NORMAL HOMOGENEOUS PARTITIONS OF THREE-DIMENSIONAL EUCLIDEAN SPACE WHICH ARE NOT PARTITIONS INTO FUNDAMENTAL REGIONS OF A SPACE GROUP.

Werner Fischer, Institut für Mineralogie, Lahnberge, D 3550 Marburg.

For $n \geq 3$, it was not known so far whether there exists a normal homogeneous space partition of R^n into convex polyhedra, which does not form a partition into fundamental regions (asymmetric units) of a space group of R^n (Delone, 1961). In this connexion, *normal* means that adjacent polyhedra share entire faces, and *homogeneous* implies that the set of all polyhedra of the space partition forms an orbit under the action of some space group. For n=3, a related problem has been solved in crystallography. There exist exactly five Wyckoff positions the point configurations (orbits of points) of which cannot be generated in the general position of a space group: $I\bar{4}3d$ (d) $x \neq 1/8$, Ia3d (f) $x \neq 0$, Fd3m (f) $x \neq 1/4$, $F4_132$ (f) $x \neq 1/4$, Fd3 (f) $x \neq 1/4$ (cp. Fischer, 1973). All corresponding space partitions into Dirichlet domains (Koch, 1972) do not form partitions into fundamental regions of a space group in R^3. These cases give rise to related solutions of the problem for $n>3$. It has been proved for Fd3m (f) x=1/8, that the corresponding Dirichlet domains (distorted octahedra) cannot be rearranged to another type of space partition. This example, therefore, is a new special type of answer to Hilbert's Problem 18 (1900). (A detailed paper has been submitted to Geometriae Dedicata.)

B.N. Delone (1961), Soviet. Math. 2, 812-815.
W. Fischer (1973), Z. Kristallogr. 138, 129-146.
D. Hilbert (1900), Nachricht von der Kgl. Gesellschaft d. Wiss. zu Göttingen, Math.-physik. Klasse, 253-297.
E. Koch (1972), Diss., Marburg.