# Enumeration of Conformers for Octahedral fac/mer- $\left[\mathrm{MX}_{3}(\mathrm{AB})_{3}\right]$ and fac/mer- $\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$ <br> Complexes on the Basis of Computational Group Theory* 

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#### Abstract

Conformers of $f a c-[\mathrm{MX} 3(\mathrm{AB}) 3]$, mer-[MX3(AB)3], fac-[MX3(ABC)3], and mer-[MX3(ABC)3] complexes have been enumerated on the basis of computational group theory, where M is the central metal ion, while $\mathrm{X}, \mathrm{AB}$, and ABC are the monoatomic, diatomic, and bent triatomic ligands, respectively, bound to M through X or A . For the fac-[MX3(AB)3] complex, 10 bisected diastereomers have been found as $1 \mathrm{C3v}, 1 \mathrm{C3}, 3 \mathrm{Cs}$, and 5 C . Based on the 10 diastereomers of the $\mathrm{fac}-\mathrm{MX} 3(\mathrm{AB}) 3$ core unit, 198 diastereomers have been found for the fac-[MX3(ABC)3] complex, which are assigned to six point groups, $1 \mathrm{C} 3 \mathrm{v}, 4 \mathrm{C} 3,11 \mathrm{Cs}$, 182 C1. On the other hand, for the mer-[MX3(AB)3] complex, 12 bisected diastereomers have been found as 12 C 1 . Based on the 12 diastereomers of the $\operatorname{mer}$-MX3(AB)3 core unit, 324


[^0]diastereomers have been found for the mer- $\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$ complex, which are assigned to the $C_{1}$ point group as $324 C_{1}$.

## 1 Introduction

Enumeration of the conformers is of importance in conformational analysis. For metal complexes, however, enumeration is not so easy because of the large number of conformers due to the large coordination numbers. The conformers of octahedral metal complexes, $\left[\mathrm{M}(\mathrm{AB})_{6}\right],\left[\mathrm{M}(\mathrm{ABC})_{6}\right],\left[\mathrm{MX}(\mathrm{AB})_{5}\right],\left[\mathrm{MX}(\mathrm{ABC})_{5}\right]$, trans $-\left[\mathrm{MX}_{2}(\mathrm{AB})_{4}\right]$, trans-$\left[\mathrm{MX}_{2}(\mathrm{ABC})_{4}\right]$, cis- $\left[\mathrm{MX}_{2}(\mathrm{AB})_{4}\right]$, and cis-[ $\left.\mathrm{MX}_{2}(\mathrm{ABC})_{4}\right]$ (Figure 1), have been enumerated [1-6] by the computational group theory (CGT) method [7], where M is the central metal ion, while $\mathrm{X}, \mathrm{AB}$, and ABC are the monoatomic, diatomic, and bent triatomic ligands, respectively, bound to M through X or A . Using the result of the enumeration, conformational analysis was conducted for some actual compounds [ $1,2,8$ ].

$\left[\mathrm{M}(\mathrm{AB})_{6}\right]$

$\left[\mathrm{M}(\mathrm{ABC})_{6}\right]$

[MX(AB) $)_{5}$ ]

[MX(ABC) $)_{5}$ ]


trans-[MX $\left.{ }_{2}(\mathrm{AB})_{4}\right]$

cis- $\left[\mathrm{MX}_{2}(\mathrm{AB})_{4}\right]$

cis- $\left[\mathrm{MX}_{2}(\mathrm{ABC})_{4}\right]$

Figure 1. Structures of octahedral metal complexes, $\left[\mathrm{M}(\mathrm{AB})_{6}\right](\mathrm{a}),\left[\mathrm{M}(\mathrm{ABC})_{6}\right](\mathrm{b}),\left[\mathrm{MX}(\mathrm{AB})_{5}\right](\mathrm{c}),\left[\mathrm{MX}(\mathrm{ABC})_{5}\right]$ (d), $\operatorname{trans}-\left[\mathrm{MX}_{2}(\mathrm{AB})_{4}\right](\mathrm{e})$, trans $-\left[\mathrm{MX}_{2}(\mathrm{ABC})_{4}\right](\mathrm{f})$, cis- $\left[\mathrm{MX}_{2}(\mathrm{AB})_{4}\right](\mathrm{g})$, and $c i s-\left[\mathrm{MX}_{2}(\mathrm{ABC})_{4}\right]$ (h)

In this study, in order to extend the target molecules in the conformational analysis, enumeration was conducted for octahedral complexes, fac-[ $\left.\mathrm{MX}_{3}(\mathrm{AB})_{3}\right]$, mer- $\left[\mathrm{MX}_{3}(\mathrm{AB})_{3}\right]$, $f a c-\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$, and $m e r-\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$ (Figure 2). The $f a c$-complexes possess central $f a c-\mathrm{MXX}_{3} \mathrm{~A}_{3}$ unit, belonging to the $C_{3 v}$ point group, while the $m e r$-complexes possess central $m e r-\mathrm{MX}_{3} \mathrm{~A}_{3}$ unit, belonging to the $C_{2 v}$ point group. The enumeration results are expected to be useful for conformational analysis for related octahedral complexes.


Figure 2. Structures of octahedral metal complexes, $f a c-\left[\mathrm{MX}_{3}(\mathrm{AB})_{3}\right]$ (a), $f a c-\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right](\mathrm{b})$, mer- $\left[\mathrm{MX}_{3}(\mathrm{AB})_{3}\right]$ (c), and mer- $\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$ (d)

## 2 Methods

Conformers were obtained based on the computational group theory (CGT) method [7], which was performed using GAP program [9] on Intel Core i5-2450 ( 2.50 GHz ) computer. Three-dimensional models were drawn by Winmostar software [10], and the point groups were ascertained by the software. In enumeration, four orientations (bisecting orientations) are considered for the rotation around the $\mathrm{M}-\mathrm{A}$ bond, and three orientations are considered for the
rotation around the $\mathrm{A}-\mathrm{B}$ bond (Figure 3a). To describe the structure, we use the following notation [1-6]. For example, the structure shown in Figure 3b is described as [ $[y+z$ ], [ - ], [ -$],[-],[-x+z],[-x+y]]$. In this notation, the orientations of the $A B$ ligands are described as $[y+z],[-x+z]$, etc., and the X ligands are described as [ - ], in the order of the numbering system [ $\mathrm{x}, \mathrm{y}, \mathrm{z},-\mathrm{x},-\mathrm{y},-\mathrm{z}]$.


Figure 3. Typical orientations considered in enumeration (a) and an example of a conformer with a numbering system (b)

## 3 Results and discussion

### 3.1 Enumeration for fac - and mer - $\left[\mathrm{MX}_{3}(\mathrm{AB})_{3}\right]$ complex

The enumeration was conducted for the $f a c-\left[\mathrm{MX}_{3}(\mathrm{AB})_{3}\right]$ complex by the GAP program [9], excluding the unfavorable structures possessing B atoms in a close distance. The resulting diastereomers are summarized in Table 1 with their point groups and are depicted in Figure 4. The obtained 10 diastereomers, F1-F10, are categorized in four-point groups as $1 C_{3 v}, 1 C_{3}, 3$ $C_{s}$ and $5 C_{1}$. Among them, the $C_{3}$ and $C_{1}$ point groups are chiral. So, each diastereomer belonging to either the $C_{3}$ or $C_{1}$ point group has an enantiomer, and the total number of the conformers is 16. (F1, F2, F2', F3, F4, F5, F6, F6', F7, F7', F8, F8', F9, F9', F10, and F10', where the symbol " '" represents the mirror image). Except for the $C_{1}$ point group, all the point
groups belong to the subgroups of the $C_{3 v}$ point group. This is consistent with the $C_{3 v}$ symmetry of the fac-MX ${ }_{3} \mathrm{~A}_{3}$ coordination geometry.

Table 1. Bisecting diastereomers for a $\mathrm{fac}-\left[\mathrm{MX}_{3}(\mathrm{AB})_{3}\right]$ complex

| No | Example | Point Group |
| :---: | :---: | :---: |
| F1 | $[[y+z],[-],[-],[-],[-\mathrm{x}+\mathrm{z}],[-\mathrm{x}+\mathrm{y}]]$ | $C_{3 v}$ |
| F2 | $[[\mathrm{y}+\mathrm{z}],[-],[-\mathrm{x}-\mathrm{y}],[-],[\mathrm{x}-\mathrm{z}],[-]]$ | $C_{3}$ |
| F3 | $[[\mathrm{y}+\mathrm{z}],[-],[\mathrm{x}-\mathrm{y}],[-],[-\mathrm{x}+\mathrm{z}],[-]]$ | $C_{s}$ |
| F4 | $[[\mathrm{y}+\mathrm{z}],[-],[-\mathrm{x}+\mathrm{y}],[-],[-\mathrm{x}+\mathrm{z}],[-]]$ | $C_{s}$ |
| F5 | $[[\mathrm{y}+\mathrm{z}],[-],[-],[-],[-\mathrm{x}+\mathrm{z}],[\mathrm{x}-\mathrm{y}]]$ | $C_{s}$ |
| F6 | $[[\mathrm{y}+\mathrm{z}],[-],[-],[-],[-\mathrm{x}+\mathrm{z}],[-\mathrm{x}-\mathrm{y}]]$ | $C_{1}$ |
| F7 | $[[\mathrm{y}+\mathrm{z}],[-],[-\mathrm{x}-\mathrm{y}],[-],[\mathrm{x}+\mathrm{z}],[-]]$ | $C_{1}$ |
| F8 | $[[\mathrm{y}+\mathrm{z}],[-],[\mathrm{x}-\mathrm{y}],[-],[-\mathrm{x}-\mathrm{z}],[-]]$ | $C_{1}$ |
| F9 | $[[\mathrm{y}+\mathrm{z}],[-\mathrm{x}-\mathrm{z}],[-\mathrm{x}+\mathrm{y}],[-],[-],[-]]$ | $C_{1}$ |
| F10 | $[[\mathrm{y}+\mathrm{z}],[-\mathrm{x}-\mathrm{z}],[-],[-],[-],[-\mathrm{x}-\mathrm{y}]]$ | $C_{1}$ |




F9


F2


F6


F10


F3


F7



F8


Figure 4. Structures of diastereomers F1-F10

The enumeration for the mer- $\left[\mathrm{MX}_{3}(\mathrm{AB})_{3}\right]$ complex was conducted in the same way, and the resulting diastereomers are summarized in Table 2 with their point groups, and are depicted in Figure 5. Although the central mer $-\mathrm{MX}_{3} \mathrm{~A}_{3}$ unit belongs to the $C_{2 v}$ point group; however, since we consider the bisecting orientations for the A-B bond, all of the obtained 12 diastereomers,

M1-M12, belong to the $C_{1}$ point group (12 $C_{1}$ ). Since the $C_{1}$ point group is chiral, the total number of the conformers is 24 , considering the enantiomers.

Table 2. Bisecting diastereomers for a mer- $\left[\mathrm{MX}_{3}(\mathrm{AB})_{3}\right]$ complex

| No | Example | Point Group |
| :---: | :---: | :---: |
| M1 | $[[y+z],[-],[-x+y],[-],[-],[-x+y]]$ | $C_{1}$ |
| M2 | $[[y+z],[-],[-x+y],[-y+z],[-],[-]]$ | $C_{1}$ |
| M3 | $[[y+z],[-],[x-y],[-],[-],[x-y]]$ | $C_{1}$ |
| M4 | $[[y+z],[-],[x-y],[-],[-],[-x-y]]$ | $C_{1}$ |
| M5 | $[[y+z],[-],[-],[-y-z],[-x+z],[-]]$ | $C_{1}$ |
| M6 | $[[y+z],[-x+z],[-],[-y+z],[-],[-]]$ | $C_{1}$ |
| M7 | $[[y+z],[-x+z],[-],[-],[-x-z],[-]]$ | $C_{1}$ |
| M8 | $[[y+z],[-],[-x-y],[-y-z],[-],[-]]$ | $C_{1}$ |
| M9 | $[[y+z],[-x+z],[-],[-y-z],[-],[-]]$ | $C_{1}$ |
| M10 | $[[y+z],[-],[-x-y],[-],[-],[-x-y]]$ | $C_{1}$ |
| M11 | $[[y+z],[-x-z],[-],[-y+z],[-],[-]]$ | $C_{1}$ |
| M12 | $[[y+z],[-],[-x-y],[-],[-],[-x+y]]$ | $C_{1}$ |





M5

M6

M7

M8




M12


Figure 5. Structures of diastereomers M1-M12

### 3.2 Enumeration for fac -[ $\left.\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$ complex

Conformers of fac-[ $\left.\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$ complex were considered using the enumeration result of $f a c-\left[\mathrm{MX}_{3}(\mathrm{AB})_{3}\right]$ complex. Based on the $f a c-\mathrm{MX}_{3}(\mathrm{AB})_{3}$ unit, each ligand, AB , was extended from atom B to atom C to consider the conformers of $f a c-\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$ complex. For the
extension of C, three directions were considered whose dihedral angles M-A-B-C were $180^{\circ}$ (anti conformer), $300^{\circ}$ (gauche conformer), and $60^{\circ}$ (gauche conformer). This is thought to be sufficient for the purpose of conformational analysis. The resulting diastereomers for the fac- $\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$ complex are summarized in Table 3. Generally, $3^{3}(=27)$ conformers are expected to be generated from each diastereomer for the $f a c-\mathrm{MX}_{3}(\mathrm{AB})_{3}$ unit, by the extension of the ligands; however, in the case of $f a c-\mathrm{MX}_{3}(\mathrm{AB})_{3}$ unit of $C_{3 v}$ point group, for example, the conformers are categorized in 7 diastereomers, including $1 C_{3 v}, 1 C_{3}, 2 C_{s}$, and $3 C_{1}$ diastereomers. On the other hand, in the case of $f a c-\mathrm{MX}_{3}(\mathrm{AB})_{3}$ unit of $C_{1}$ point group, there are 27 diastereomers, belonging to $C_{1}$ point group. Consequently, the number of diastereomers becomes 198 in total, including $1 C_{3 v}, 4 C_{3}, 11 C_{s}$, and $182 C_{1}$ diastereomers, as listed in Table 4. Since the $C_{3}$ and $C_{1}$ point groups are chiral, the number of conformers becomes 384 in total, including $1 C_{3 v}, 8 C_{3}, 11 C_{s}$, and $364 C_{1}$ conformers. Except for the $C_{1}$ point group, all the point groups belong to the subgroups of the $C_{3 v}$ point group, to which the $f a c-\mathrm{MX}_{3} \mathrm{~A}_{3}$ unit belongs.

Table 3. Bisecting diastereomers of $f a c-\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$, derived from $f a c-\mathrm{MX}_{3}(\mathrm{AB})_{3}$ core unit

| No | Point group of <br> $f a c-\mathrm{MX}_{3}(\mathrm{AB})_{3}$ core <br> unit | Point groups of <br> $f a c-\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$ <br> diastereomer | Total number <br> of <br> diastereomers | Total number <br> of conformers |
| :---: | :---: | :---: | :---: | :---: |
| F1 | $C_{3 v}$ | $1 C_{3 v}, 1 C_{3}, 2 C_{s}, 3 C_{1}$ | 7 | 11 |
| F2 | $C_{3}$ | $3 C_{3}, 8 C_{1}$ | 11 | 22 |
| F3 | $C_{s}$ | $3 C_{s}, 12 C_{1}$ | 15 | 27 |
| F4 | $C_{s}$ | $3 C_{s}, 12 C_{1}$ | 15 | 27 |
| F5 | $C_{s}$ | $3 C_{s}, 12 C_{1}$ | 15 | 27 |
| F6 | $C_{1}$ | $27 C_{1}$ | 27 | 54 |
| F7 | $C_{1}$ | $27 C_{1}$ | 27 | 54 |
| F8 | $C_{1}$ | $27 C_{1}$ | 27 | 54 |
| F9 | $C_{1}$ | $27 C_{1}$ | 27 | 54 |
| F10 | $C_{1}$ | $27 C_{1}$ |  | 27 |
|  |  |  | Total | 198 |

Table 4. Bisecting conformers of $f a c-\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$

| No | Point group | Total number of <br> diastereomers | Total number of <br> conformers |
| :---: | :---: | :---: | :---: |
| 1 | $C_{3 v}$ | 1 | 1 |
| 2 | $C_{3}$ | 4 | 8 |
| 3 | $C_{s}$ | 11 | 11 |
| 4 | $C_{1}$ | 182 | 364 |
|  |  | Total | 198 |

### 3.3 Enumeration for $\boldsymbol{m e r}$ - $\left[\mathrm{MX}_{3}(\mathbf{A B C})_{3}\right]$ complex

Conformers of mer-[ $\left.\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$ complex were considered using the enumeration result of mer- $\left[\mathrm{MX}_{3}(\mathrm{AB})_{3}\right]$ complex, and the resulting diastereomers are summarized in Table 5. The number of diastereomers is 324 in total, corresponding $324 C_{1}$ diastereomers, as listed in Table 6. Since the $C_{1}$ point group is chiral, the number of conformers becomes 648 in total.

Table 5. Bisecting diastereomers of mer- $\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$, derived from mer $-\mathrm{MX}_{3}(\mathrm{AB})_{3}$ core unit

| No | Point group of <br> $m e r-\mathrm{MX}_{3}(\mathrm{AB})_{3}$ <br> core unit | Point groups of <br> $m e r-\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$ <br> diastereomer | Total number <br> of <br> diastereomers |
| :---: | :---: | :---: | :---: | :---: |
| of conformers number |  |  |  |

Table 6. Bisecting conformers of mer- $\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$

| No | Point group | Total number of <br> diastereomers | Total number of <br> conformers |
| :---: | :---: | :---: | :---: |
| 1 | $C_{1}$ | 324 | 648 |
|  | Total | 324 | 648 |

As summarized in Tables 1-6, possible diastereomers are enumerated for fac/mer- $\left[\mathrm{MX}_{3}(\mathrm{AB})_{3}\right]$ and fac/mer- $\left[\mathrm{MX}_{3}(\mathrm{ABC})_{3}\right]$ complexes. As the previous enumeration results [1-3] were useful for the conformational analyses of some metal complexes [1,2,8], the enumeration results in this study are expected to be useful for conformational analysis of related octahedral complexes.

## 4 Concluding remarks

In this study, conformers of octahedral fac/mer-[MX3(AB)3] and fac/mer-[MX3(ABC)3] complexes were enumerated on the basis of computational group theory. For the fac-[MX3(AB)3] complex, 10 bisecting diastereomers have been found as $1 \mathrm{C} 3 \mathrm{v}, 1 \mathrm{C} 3,3 \mathrm{Cs}$ and 5 C 1 . For the fac-[MX3(ABC)3] complex, 198 bisecting diastereomers have been found as $1 \mathrm{C} 3 \mathrm{v}, 4 \mathrm{C} 3,11 \mathrm{Cs}$, and 182 C 1 . For the mer-[MX3(AB)3] complex, 12 bisecting diastereomers have been found as 12 C 1 . For the mer-[MX3(AB)3] complex, 324 bisecting diastereomers have been found as 324 C 1 . The results were summarized in tables, which are useful in conformational analysis of the related complexes.

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