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Graphs Equienergetic with Their Complements*

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Abstract

The energy $E(G)$ of a graph G is the sum of the absolute values of its eigenvalues. In this paper, we present several classes of non-self-complementary graphs, satisfying $E(G) = E(\overline{G})$, where $\overline{G}$ is the complement of G.

1 Introduction

Let G be a simple graph with n vertices. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ be the vertex set of G. The *complement* of a graph G is the graph $\overline{G}$ with vertex set $V(\overline{G}) = V(G)$ and two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G. A graph G is said to be *self-complementary* if it is isomorphic to its complement. The *line graph* of G, denoted by $L(G)$ is a graph whose vertex set has one-to-one correspondence with the edges of G and two vertices are adjacent in $L(G)$ if and only if the corresponding edges are adjacent in G [11].

The *adjacency matrix* of a graph G is a square matrix $A(G) = [a_{ij}]$ of order n, in which $a_{ij} = 1$ if the vertices v_i and v_j are adjacent and $a_{ij} = 0$, otherwise. The

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eigenvalues of $A(G)$ are called the *eigenvalues* of G and their collection is called the *spectrum* of G. Since $A(G)$ is a real symmetric matrix, its eigenvalues can be labeled as $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$. If $\lambda_1, \lambda_2, \ldots, \lambda_k$ are the distinct eigenvalues of G with respective multiplicities $m_1, m_2, \ldots, m_k$, then the spectrum of G is denoted by

$$Spec(G) = \begin{pmatrix} \lambda_1 & \lambda_2 & \cdots & \lambda_k \\ m_1 & m_2 & \cdots & m_k \end{pmatrix}.$$

The spectrum of the union of two graphs is the union of their spectra. Two graphs are said to be *cospectral* if they have same spectra. More details about spectra of graphs can be found in [7].

The *energy* of a graph G, denoted by $E(G)$, is defined as the sum of the absolute values of the eigenvalues of G. That is,

$$E(G) = \sum_{i=1}^{n} |\lambda_i|.$$

This concept was introduced by Gutman [9] in 1978. The energy of a graph is a quantity closely related to the Huckel molecular orbital total π-electron energy [10]. For more about the graph energy, one can refer the book [15].

Two graphs G_1 and G_2 of the same order are said to be *equienergetic* if $E(G_1) = E(G_2)$. Cospectral graphs are always equienergetic. Several papers dealing with the non-cospectral, equienergetic graphs have been appeared. Balakrishnan [2] constructed equienergetic graphs on n vertices, using tensor product of graphs, for all $n \equiv 0 (\mathrm{mod}\, 4)$. Stevanović [24] gave the construction of equienergetic graphs for all $n \equiv 0 (\mathrm{mod}\, 5)$. Ramane and Walikar [20] constructed equienergetic graphs by join of two graphs for all $n \geq 9$. Indulal and Vijayakumar [14] gave equienergetic self-complementary graphs. Xu and Hou [25] constructed equienergetic bipartite graphs. Ramane et al. [21] obtained the energy of iterated line graphs of regular graphs and thus gave infinitely many pairs of non-cospectral equienergetic graphs. In [19], the energy of the complement of iterated line graphs of regular graphs is obtained and thus found equienergetic graphs. Bronkov et al. [5] listed some equienergetic trees. Other results on equienergetic graphs can be found in $[1, 4, 8, 13, 16, 17, 23]$. The purpose of this paper is to investigate the graphs satisfying $E(G) = E(\overline{G})$.

2 Graphs with $E(G) = E(\overline{G})$

If G is self-complementary then it is obvious that $E(G) = E(\overline{G})$. Hence it is less trivial to find the non-self-complementary graphs satisfying $E(G) = E(\overline{G})$. We need following theorem.

Theorem 2.1. [22] *Let G be an r-regular graph of order n with the eigenvalues $r, \lambda_2, \ldots, \lambda_n$. Then the eigenvalues of $\overline{G}$ are $n-r-1, -\lambda_2-1, \ldots, -\lambda_n-1$.* ■

Let K_n be the complete graph on n vertices, C_n be the cycle on n vertices and $K_{p,q}$ be the complete bipartite graph on $n = p+q$ vertices. The smallest non-self-complementary graph satisfying $E(G) = E(\overline{G})$ is the cycle C_4. Its eigenvalues are $2, 0, 0, -2$ and the eigenvalues of its complement, $\overline{C_4} = 2K_2$, are $1, 1, -1, -1$. Thus $E(C_4) = E(\overline{C_4}) = 4$. The graph C_4 is connected, whereas its complement is disconnected. The smallest non-self-complementary graph satisfying $E(G) = E(\overline{G})$, where both G and $\overline{G}$ are connected is the cycle C_6. The eigenvalues of C_6 are $2, 1, 1, -1, -1, -2$ and the eigenvalues of $\overline{C_6}$ are $3, 1, 0, 0, -2, -2$. Thus $E(C_6) = E(\overline{C_6}) = 8$.

Theorem 2.2. *For $n \geq 2$, if $G = nK_n$, the union of n copies of K_n, then $E(G) = E(\overline{G})$.*

Proof. The eigenvalues of K_n are $n-1$ and -1 ($n-1$ times). Hence the spectrum of $G = nK_n$ is

$$Spec(G) = \begin{pmatrix} n-1 & -1 \\ n & n(n-1) \end{pmatrix}.$$

Therefore $E(G) = |n-1|(n) + |-1|n(n-1) = 2n(n-1)$.

Graph $G = nK_n$ is a regular graph of degree $n-1$ on n^2 vertices. By Theorem 2.1, the spectrum of $\overline{G}$ is

$$Spec(\overline{G}) = \begin{pmatrix} n^2-n & -n & 0 \\ 1 & n-1 & n(n-1) \end{pmatrix}.$$

Therefore $E(\overline{G}) = |n^2-n| + |-n|(n-1) + |0|n(n-1) = 2n(n-1)$. Hence $E(G) = E(\overline{G})$. ■

The *tensor product* $M \otimes N$ of the $r \times s$ matrix $M = [m_{ij}]$ and $t \times u$ matrix $N = [n_{ij}]$ is defined as $rt \times su$ matrix got by replacing each entry m_{ij} of M by the double array $m_{ij}N$. If α and β are the eigenvalues of the square matrices M and N respectively, then $\alpha\beta$ is the eigenvalue of $M \otimes N$. Suppose M and N commute. Then there is an ordering $\alpha_1, \alpha_2, \ldots, \alpha_n$ of the eigenvalues of M and an ordering $\beta_1, \beta_2, \ldots, \beta_n$ of the eigenvalues of N such that the eigenvalues of $M+N$ are $\alpha_1+\beta_1, \alpha_2+\beta_2, \ldots, \alpha_n+\beta_n$ [12].

Lemma 2.3. *The spectrum of the line graph of a complete bipartite graph $K_{p,q}$ is*

$$Spec(L(K_{p,q})) = \begin{pmatrix} p+q-2 & p-2 & q-2 & -2 \\ 1 & q-1 & p-1 & (p-1)(q-1) \end{pmatrix}.$$

Proof. The adjacency matrix of $L(K_{p,q})$ can be written in the form of blocks as

$$A(L(K_{p,q})) = \begin{bmatrix} J_q - I_q & I_q & \cdots & I_q \\ I_q & J_q - I_q & \cdots & I_q \\ \vdots & & \ddots & \vdots \\ I_q & I_q & \cdots & J_q - I_q \end{bmatrix}_{pq \times pq},$$

where J_q is the square matrix of order q with all entries equal to 1 and I_q is the identity matrix of order q. Each row and column of $A(L(K_{p,q}))$ contains p blocks. Here, $A(L(K_{p,q}))$ can be expressed as

$$A(L(K_{p,q})) = I_p \otimes (J_q - 2I_q) + J_p \otimes I_q. \qquad (1)$$

Since the eigenvalues of I_p are all ones and the eigenvalues of J_p are p and 0 ($p-1$ times), by Eq. (1), the spectrum of $L(K_{p,q})$ is

$$Spec(L(K_{p,q})) = \begin{pmatrix} p+q-2 & p-2 & q-2 & -2 \\ 1 & q-1 & p-1 & (p-1)(q-1) \end{pmatrix}.$$

■

Theorem 2.4. *For $p, q \geq 2$, $E\left(L(K_{p,q})\right) = E\left(\overline{L(K_{p,q})}\right)$.*

Proof. From Lemma 2.3,

$$\begin{aligned} E\left(L(K_{p,q})\right) &= |p+q-2| + |p-2|(q-1) + |q-2|(p-1) + |-2|(p-1)(q-1) \\ &= 4(pq-p-q+1). \end{aligned}$$

The line graph of $K_{p,q}$ has pq vertices and it is a regular graph of degree $p+q-2$. Therefore by Theorem 2.1, the spectrum of its complement is

$$Spec\left(\overline{L(K_{p,q})}\right) = \begin{pmatrix} pq-p-q+1 & 1-p & 1-q & 1 \\ 1 & q-1 & p-1 & (p-1)(q-1) \end{pmatrix}.$$

Therefore

$$\begin{aligned} E\left(\overline{L(K_{p,q})}\right) &= |pq-p-q+1| + |1-p|(q-1) + |1-q|(p-1) + |1|(p-1)(q-1) \\ &= 4(pq-p-q+1). \end{aligned}$$

Hence $E\left(L(K_{p,q})\right) = E\left(\overline{L(K_{p,q})}\right)$. ■

The *Cartesian product* of two graphs G_1 and G_2 is the graph $G_1 \times G_2$ with vertex set $V(G_1) \times V(G_2)$ and in which the vertices (u_1, u_2) and (v_1, v_2) are adjacent if either u_1 is adjacent to v_1 in G_1 and $u_2 = v_2$ or $u_1 = v_1$ and u_2 is adjacent to v_2 in G_2.

The *tensor product* of two graphs G_1 and G_2 is the graph $G_1 \otimes G_2$ with vertex set $V(G_1) \times V(G_2)$ and in which the vertices (u_1, u_2) and (v_1, v_2) are adjacent if and only if u_1 is adjacent to v_1 in G_1 and u_2 is adjacent to v_2 in G_2.

Theorem 2.5. [3] *If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of G_1 and $\mu_1, \mu_2, \ldots, \mu_m$ are the eigenvalues of G_2, then*
(i) the eigenvalues of $G_1 \times G_2$ are $\lambda_i + \mu_j$, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$.
(ii) the eigenvalues of $G_1 \otimes G_2$ are $\lambda_i \mu_j$, $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$. ■

In [4] it has been proved that $E(K_p \times K_q) = E(K_p \otimes K_q)$. In the following theorem we show that $L(K_{p,q}) \cong K_p \times K_q$ and $\overline{L(K_{p,q})} \cong K_p \otimes K_q$.

Theorem 2.6. *$L(K_{p,q}) \cong K_p \times K_q$ and $\overline{L(K_{p,q})} \cong K_p \otimes K_q$.*

Proof. Suppose V_1 and V_2 be the partite sets of the vertex set of $K_{p,q}$, where $|V_1| = p$ and $|V_2| = q$. Corresponding to each vertex of V_1, there is a clique of order q in $L(K_{p,q})$. Without loss of generality consider two cliques of $L(K_{p,q})$ of order q with vertices labeled as $u_1^i, u_2^i, \ldots, u_q^i$ and $v_1^j, v_2^j, \ldots, v_q^j$ respectively. In $L(K_{p,q})$ the vertex u_k^i is adjacent to v_k^j, $k = 1, 2, \ldots, q$. It is true with all p cliqes of order q. Hence $L(K_{p,q}) \cong K_p \times K_q$.

In $\overline{L(K_{p,q})}$, no two vertices among $u_1^i, u_2^i, \ldots, u_q^i$ are adjacent. Similary no two vertices among $v_1^j, v_2^j, \ldots, v_q^j$ are adajcent in $\overline{L(K_{p,q})}$. Where as the vertices u_k^i and v_l^j are adjacent in $\overline{L(K_{p,q})}$ for $k, l = 1, 2, \ldots, q$ and $k \neq l$. Hence $\overline{L(K_{p,q})} \cong K_p \otimes K_q$. ■

In [18] it was reported that $E\left(L(K_{p,p})\right) = E\left(\overline{L(K_{p,p})}\right)$, for $p \geq 2$.

Remark: The graph $L(K_{3,3})$ is self-complementary.

Proposition 2.7. *If either $p = 2$ and $q \geq 2$ or $p \geq 2$ and $q = 2$, then $\overline{L(K_{p,q})}$ is bipartite.*

Proof. A graph G is bipartite if and only if for each eigenvalue λ of G, $-\lambda$ is also its eigenvalue [7]. Thus if either $p = 2$ and $q \geq 2$ or $p \geq 2$ and $q = 2$, by spectrum of $\overline{L(K_{p,q})}$ given in the proof of Theorem 2.4, we see that for every eiegnvalue λ of $\overline{L(K_{p,q})}$, there is an eigenvalue $-\lambda$ of $\overline{L(K_{p,q})}$. Hence the result. ■

Theorem 2.8. *Let $G = \overline{nK_n}$, $n \geq 2$. Then for $m \leq n$, $E(G \times K_m) = E(\overline{G \times K_m})$.*

Proof. The spectrum of $G = \overline{nK_n}$ is

$$Spec(G) = \begin{pmatrix} n^2 - n & -n & 0 \\ 1 & n-1 & n(n-1) \end{pmatrix}$$

and the spectrum of K_m is

$$Spec(K_m) = \begin{pmatrix} m-1 & -1 \\ 1 & m-1 \end{pmatrix}.$$

Therefore by Theorem 2.5, the spectrum of $G \times K_m$ is

$$\begin{pmatrix} n^2 - n + m - 1 & n^2 - n - 1 & -n + m - 1 & -n - 1 & m - 1 & -1 \\ 1 & m - 1 & n - 1 & (n-1)(m-1) & n^2 - n & (n^2 - n)(m-1) \end{pmatrix}.$$

Therefore,

$$\begin{aligned} E(G\times K_m) &= |n^2-n+m-1|+|n^2-n-1|(m-1)+|-n+m-1|(n-1) \\ &\quad +|-n-1|(n-1)(m-1)+|m-1|(n^2-n)+|-1|(n^2-n)(m-1) \\ &= 2n(n-1)(2m-1). \end{aligned}$$

The graph $G\times K_m$ is a regular graph on mn^2 vertices with regularity $n^2-n+m-1$. Therefore by Theorem 2.1 and spectrum of $G\times K_m$, the spectrum of $\overline{G\times K_m}$ is

$$\begin{pmatrix} mn^2-n^2+n-m & -n^2+n & n-m & n & -m & 0 \\ 1 & m-1 & n-1 & (m-1)(n-1) & n^2-n & (n^2-n)(m-1) \end{pmatrix}.$$

Therefore

$$\begin{aligned} E(\overline{G\times K_m}) &= |mn^2-n^2+n-m|+|-n^2+n|(m-1)+|n-m|(n-1) \\ &\quad +|n|(m-1)(n-1)+|-m|(n^2-n)+|0|(n^2-n)(m-1) \\ &= 2n(n-1)(2m-1). \end{aligned}$$

Hence, $E(G\times K_m)=E(\overline{G\times K_m})$. ■

Remark: If $G=\overline{nK_n}$, $n\geq 2$ then for $m>n$, $E(G\times K_m)=4mn^2-4n^2-2mn+2n-2m+2$ and $E(\overline{G\times K_m})=4mn^2-4n^2-2mn+4n-2m$. Therefore $E(\overline{G\times K_m})-E(G\times K_m)=2(n-1)$. This shows that the energy difference of the graphs $G\times K_m$ and $\overline{G\times K_m}$ is independent of m.

A *strongly regular graph* with parameters (n,r,a,b) is an r-regular graph $(0<r<n-1)$ on n vertices in which any two adjacent vertices have exactly a common neighbours and any two non-adjacent vertices have exactly b common neighbours. If G is a strongly regular graph with parameters (n,r,a,b) then its complement $\overline{G}$ is also strongly regular graph with parameters $(n,n-r-1,n-2r+b-2,n-2r+a)$. The strongly regular graph has only three distinct eigenvalues [7].

Theorem 2.9. [7] *If G is a strongly regular graph with parameters (n,r,a,b), then the spectrum of G is*

$$\begin{pmatrix} r & \frac{1}{2}(a-b+t) & \frac{1}{2}(a-b-t) \\ 1 & \frac{1}{2}(n-1-\frac{\Delta}{t}) & \frac{1}{2}(n-1+\frac{\Delta}{t}) \end{pmatrix},$$

where $t=\sqrt{(a-b)^2+4(r-b)}$ and $\Delta=2r+(n-1)(a-b)$. ■

Theorem 2.10. *If G is a strongly regular graph with parameters $(4n^2,2n^2-n,n^2-n,n^2-n)$, $n>1$, then $E(G)=E(\overline{G})$.*

Proof. By Theorem 2.9,

$$Spec(G) = \begin{pmatrix} 2n^2-n & n & -n \\ 1 & 2n^2-1 & 2n^2+n-1 \end{pmatrix}.$$

Therefore,

$$\begin{aligned} E(G) &= |2n^2-n| + |n|(2n^2-n) + |-n|(2n^2+n-1) \\ &= 2n(2n-1)(n+1). \end{aligned}$$

The graph G is a regular graph on $4n^2$ vertices with regularity $2n^2-n$. Therefore by Theorem 2.1 and the spectrum of G, we have

$$Spec(\overline{G}) = \begin{pmatrix} 2n^2+n-1 & -n-1 & n-1 \\ 1 & 2n^2-n & 2n^2+n-1 \end{pmatrix}.$$

Therefore

$$\begin{aligned} E(\overline{G}) &= |2n^2+n-1| + |-n-1|(2n^2-n) + |n-1|(2n^2+n-1) \\ &= 2n(2n-1)(n+1). \end{aligned}$$

Hence, $E(G) = E(\overline{G})$. ■

Remark: If $n = 2$ in Theorem 2.10, then we get a strongly regular graph with parameters $(16, 6, 2, 2)$, which is a *Shrikhande graph* [6]. Energy of Shrikhande graph and of its complement is 36.

Theorem 2.11. *If G is a strongly regular graph with parameters $(n^2, 3n-3, n, 6)$, $n > 2$, then $E(G) = E(\overline{G})$.*

Proof. By Theorem 2.9,

$$Spec(G) = \begin{pmatrix} 3n-3 & n-3 & -3 \\ 1 & 3n-3 & n^2-3n+2 \end{pmatrix}.$$

Therefore,

$$\begin{aligned} E(G) &= |3n-3| + |n-3|(3n-3) + |-3|(n^2-3n+2) \\ &= 6(n-1)(n-2). \end{aligned}$$

The graph G is a regular graph on n^2 vertices with regularity $3n-3$. Therefore by Theorem 2.1 and the spectrum of G, we have

$$Spec(\overline{G}) = \begin{pmatrix} n^2-3n+2 & 2-n & 2 \\ 1 & 3n-3 & n^2-3n+2 \end{pmatrix}.$$

Therefore

$$\begin{aligned} E(\overline{G}) &= |n^2-3n+2|+|2-n|(3n-3)+|2|(n^2-3n+2) \\ &= 6(n-1)(n-2). \end{aligned}$$

Hence, $E(G)=E(\overline{G})$. ■

3 Some more graphs satisfying $E(G)=E(\overline{G})$

$E(L(K_n))=E\left(\overline{L(K_n)}\right)$ if and only if $n=6$ [19].

The spectrum of $C_4 \times K_2$ is

$$Spec(C_4\times K_2)=\begin{pmatrix} 3 & 1 & -1 & -3 \\ 1 & 3 & 3 & 1 \end{pmatrix}.$$

By Theorem 2.1,

$$Spec(\overline{C_4\times K_2})=\begin{pmatrix} 4 & 2 & 0 & -2 \\ 1 & 1 & 3 & 3 \end{pmatrix}.$$

Therefore $E\left(C_4\times K_2\right)=E\left(\overline{C_4\times K_2}\right)=12$.

The strongly regular graph with parameters $(50,7,0,1)$ is a *Moore graph MG* and its spectrum is

$$Spec(MG)=\begin{pmatrix} 7 & 2 & -3 \\ 1 & 28 & 21 \end{pmatrix}$$

and

$$Spec(\overline{MG})=\begin{pmatrix} 42 & 2 & -3 \\ 1 & 21 & 28 \end{pmatrix}.$$

By Theorem 2.5 the spectrum of $\overline{MG}\times K_2$ is

$$Spec\left(\overline{MG}\times K_2\right)=\begin{pmatrix} 43 & 41 & 3 & 1 & -2 & -4 \\ 1 & 1 & 21 & 21 & 28 & 28 \end{pmatrix}.$$

The graph $\overline{MG}\times K_2$ is a regular graph on 100 vertices with regularity 43. By Theorem 2.1

$$Spec\left(\overline{\overline{MG}\times K_2}\right)=\begin{pmatrix} 56 & 3 & 1 & -2 & -4 & -42 \\ 1 & 28 & 28 & 21 & 21 & 1 \end{pmatrix}.$$

Therefore $E\left(\overline{MG}\times K_2\right)=E\left(\overline{\overline{MG}\times K_2}\right)=336$.

The strongly regular graph with parameters $(16,5,0,2)$ is a *Clebsch graph CG* and its spectrum is

$$Spec(CG) = \begin{pmatrix} 5 & 1 & -3 \\ 1 & 10 & 5 \end{pmatrix}$$

and

$$Spec(\overline{CG}) = \begin{pmatrix} 10 & -2 & 2 \\ 1 & 10 & 5 \end{pmatrix}.$$

By Theorem 2.5 the spectrum of $\overline{CG} \times K_2$ is

$$Spec\left(\overline{CG} \times K_2\right) = \begin{pmatrix} 11 & 9 & 3 & 1 & -1 & -3 \\ 1 & 1 & 5 & 5 & 10 & 10 \end{pmatrix}.$$

The graph $\overline{CG} \times K_2$ is a regular graph on 32 vertices with regularity 11. By Theorem 2.1

$$Spec\left(\overline{\overline{CG} \times K_2}\right) = \begin{pmatrix} 20 & 2 & 0 & -2 & -4 & -10 \\ 1 & 10 & 10 & 5 & 5 & 1 \end{pmatrix}.$$

Therefore $E\left(\overline{CG} \times K_2\right) = E\left(\overline{\overline{CG} \times K_2}\right) = 80$.

4 Conclusion

In this paper we have attempted to give the non-self-complementary graphs whose energy is equal to the energy of its complement. Several classes of non-self-complementary graphs, satisfying $E(G) = E(\overline{G})$ are reported. All graphs given in this paper are regular. Out of which many are strongly regular graphs. Following problems can be taken for further study on this topic.

(i) Discussion of the non-self-complementary, non-regular graphs satisfying $E(G) = E(\overline{G})$.

(ii) Finding structural and spectral properties of graphs satisfying $E(G) = E(\overline{G})$.

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