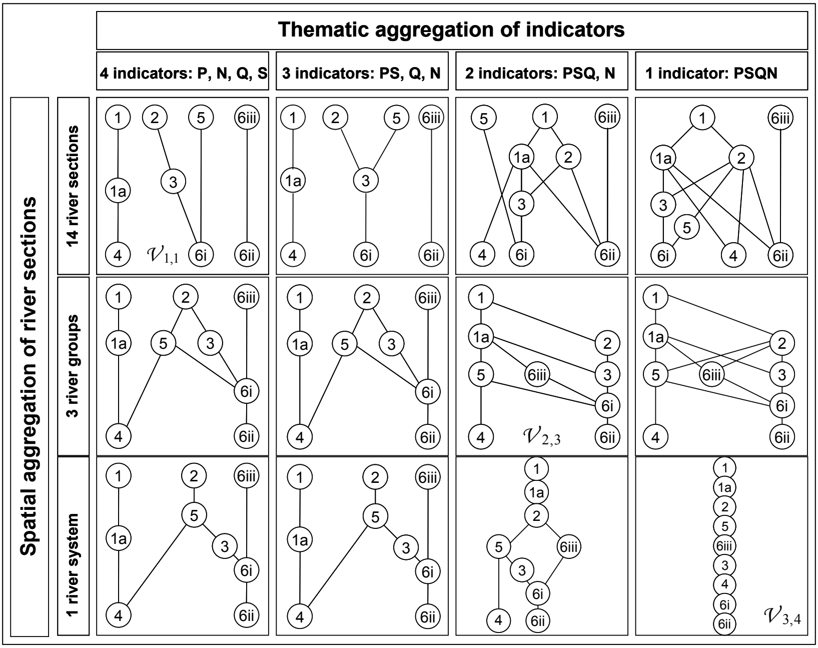

Figure 5: Evaluation result of the aggregation scheme

Graphical display of the matrix $\mathcal{V}$. Entries $\mathcal{V}_{1,1}$ $\mathcal{V}_{1,1}$ and $\mathcal{V}_{1,1}$ discussed in section 3. are highlighted.

2.5 Number of comparabilities

The application of the twofold aggregation scheme will cause structural changes within the sequence of HD’s towards a linear ranking. To describe such changes the number of comparabilities is an appropriate measure. As each aggregation step will reproduce or increase the number of comparabilities, the steps downwards or to the right hand side of the aggregation scheme (Figure 4) can mathematically be characterized as follows: Let *IB*(i),(j) be that specific information base which includes the indicators and aggregated indicators, corresponding to the i^{th} row and j^{th} column. We define the number of comparabilities (VT):

$VT_{IB(i),(j)}$ = card $\{(x,y) \in E,\ x<y \text{ corresponding to } (E, IB(i),(j))\}$.

Then: $\Delta_{(i,j),(k,l)} := VT_{IB(i),(j)} - VT_{IB(k),(l)}$ with $VT_{IB(i),(j)} \geq VT_{IB(k),(l)}$.

The Δ values describe the transitions from one entry of matrix $\mathcal{V}$ to another one following an increasing fusion, either in spatial, in thematic or on both levels simultaneously. Thus the additional comparabilities are counted, arising from a certain aggregation process. Note that additivity $\Delta_{(i,j),(k,l)} + \Delta_{(k,l),(m,n)} = \Delta_{(i,j),(m,n)}$ holds, which follows trivially from the definition of $\Delta_{(i,j),(k,l)}$. As within a linear ranking each of the scenarios is mutually comparable the number of comparabilities is: n*(n-1)/2, n being the number of scenarios. In our example of the evaluation of nine scenarios, we thus have $\frac{9*8}{2}$ comparabilities, and $VT_{IB(3)(4)}=36$.

3. Results and discussion

The discussion and interpretation of the evaluation results (Hasse Diagrams) of the two fold aggregation procedure is supported by several concepts, of which the number of comparabilities and the so called jumps and bumps will be discussed in the following section. For a more general critical discussion of characterising quantities see Pavan (2003).

Number of comparabilities within a HD is as a measure of structural changes among Hasse Diagrams. An increase in the number of comparabilities indicates an unspecified structural change of the evaluation result towards a linear ranking, and thus towards a unique decision. By checking the increase of comparabilities sensitive aggregation steps can be identified. Such sensitive steps, causing remarkable structural changes are highlighted by an abrupt rise of the number of comparabilities. Their identification may give reason to a thorough check of indicator weights. The comparative discussion of the number of comparabilities will be exemplified for three evaluation results: The first example is the very basic evaluation result of

$\mathcal{V}_{1,1}$ without aggregation and weighting of indicators (Figure 5). The HD consists of three hierarchies. Three out of the total of nine scenarios are identified as possible solutions (minimal elements), which, however, are incomparable. These are the scenarios: 4, 6i and 6ii. The number of comparabilities is 8 (Figure 6). This small number (compared to a total of maximal 36 within a linear ranking) indicates, that there are many conflicts among the indicators. Thus this basic evaluation result provides only little decision support in terms of identification of one best solution.

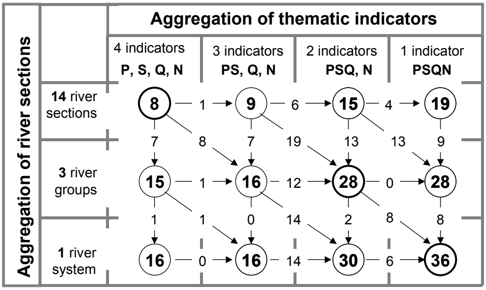

Figure 6: Number of combarabilities within the Hasse Diagrams

Circles show the $VT_{IB(i)(j)}$-values (number of comparabilities), the transition arrows are characterised by $\Delta_{(i,j),(k,l)}$. The number of comparabilities of the $\mathcal{V}_{1,1}$, $\mathcal{V}_{2,3}$ and $\mathcal{V}_{3,4}$ entries of the aggregation scheme are highlighted with bold lines.

The second example is an evaluation result after steps of indicator aggregation causing strong changes in the structure of the HD. In our example the $\mathcal{V}_{2,3}$ entry (Figure 5) is identified as the HD resulting from the most sensitive steps of aggregation. Compared to the previous aggregation steps $\mathcal{V}_{1,2}$ the number of comparabilities is enhanced by 19 to a total of 28 (Figure 6). When compared to the basic evaluation result of $\mathcal{V}_{1,1}$ the number of comparabilities is enhanced by 20 (Figure 6). On the spatial level in $\mathcal{V}_{2,3}$ the original 14 river sections are grouped into three river groups. By this aggregation the incomparability of the scenarios 6i and 6ii is removed, because the tributaries Erpe (river section 4) and Wuhle (river section 5) are aggregated to one group. Based on the analysis of antagonistic indicators it is known, that scenario 6i would have negative impacts to the river Wuhle. The closing of the wwtp Fkb (Figure 1) would increase the phosphorus concentration and would cause a significant reduction of the discharge. Note that by advanced waste water purification the discharge of waste water causes a dilution of the phosphorus concentration in the river and that some of the small rivers contain more than 80% of purified waste water, which will be missing after the closing of the wwpt. In contrast scenario 6ii comprises the closing of the wwtp Mhn. This would affect the discharge of the Erpe river negatively. On the thematic level in $\mathcal{V}_{2,3}$ the original Q indicator is aggregated with the aggregated PS indicator. Analysing the indicator values we found, that the indicators P, S and N have the same trend. Measures of module (B) waste water treatment would cause a reduction of the P and N immission into the surface waters, same as the measures of module (C) entry of storm water, indicated by the S indicator. In contrast, the indicator Q is anti-correlated to the other indicators, as the closing of a wwtp will often reduced the discharge into the respective river section significantly. By aggregation these two counteracting effects are compensated for. The importance to the Q indicator should be the reason to check it's ecological significance, and possibly also it's weighting compared to chemical parameters such as nutrient loads.

The third example is the evaluation result after steps of aggregation which lead to a linear ranking of the options. In our example, a linear ranking of all scenarios is not obtained until the very last steps of aggregation ($\mathcal{V}_{3,4}$): 6ii < 6i < 4 < 3 < 6iii < 5 < 2 < 1a < 1, with scenario 6ii being ranked best and scenario 1 the worst (Figure 5). Compared to the neighbored entries $\mathcal{V}_{3,4}$, $\mathcal{V}_{3,4}$ and $\mathcal{V}_{3,4}$ the main enhancement of comparabilities arises from the assignment of the river sections into three groups. The thematic aggregation of PSQ ⊕ N, however, plays a slightily minor role. From Figure 5 one is motivated to discuss the following three questions:

(1) How long does a cover-relation starting from $\mathcal{V}_{1,1}$ persists?

A cover relation is an order relation x < y, if there is no object (here scenario) $z \in E$, for which x < z and z < y in (*E*, *IB*) (Schröder 2003).

(2) When will a ||-relation be transformed into a < -relation?

(3) Does the final linear order belongs to the set of jump-minimal linear extensions? (see Figure 7 for an explanation).

(1) In our example the cover-relation of the scenarios (1, 1a) persists throughout the whole aggregation procedure. By the weighting and aggregation of indicators, no other scenarios are ranked such that the cover-relation 1 > 1a is destroyed. The reason for persistance of the cover-relation of 1 > 1a can be that, both scenarios comprise the same measures within the modules (B) "waste water treatment" and (C) "entry of storm water". The only advantage of scenario 1a is attributed to the better evaluation of the Q indicator. This advantage, however, is addressed to the (A) module "hydrological boundary conditions". In scenario 1a the amount of water entering the study site is not reduced, because fooding of open pit casts has not yet started. All other scenarios 2, 3, 4, 5, 6i, 6ii and 6iii share this disadvantage of scenario 1, namely the reduction of water discharge. However, compared to 1a they have several advantages such as advanced waste water treatment. For these reasons the weighting of indicators cannot resolve the cover relation and the final linear order $\mathcal{V}_{3,4}$ contains scenarios 1 > 1a as a "cover bump".

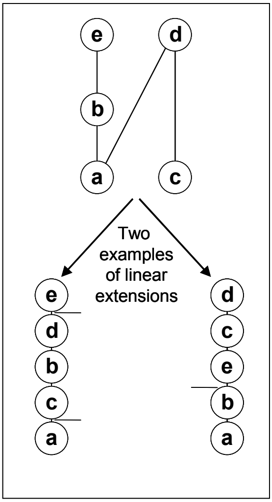

Figure 7: Demonstration of the jump, bump concept.

In the linear extension, see Trotter (1991) for details, on the left hand side, the chain-structure is heavily perturbed, the number of jumps equals 2 (Bouchitte & Habib 1989). On the right hand side a jump-minimal linear extension is shown, as this linear order reproduce optimally the chains of the original poset. Here both chains b > a and d > c are still visible in the linear extension, corresponding to a jump number of only 1.

(2) In the $\mathcal{V}_{1,1}$ matrix, scenario 4 is incomparable to 6ii. This ||-relation is maintained until the very last steps of spatial and thematic aggregation, forcing a linear order with scenarios 4 > 6i > 6ii. The reasons for the persistence of the 4 || 6ii -relation can be explained by the antagonistic indicators, and by the order of the indicator aggregation. From the analysis of the

antagonistic indicators we know that the incomparability between scenarios 4 and 6ii arises from both, spatial and thematic indicators. Scenario 4 is less favourable with respect to the N indicator for 11 of the 14 river sections, which belong to all three river groups. Scenario 6ii is less favourable with respect to the Q indicator only in the Wuhle river section. By exclusive thematic aggregation of the antagonistic indicators Q and N ($\mathcal{V}_{1,1}$ to $\mathcal{V}_{1,4}$) the conflict between N and Q will disappear, nevertheless the 4 || 6ii relation cannot be resolved, because spatial antagonisms remain. The 11 conflictive river sections are spread over all three river groups. By exclusive spatial aggregation of river sections ($\mathcal{V}_{1,1}$ to, $\mathcal{V}_{3,1}$) the 4 || 6ii relation is also maintained because of the above-mentioned thematic antagonisms. Consequently the relation of scenarios 4 and 6ii is not destroyed until the very last steps of aggregation $\mathcal{V}_{3,4}$.

(3) If the chain of scenarios 6iii > 6ii > 6i, in $\mathcal{V}_{2,3}$ all comprising alternative sanitary technique, is compared with the final linear ranking in $\mathcal{V}_{3,4}$, then it can be observed that the cover-relations 6iii > 6ii and 6ii > 6i are lost. Generalising this observation one may ask whether or not the weighting schemes tends to preserve chains, found for example in the interim aggregation step $\mathcal{V}_{2,3}$. Expressed in technical terms: Does the weighting scheme lead to a linear ranking as a greedy linear extension with respect to the poset in $\mathcal{V}_{2,3}$ (i.e. a linear extension with a minimal number of jumps)? With respect to $\mathcal{V}_{2,3}$, 6 jumps are found in $\mathcal{V}_{3,4}$. It is easy to show that the final ranking of the scenarios 1 > 2 > 1a > 5 > 3 > 4 > 6iii > 6i > 6ii is compatible with the HD in $\mathcal{V}_{2,3}$ however, with only 4 jumps (compared to the poset in $\mathcal{V}_{2,3}$). Therefore $\mathcal{V}_{3,4}$ is not a jump-minimal linear extension. It does not necessarily preserve chains found during interim aggregation steps, which in turn may be of contextual interest.

4. Conclusion

METEOR is a new elementary decision support tool based on Discrete Mathematics. The approach combines the advantages of "classical" DSS such as AHP (Saaty 1982, 1994), PROMETHEE (Brans & Vincke 1985), NAIADE (Matarazzo & Munda 2001) and MAUT (Schneeweiss, 1991) which allow participation of stakeholders and always provide a linear ranking with one winner solution, and the advantages associated with discrete approaches such as the HDT, which provides high transparency throughout the whole evaluation process. Discussion and interpretation of the data is supported by several concepts such as the width of the HD, the number of comparabilities or jumps and bumps; for a critical discussion of characterising quantities see for example Pavan (2003). We would like to encourage stakeholders and researchers to carefully investigate the effects of step-by-step indicator aggregation, even though it might be a more time consuming method of decision making, compared to simultaneous aggregation of all indicators. Step-by-step aggregation of indicators provides the advantage to unmask the weighting camouflage in decision support systems. We propose furthermore that the concept of jumps and bumps has to be studied in detail, as it may be a useful tool for the characterisation - with respect to order preserving aggregation - of the different partial orders in METEOR. The meaning of robust ranking relations of options in practical application has to be investigated.

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Appemdix: Basic evaluation matrix

	1-P	1-N	1-Q	1-S	2-P	2-N	2-Q	2-S	3-P	3-N	3-Q	3-S	4-P	4-N	4-Q	4-S	5-P	5-N	5-Q	5-S	6-P	6-N	6-Q	6-S	7-P	7-N	7-Q	7-S
1a	0.07	3.03	0.00	2.00	0.11	4.45	0.00	2.00	0.15	5.05	0.00	2.00	0.37	13.80	0.00	2.00	0.49	22.90	0.00	2.00	0.00	31.47	0.00	2.00	0.01	1.99	0.00	0.00
1	0.08	3.40	0.00	2.00	0.14	4.96	0.00	2.00	0.19	5.62	0.00	2.00	0.37	13.80	0.00	2.00	0.49	22.90	0.00	2.00	0.00	31.47	0.00	2.00	0.02	1.99	0.00	0.00
2	0.07	3.03	0.00	2.00	0.10	3.00	0.00	2.00	0.15	3.65	0.00	2.00	0.23	7.60	0.00	2.00	0.15	1.93	0.80	2.00	0.00	21.07	0.00	2.00	0.02	1.99	0.00	0.00
3	0.02	2.70	0.00	1.00	0.04	2.60	0.00	1.00	0.05	3.65	0.00	1.00	0.03	6.70	0.00	1.00	0.15	1.93	0.80	1.00	0.00	11.17	0.00	1.00	0.00	1.99	0.00	0.00
4	0.02	2.70	0.00	1.00	0.04	3.30	0.00	1.00	0.05	4.18	0.00	1.00	0.03	6.70	0.00	1.00	0.04	8.99	0.00	1.00	0.00	11.17	0.00	1.00	0.00	1.99	0.00	0.00
5	0.02	3.03	0.00	1.00	0.04	3.00	0.00	1.00	0.05	3.89	0.00	1.00	0.03	6.70	0.00	1.00	0.15	1.93	0.80	1.00	0.00	11.17	0.00	1.00	0.01	3.10	0.00	0.00
6I	0.02	2.50	0.00	1.00	0.04	2.60	0.00	1.00	0.05	2.83	0.00	1.00	0.03	2.50	0.00	1.00	0.15	1.93	0.80	1.00	0.00	3.83	0.00	1.00	0.00	1.99	0.00	0.00
6II	0.02	2.50	0.00	1.00	0.04	2.60	0.00	1.00	0.05	2.83	0.00	1.00	0.03	2.50	0.20	1.00	0.04	1.93	0.00	1.00	0.00	3.83	0.00	1.00	0.00	1.99	0.00	0.00
6III	0.02	2.50	0.00	1.00	0.04	2.60	0.00	1.00	0.05	2.83	0.00	1.00	0.03	2.50	0.20	1.00	0.04	1.93	0.00	1.00	0.10	3.83	0.80	1.00	0.00	1.99	0.00	0.00

	8-P	8-N	8-Q	8-S	9-P	9-N	9-Q	9-S	10-P	10-N	10-Q	10-S	11-P	11-N	11-Q	11-S	12-P	12-N	12-Q	12-S	13-P	13-N	13-Q	13-S	14-P	14-N	14-Q	14-S
1a	0.21	8.10	0.00	2.00	0.03	4.86	0.00	2.00	0.14	5.90	0.00	2.00	0.12	5.35	0.00	2.00	0.08	3.55	0.00	2.00	0.18	6.15	0.00	2.00	0.15	5.60	0.00	2.00
1	0.28	9.21	0.00	2.00	0.03	4.86	0.00	2.00	0.16	5.90	0.00	2.00	0.15	5.35	0.00	2.00	0.08	3.55	0.00	2.00	0.18	6.15	0.00	2.00	0.15	5.60	0.00	2.00
2	0.28	8.10	0.00	2.00	0.00	3.91	0.00	2.00	0.13	4.70	0.00	2.00	0.12	4.40	0.00	2.00	0.08	3.55	0.00	2.00	0.14	4.78	0.00	2.00	0.13	4.55	0.00	2.00
3	0.05	5.70	0.00	1.00	0.00	2.70	0.00	1.00	0.03	3.77	0.00	1.00	0.03	3.63	0.00	1.00	0.04	3.20	0.00	1.00	0.03	3.85	0.00	1.00	0.03	3.70	0.00	1.00
4	0.05	4.90	0.00	1.00	0.00	2.24	0.00	1.00	0.03	3.77	0.00	1.00	0.03	3.63	0.00	1.00	0.04	3.20	0.00	1.00	0.03	3.85	0.00	1.00	0.03	3.70	0.00	1.00
5	0.05	4.90	0.00	1.00	0.00	2.70	0.00	1.00	0.03	3.77	0.00	1.00	0.03	3.63	0.00	1.00	0.04	3.55	0.00	1.00	0.03	3.85	0.00	1.00	0.03	3.70	0.00	1.00
6I	0.05	3.00	0.00	1.00	0.00	1.63	0.00	1.00	0.03	2.50	0.00	1.00	0.03	2.60	0.00	1.00	0.04	3.20	0.00	1.00	0.03	2.50	0.00	1.00	0.03	2.50	0.00	1.00
6II	0.05	3.00	0.00	1.00	0.00	1.63	0.00	1.00	0.03	2.50	0.00	1.00	0.03	2.60	0.00	1.00	0.04	3.20	0.00	1.00	0.03	2.50	0.00	1.00	0.03	2.50	0.00	1.00
6III	0.05	3.00	0.00	1.00	0.00	1.63	0.00	1.00	0.03	2.50	0.00	1.00	0.03	2.60	0.00	1.00	0.04	3.20	0.00	1.00	0.03	2.50	0.00	1.00	0.03	2.50	0.00	1.00