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FORMAL CONCEPT ANALYSIS AND CHEMOMETRICS: CHEMICAL COMPOSITION OF ANCIENT EGYPTIAN BRONZE ARTIFACTS

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Abstract Formal concept analysis (FCA) developed by R. WILLE and his coworkers in Darmstadt/Germany [1–3] (see also [11–25]) is a very effective method for exploring all kinds of data. Therefore, it have been used in archeometry [4, 5], too. This paper also seeks to introduce the basic ideas of FCA. Thereafter, a new method for classifying objects developed recently is represented. As an example, it is used to classify ancient Egyptian bronze artifacts according to their chemical composition.

Formal Concept Analysis

The basis notion in FCA is a triple (G, M, I) called formal context. G is the set of objects under consideration and M the set of their (original or derived, see below) attributes. An element gIm ($g \in G$, $m \in M$) of the binary relation $I \subseteq G \times M$ ($gIm \equiv (g, m) \in I$) is read: "Object g has attribute m". Another form for representing a context (G, M, I) is a cross table. Its rows are labeled by the objects G and its columns by the attributes M. An "**X**" at the position of row g with column m means that gIm holds.

Usually, we start in data analysis with more than only a single value ("**X**") in the data matrix or table. This data structure is given by a many-valued context (G, M, Z, R). G and M are the sets like those mentioned above. The elements of set Z are the values (with $|Z| \geq 2$). The relation $R \subseteq G \times M \times Z$ now is a ternary one. Its element (g, m, z) is read: "Object g has the value z for the attribute m". If (g, m, z_1) and (g, m, z_2) are true, then $z_1 = z_2$.

Let us briefly summarize other basic ideas and definitions of FCA using the example of ten Middle-Kingdom (XI or XII dynasty) gold and silver alloys taken from [6]. The contents of gold (Au), silver (Ag) and copper (Cu) can be seen in Table 1 and in Figure 1. For our purpose, the four averages shown in Table 1 are sufficient for constructing the many-valued context (G_A, M_A, Z_A, R_A) (see Table 2). The four objects of G_A now represent the classes of gold and silver alloys known in Ancient Egypt: G_A = {*nb* [gold], *nb ḥḏ* [white gold], *ḏʿm* [electrum], *ḥḏ* [silver]}. M_A contains the contents of gold, silver and copper, respectively. These attributes are denoted by their corresponding symbols of the chemical element: M_A = {Au, Ag, Cu}. The values of Z_A are the numbers given in Table 2 and the sign "–" (none).

The aim of FCA is to find (formal) concepts and to construct the hierarchy between them. For doing that, the many-valued context must be transformed by conceptual scaling to a single-valued one: $(G_A, M_A, Z_A, R_A) \rightarrow (G_A, A_A, I_A)$ [9, 4]. For each of the attributes $m \in M_A$, a context (G_m A_m, I_m) called its scale must be given. The object set G_m is equal to the set $z(m) \in Z$ of all the values belonging to m. G_m covers all different values of the column m in the many-valued context. M_m and I_m should reflect some conceptual meaning connected with the attribute m. Thus, scaling is always an action of interpretation of the data given in the many-valued context (data matrix).

Table 1
Ten gold and silver objects from [6]
(* average values)

sample	***content*** [%]					
n^o^	Au	Au^*	Ag	Ag^*	Cu	Cu^*
1	94.8	92.7	3.7	4.4	–	–
2	92.7		4.9		–	
3	90.5		4.5		–	
4	85.9	84.4	13.8	15.2	0.3	0.4
5	82.9		16.6		0.5	
6	80.1	78.6	20.3	21.2	–	–
7	78.7		20.9		–	
8	78.2		21.1		–	
9	77.3		22.3		–	
10	0.7	0.7	91.8	91.8	–	–

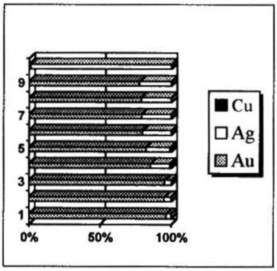
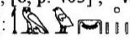


Fig. 1 Relative contents of the ten samples

For scaling the example context the scales shown in the first row of Table 3 are chosen. Their attributes are denoted by "w" (little), "m" (middle), "v" (much), "sw" (very little) and "0" (none), respectively. The meaning of the nominal scale SCCu is classifying (classes "none" and "little"). Biordinal scaling means ranking and classifying the values of the corresponding attribute. This type of scale was used for scaling the attributes Au and Ag (class "very little" and class "at least little" subdivided by the ordination "little < middle < much"). For instance, the four attributes swAu, wAu, mAu and vAu will be in the derived context instead of Au in the original one. Thus, the value 84.4 [% Au] will be replaced with the pattern: $84.4 \rightarrow \begin{array}{cccc} (\mathrm{swAu} & \mathrm{wAu} & \mathrm{mAu} & \mathrm{vAu}) \\ & \mathrm{X} & \mathrm{X} & \end{array}$, because an alloy that has a content of 84.4 mass-% Au contains both little and middle amounts of this metal, but it does not fall into the class "very little content of gold". The derived single-valued context AUAGCU is shown at the bottom of Table 3.

Table 2 The many-value example context

				attributes		
				content [%]		
	sample n°	***name***	***translit.***	Au	Ag	Cu
o b j e c t s	*1...3*		*nb*(*w*)[a]	92.7	4.4	–
	4...5	, 	*nb ḥḏ*[b]	84.4	15.2	0.4
	6...9		*ḏꜥm*[c]	78.6	21.2	–
	10	, 	*ḥḏ*[d]	0.7	91.8	–

[a] Wb. II 237, [8, p. 403] , [b] Wb. II 237 9.10, [8, p. 403],
[c] Wb. V 537, [8, p. 999]:  *ḏꜥmw*, [d] Wb. III 209, [8, p. 574]. [*] [7]

Now we define a (formal) concept corresponding to a given context ((G, M, I)): If X' is the set of attributes shared by all objects of the subset $X \subseteq G$,

$$X' := \{m \in M \mid gIm \text{ for all } g \in X\},$$

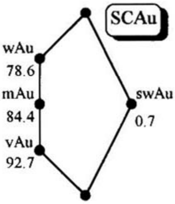
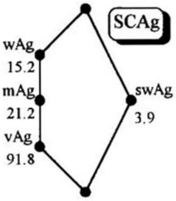
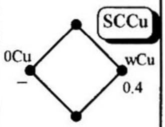
and Y' the set of objects sharing all attributes of the subset $Y \subseteq M$,

$$Y' := \{g \in G \mid gIm \text{ for all } m \in Y\},$$

then a formal concept can be defined as the pair (X, Y), if $X' = Y$ and $Y' = X$ hold. The subset X is called the extent and the subset Y the intent of the concept (X, Y).

In the case of the context AUAGCU, 13 concepts were found. For instance, ({*ḥḏ*, *ḏꜥm*}, {wAg, mAg, 0Cu}) is the concept with at least a middle content of silver and no copper. It has two metal types in its extent: silver (*ḥḏ*) and electrum (*ḏʿm*).

Table 3 Scaling of the example context and the derived context AUAGCU

<table>
<tr><td><i>scale name</i></td><td>SCAu</td><td>SCAg</td><td>SCCu</td></tr>
<tr><td><i>context</i></td><td><pre>
      s
      wwmv
      AAAA
      uuuu
 0.7| X...
78.6| .X..
84.4| .XX.
92.7| .XXX
</pre></td><td><pre>
      s
      wwmv
      AAAA
      gggg
 4.4| X...
15.2| .X..
21.2| .XX.
91.8| .XXX
</pre></td><td><pre>
       0w
       CC
       uu
   - | X.
 0.4 | .X
</pre></td></tr>
<tr><td><i>scale</i></td><td>biordinal</td><td>biordinal</td><td>nominal</td></tr>
<tr><td><i>line diagram</i></td><td>
</td><td>
</td><td>
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</table>

⇓ ⇓ ⇓

AUAGCU	Au				Ag				Cu	
	swAu	wAu	mAu	vAu	swAg	wAg	mAg	vAg	0Cu	wCu
nb		X	X	X	X				X	
nb ḥḏ		X	X			X				X
ḏʿm		X				X	X		X	
ḥḏ	X					X	X	X	X	

In the set of the concepts $\mathfrak{B}(G, M, I)$ belonging to the context (G, M, I) a partial ordering denoted by $\leq$ (subconcept-superconcept relation) can be defined: concept (X, Y) is subconcept of the (super)concept (U, V) if the extent U contains all the objects of the extent X and the intent Y has all the attributes of the intent V,

$$(X, Y) \leq (U, V) :\Leftrightarrow X \subseteq U \; (\Leftrightarrow V \subseteq Y).$$

The partially ordered set $(\mathfrak{B}(G,M,I), \leq)$ has the mathematical structure of a complete lattice, and is called the concept lattice of the context (G,M,I). It reflects the generalized hierarchy and conceptual structure corresponding to (G,M,I).

Concept lattices are effectively visualized by labeled line diagrams. Little circles or dots represent the concepts and the ascending paths of line segments represent the relation $\leq$. A name of an object g is attached to the circle representing that concept which is the smallest one having g in its extent (its object concept), while a name of an attribute m is attached to the circle representing that concept which is the greatest one having m in its intent (its attribute concept). To construct the intent of any concept in this line diagram one combines all attributes attached to dots which may be reached by ascending paths. For the construction of the extent of any concept you have to combine all objects attached to dots which may be reached by descending paths.

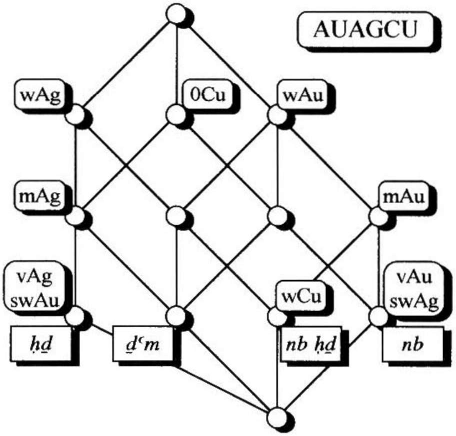

Fig. 2 **Line diagram of the context AUAGCU**

The line diagram of the context AUAGCU is shown in Figure 2. The line diagrams of the two biordinal scales SCAu and SCAg, respectively, and the nominal one SCCu are represented in Table 3. Their conceptual meanings (ordination and classification) can clearly be seen. Figure 2 shows a part of a so-called grid scale.

Because all the object concepts are the nearest (upper) neighbors of the lowest concept that contains no object, it is possible to conclude that the four metal types represent four different classes. In contrast to this result of FCA, both LUCAS & HARRIS [6] and the "Wörterbuch" [7] (Wb. II 237.9,10) put *nb ḥḏ* (white gold) into one class together with *nb* (gold)[1], but it can clearly be distinguished from this metal type. Thus, two pure metal types (*nb, ḥḏ*) and two alloy ones (*nb ḥḏ, ḏʿm*) can be determined in the "universe" of ancient Egyptian gold and silver objects. If Figures 1 and 2 are compared it is easily seen that the line diagram of the lattice structure yields more information than the usual statistical plot. But both figures show that *nb ḥḏ* can be distinguished from *ḏʿm* with respect to the fact that copper only occurs in the former one. Note, that FCA already yields useful results in this example case considered here.

A New Method of Classification Based on Formal Concept Analysis

Using the contents of copper (Cu), tin (Sn), lead (Pb) and arsenic (As) of 56 samples of Egyptian bronze[2] artifacts, it will be demonstrated how classes of the objects can be generated by a method developed in 1995.[3] The element concentrations measured by J. RIEDERER (RADTKEN Laboratory, Berlin) [10] are shown in Table 4. This 56 samples are the objects of *G* of the many-valued context BRONZE = (*G, M, Z, R*) that we now have to study. Its attributes are the four element contents, *M* = {Cu, Sn, Pb, As}. The order in Table 4 is a chronological one.

Before scaling, intervals of the several contents (columns) were constructed as is shown in Table 5. There can also be seen what scales were chosen[4]. Table 6 represents the derived context BRONZEDV.

The attributes now are denoted by "*iE*", where *i* the interval number given in Table 6 and *E* the corresponding element symbol. Using the program ConImp by P. BURMEISTER [11], 196 concepts were found (|𝔅(BRONZDV)| = 196). This high degree of complexity is a usual result when investigating contexts with a large number of both, objects and attributes.

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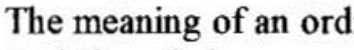
[1] HANNIG [8, p. 403] writes "*Weißgold" where the asterisk means "uncertain, having to be proved".

[2]  … *ḥzmn*, [7] (Wb. III 163), [8, p. 562].

[3] For the first time it was represented at the *First International Conference on Ancient Egyptian Mining, Metallurgy and Conservation of Metallic Artifacts* in Cairo/Egypt (April 1995).

[4] The meaning of an ordinal scale used for Cu and Sn is only ranking. The line diagram of such a scale is a chain.

Table 4 The many-valued context BRONZE (The values are rounded)

n°	*Cu* %	*Sn* %	*Pb* %	*As* 0.01%	***n°***	*Cu* %	*Sn* %	*Pb* %	*As* 0.01%
17	82	3	13	47	***33***	91	2	7	13
54	84	4	12	70	***29***	86	12	2	11
16b	81	5	14	30	***65***	87	7	6	14
1	81	5	13	55	***28a***	89	5	6	12
23	80	4	15	47	***28b***	90	6	4	12
58	82	5	12	54	***28c***	92	6	2	9
66a	82	4	13	42	***5***	86	10	3	10
4	84	2	13	31	***32b***	87	9	3	11
20b	74	4	22	12	***69***	87	8	4	15
20a	76	5	18	16	***2***	89	7	3	10
52	83	6	10	12	***53***	88	7	5	16
32a	73	6	18	51	***67***	90	8	2	22
57	78	6	14	9	***19***	88	11	0	8
7	77	10	12	19	***8***	90	9	0	16
63	82	4	13	11	***22***	90	10	0	26
60	84	6	10	19	***21***	80	12	8	0
15	84	9	6	19	***62***	82	8	10	0
24	85	8	7	60	***56***	84	10	6	0
3	80	9	10	13	***12***	88	10	0	0
11	83	7	9	59	***59***	87	1	0	0
61	85	7	7	55	***27***	89	10	1	0
64	83	7	9	17	***55***	87	9	3	0
49	88	6	6	12	***31***	85	8	5	74
51	89	1	9	77	***36***	69	5	23	163
16a	87	3	9	39	***50***	74	22	3	18
25	87	6	7	11	***26***	58	8	34	0
9	88	3	8	9	***18***	73	2	18	80
66b	89	3	7	44	***48***	95	0	2	36

Table 5 Intervals for scaling the context BRONZE
(*nr*: number of objects in the interval)

element	*Cu*		*Sn*		*Pb*		*As*	
	value(s)	*nr*	*value(s)*	*nr*	*value(s)*	*nr*	*value(s)*	*nr*
Interval 0					0	5	0	8
Interval 1	58 - 69	2	0 - 3	10	1 - 5	14	8 - 22	28
Interval 2	73 - 78	7	4 - 6	19	6 - 10	20	26 - 36	4
Interval 3	80 - 85	21	7 - 9	17	12 - 15	11	39 - 60	11
Interval 4	86 - 90	23	10 - 12	9	18	3	70 - 80	4
Interval 5	91 - 95	3	22	1	22 - 34	3	163	1
scaling	ordinal		ordinal		biordinal		biordinal	

Table 6 The context BRONZEDV[5]

1234512345012345012345 CCCCCSSSSSPPPPPPAAAAAA uuuuunnnnnbbbbbbssssss	
XXXX.XXX...X.....X....	67
XXX..X.....XXX...XX...	4
XXX..XXX...X.....XXXX.	31
XXXXXX.....XX....X....	33
X....XX....XXXXX.XXXXX	36
XXX..XX....XX....X....	60
XXXX.XXX...X.....X....	53
XXX..XXX...XX....X....	64
XXXX.XXX...X.....X....	2
XXXX.X.....XX....X....	9
XXX..XXX...XX....XXX..	11
XXXX.XXXX.X......XX...	22
XXXX.XX....XX....X....	25
X....XXX...XXXXXX.....	26
XXXX.XXX...X....X.....	55
XXX..XXXX..XX...X.....	56
XXX..XX....XXX...XXX..	58
XXXX.X....X.....X.....	59
XXX..XXX...XX....XXX..	61
XXX..XX....XXX...X....	63
XXX..XX....XXX...XXX..	66a
XXXX.X.....XX....XXX..	66b
XXXX.XXX...X.....X....	69
XXX..XX....XXX...XXX..	1
XX...XXXX..XXX...X....	7
XXX..XX....XXX...XXX..	23
XXX..XXX...XX...X.....	62
XXXX.X.....XX....XXX..	16a
XXX..XX....XXX...XX...	16b
XXX..XXXX..XX...X.....	21
XX...XXXXX.X.....X....	50
XX...XX....XXX...X....	57
XXXX.XXXX..X.....X....	5
XXXX.XXXX.X.....X.....	12
XXX..X.....XXX...XXX..	17
XX...XX....XXXX..XXX..	32a
XXXX.XXX...X.....X....	32b
XXXX.X.....XX....XXXX.	51
XXX..XX....XXX...XXXX.	54
XXX..XXX...XX....X....	3
XXX..XX....XX....X....	52
XX...XX....XXXX..X....	20a
XX...XX....XXXXX.X....	20b
XXXX.XXX..X......X....	8
XXXX.XX....XX....X....	49
XXX..XXX...XX....XXX..	24
XXXX.XXXX..X....X.....	27
XX...X.....XXXX..XXXX.	18
XXXX.XXXX.X......X....	19
XXXX.XX....XX....X....	28a
XXXX.XX....X.....X....	28b
XXXXXXX....X.....X....	28c
XXXX.XXXX..X.....X....	29
XXXX.XXX...XX....X....	65
XXXXXX.....X.....XX...	48
XXX..XXX...XX....X....	15

The method that should be described here was developed to classify the objects and to perceive the intentional background of the grouping by only studying the so-called object ordering. The graph of this ordering is a part of the line diagram of the concept lattice. In contrast with the complete line diagram, the only vertices (dots) are the object concepts and their predecessors (lower neighbors) and successors (upper neighbors). A concept is called an object concept γg of an object g if it has the smallest extent which contains g: $\gamma g = (\{g\}'', \{g\}')$. The neighborhood relation is represented by an edge (line) in the graph of object ordering.

If a concept does not have any successor in the object ordering diagram (as the gray dots in Figure 3), its associated object is the representative one for a class, and its intent is the minimal one of this class. Objects of a concept which have a single successor belong to the class of this upper neighbor (bold lines in Figure

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[5] The attribute codes $\overset{i}{E}$ are equivalent to iE, e.g. $\begin{matrix}\mathbf{3}\\ \mathbf{S}\\ \mathbf{n}\end{matrix} \equiv \mathbf{3\,Sn}$.

3). In Figure 3, the numbers given in the circles represent that one of the sample (see Table 4). Table 7 shows lists of identical objects having the same attribute pattern in the context. Of course, they must be put in that class to which their corresponding representative belongs. As it can be seen from Figure 3 that 15 classes exist.

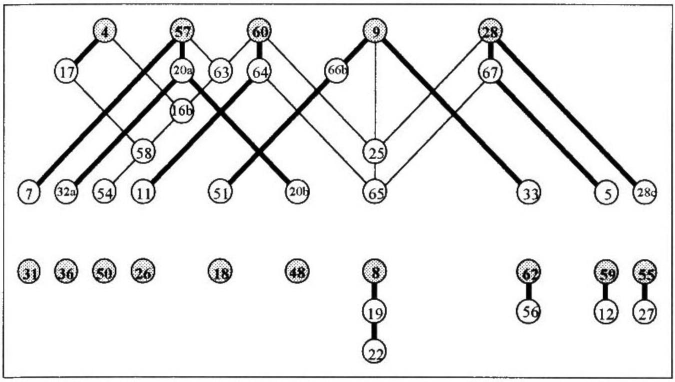

Fig. 3 **The object ordering diagram of the context BRONZEDV**

Table 7 **Identical objects**

object(s)		
repres.	*identical*	*class*
58	1 23 66a	1
60	52	2
11	24 61	3
64	3 15	3
25	49	4
66b	16a	4
5	28a 29	5
67	2 32b 53 69	5
56	21	7

For constructing disjoint classes, each object of a concept with two or more successors must be attached to one and only one of its upper neighbors. The surprise values $s(m)$ calculated for each attribute $m \in M$ should be used to solve this problem. Let $p(m)$ be the frequency of the attribute m, then its surprise value is defined as $s(m) = -\ p(m) \cdot \ln p(m)$ or $s(m) = -100 \cdot e \cdot p(m) \cdot \ln p(m)$ [%] ($e \approx 2.71828$). For calculating the frequency, the extensity $v(m)$ of the attribute m must be determined: $v(m) = |\{m\}'|/|G^*|$. Here $|\{m\}'|$ is the number of objects which have the attribute m and $|G^*|$ is the number of (representative) objects with respect to the context (G, M, I).

Using this attribute extensities as frequencies, the extensive surprise value $s_e(m)$ can be calculated: $s_e(m) = -100\cdot e\cdot v(m)\cdot \ln v(m)$. Because a concept always has a greater attribute number in its intent than each of its predecessors, the sum of the extensive surprise values of the attributes which are lost ascending in the diagram of the object ordering from a concept to its successor may be regarded as a number measuring the degree of neighborhood between the concept considered and its predecessor. In this way, the extensive surprise values may be used as a criterion which allows choosing that class to which an object is to be assigned: Objects must be put in the same class if the degree of neighborhood between them has minimal value. The intents of the lowest object concepts belonging to a class represent its maximal intent.

Table 8 Extensities and extensive surprise values of attributes m

m	3Cu	4Cu	2Sn	3Sn	2Pb	3Pb	2As	3As	4As
$39v(m)$	30	18	30	17	25	14	14	10	5
$s_e(m)$ [%]	55	97	55	98	78	100	100	95	72

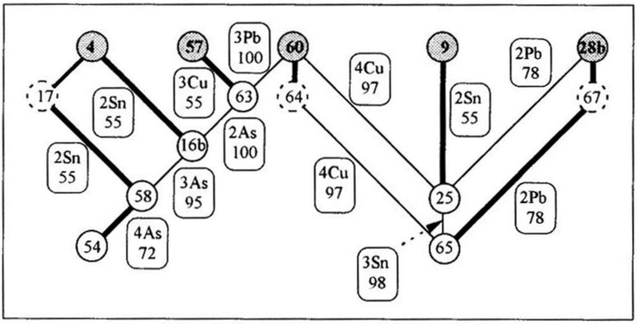

Fig. 4 Finding of the class for objects with more than one successor

The extensive surprise values of interest are given in Table 8 ($|G^*| = 39$). Figure 4 shows the part of object ordering graph which can be obtained using these data. The classification of the bronze samples is complete now.

Archaeometric Conclusions

The results derived are represented in Figure 5 and Table 9. It can be concluded that the classification obtained does not correlate with the period of production of the artifacts. But it can be suggested that it reflects different provenance of the bronzes and technological aspects including perhaps recycling.

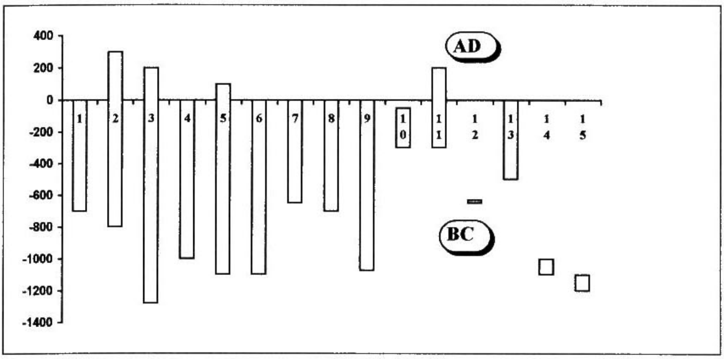

Fig. 5 The classes and periods of production

Table 9 The 15 classes and their corresponding concentration intervals

	Cu			Sn			Pb			As		
class	max.	counts	min.	max.	counts	min.	max.	counts	min.	max.	counts	min.
1		3		2	–	1		3		4	–	2
2	3	–	2	4	–	2	4	–	3	3	–	1
3		3		3	–	2		2		3	–	1
4	5	–	4	2	–	1		2		4	–	1
5	5	–	4	4	–	2	2	–	1		1	
6		4		4	–	3		0		2	–	1
7		3		4	–	3		2			0	
8		4		4	–	1		0			0	
9		4		4	–	3		1			0	
10		4			3			1			4	
11		1			2			5			5	
12		2			5			1			1	
13		1			3			5			0	
14		2			1			4			4	
15		5			1			1			2	

Appendix: Advantages of Formal Concept Analysis

To uncover the main differences between graphical methods in data analysis such as factor analysis, principal component analysis, correspondence analysis, cluster analysis, partial order scalogram analysis, multidimensional scaling and FCA, their principles, advantages and disadvantages are discussed in [26–30]. As a general result it could be shown that only FCA may represent the original data without loss of information.

FCA is a set-theoretic method based on both the philosophical understanding of a concept and German standards for concepts and conceptual systems. Since this concept consists of two parts (extension and intention) the advantageous peculiarities of FCA concern not only the mentioned visualization of data without loss of information but also the characterization of both the extensional and intensional data structure connected with the data under consideration.

Using FCA, the complete set of "*if ... then ...*" rules (attribute implications) corresponding to the conceptual structure of the data may be derived. There is no other method which is able to yield such results in the analysis of given data.

Finally, the conclusion obtained by T.P. REINARTZ and M. ZICKWOLFF comparing hierarchical conceptual clustering and FCA should be mentioned: *"All in all, we do not emphasize one method or another. Instead, we conclude that specific application requirements must guide the decision of appropriateness. ... each user may decide if the extra effort of formal concept analysis is worth in the specific application scenario."* [30, p. 414].

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