# Analysis of the Coupled Homogeneous-Catalytic Reaction by the Adomian Decomposition Method 

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(Received May 7, 2018)


#### Abstract

In this paper, the analysis is performed for a mathematical model of the coupled homogeneous-catalytic reaction, which consists of two strongly nonlinear secondorder partial differential equations. Based on the Adomian decomposition method, the approximate analytical solutions of the model can be obtained by making use of the boundary conditions appropriately and selecting the search direction of solution accurately. Assigning values to the dimensionless parameters properly in the model, we can get an objective description of the solutions for the distributions of the temperature and fluid concentration.


## 1 Introduction

Nonlinear partial differential equations (PDEs) are of vital significance in science and engineering because the great majority of physical systems are essentially nonlinear. In order to gain the solutions of nonlinear PDEs, a large number of numerical methods have been developed in the past few decades, such as finite difference method (FDM) [1], finite elements method (FEM), finite volume method (FVM) [2,3], spectral method [4,5], Taylor meshless method (TMM) [6], homotopy analysis method (HAM) [7] and so on. Among them, Adomian decomposition method (ADM) is regarded as one of the most effective

[^0]ways to obtain the solutions of many nonlinear PDEs, which can solve various equations in the absence of discretization or invalidation. In this article, we mainly employ ADM to deal with a model which is composed of nonlinear PDEs. It has attracted much attention in recent years in the applications of ADM. For example, ADM has been widely applied to deal with some sorts of systems of equations in algebraic, differential, partial differential, differential-difference, linear, nonlinear, integral and integro-differential operations [8-23]. The concept of ADM was originally proposed by Adomian [8, 24, 25] in the early 1980s. The major technique of ADM is that the solution of a given equation represented in the form of the sum of an infinite series converges to exact solutions rapidly.

On the other side, due to the complexity of solving the second-order PDEs [18, 19, 22], there are few papers to solve them. However, as a matter of fact, some problems are simulated by second-order PDEs or systems of equations which usually accompany boundary conditions. To solve second-order PDEs by ADM, it is of vital importance to study and analyze boundary conditions. In a coordinate direction, the given boundary conditions can be the standard boundary conditions or the nonstandard boundary conditions. If the boundary condition is a standard boundary condition, then the solution of the equations is existent and unique. Obviously, in order to obtain a fully deterministic approximate analytical solution, we should choose standard boundary conditions instead of non-standard boundary conditions. In addition, if more than one boundary condition is given in a model, we can select the conditions needed to solve the model according to the influence of the boundary condition on the model to some extent.

The mathematical model of the coupled homogeneous-catalytic reaction is one of the classical models in chemical reaction engineering [18, 19, 26-28]. Because of its strong nonlinearity, numerical solution is difficult to realize. Unlike classical two-phase models that contain a single inter-phase transfer coefficient, the 3-mode models contain three transfer coefficients representing the inter and intra-phase heat/mass transfer [26]. For this situation, a reduced order 3-mode model which analyzes the problem of flow in a channel with homogeneous reaction(s) in the bulk and catalytic reaction(s) on the wall is needed, in order to forecast the temperature and fluid concentration distribution in the reactor in more detail. With reference to the classical coupled homogeneous-catalytic reaction model [26], a dimensionless coupled homogeneous-catalytic reaction model is established, which describes the distributions of the dimensionless temperature $\theta(x, y)$ and the fluid concentration $c(x, y)$ in the homogeneous-catalytic reactor. The model is
given by

$$
\left\{\begin{array}{l}
f(y) \frac{\partial c}{\partial x}=\frac{1}{P e} \frac{\partial^{2} c}{\partial x^{2}}+\frac{1}{p}\left(\frac{\partial^{2} c}{\partial y^{2}}+\frac{s}{y} \frac{\partial c}{\partial y}\right)-\frac{\phi^{2}}{p} c \exp \left[\frac{\gamma_{h} \theta}{1+\theta}\right]  \tag{1}\\
f(y) \frac{\partial \theta}{\partial x}=\lambda \frac{L e_{f}}{P e} \frac{\partial^{2} \theta}{\partial x^{2}}+\frac{L e_{f}}{p}\left(\frac{\partial^{2} \theta}{\partial y^{2}}+\frac{s}{y} \frac{\partial \theta}{\partial y}\right)+\beta \frac{\phi^{2}}{p} c \exp \left[\frac{\gamma_{h} \theta}{1+\theta}\right]
\end{array}\right.
$$

where $x$ and $y$ are the dimensionless coordinate along the length of the channel and dimensionless first transverse coordinate, $P e$ is the axial Peclet number, $p$ is the transverse Peclet number, $s$ is the solid phase, $\phi$ is the Thiele modulus for the homogeneous reaction, $\gamma_{h}$ is the dimensionless activation energy for the homogeneous reaction, $\lambda$ is the ratio of effective thermal conductivity in the direction of flow to fluid thermal conductivity with boundary conditions, $L e_{f}$ is the fluid Lewis number, $\beta$ is the dimensionless adiabatic temperature rise, and all of these are positive numbers. $f(y)$ is the velocity profile. It should be pointed out that for the homogeneous-catalytic reaction, there is a coefficient of $\frac{s}{y}$ and this coefficient has a singularity [29].

The given boundary conditions are as follows

$$
\begin{align*}
& \frac{1}{P e} \frac{\partial c}{\partial x}=f(y)(c-1),  \tag{2}\\
& x=0, \quad \lambda \frac{L e_{f}}{P e} \frac{\partial \theta}{\partial x}=f(y)\left(\theta-\theta_{i n}\right),  \tag{3}\\
& x=1, \quad \frac{\partial c}{\partial x}=0,  \tag{4}\\
& x=1, \quad \frac{\partial \theta}{\partial x}=0,  \tag{5}\\
& \frac{\partial c}{\partial y}=-\frac{\phi_{c}^{2}}{(1+s)} c \exp \left[\frac{\gamma_{c} \theta}{1+\theta}\right],  \tag{6}\\
& y=1, \quad \frac{\partial \theta}{\partial y}=\frac{\beta}{L e_{f}} \frac{\phi_{c}^{2}}{(1+s)} c \exp \left[\frac{\gamma_{c} \theta}{1+\theta}\right],  \tag{7}\\
& y=0, \quad \frac{\partial c}{\partial y}=0,  \tag{8}\\
& y=0, \quad \frac{\partial \theta}{\partial y}=0,  \tag{9}\\
& c(0, y)=b_{0},  \tag{10}\\
& \theta(0, y)=d_{0}, \tag{11}
\end{align*}
$$

where $\theta_{i n}$ is the inlet temperature, $\phi_{c}$ is the Thiele modulus for the catalytic reaction, and $\gamma_{c}$ is the dimensionless activation energy for the catalytic reaction.

Eqs. (1)-(11) establish a dimensionless mathematical model for the coupled homoge-neous-catalytic reaction. There may not be an exact mathematical solution to this model, that is to say, $c$ and $\theta$ that satisfy exactly Eqs.(1)-(11) and are continuous in the closed
domain $\{(x, y) \mid 0 \leq x, y \leq 1\}$ are nonexistent, and there are no $c$ and $\theta$ that make their second-order partial derivatives continuous in the domain $\{(x, y) \mid 0<x, y<1\}$. In order to find out the solution of Eq.(1), it is necessary to satisfy the boundary conditions (2)-(11) to various degrees. The boundary condition (8) must be satisfied when the first-order partial derivative of $c(x, y)$ is continuous on the closed domain $\{(x, y) \mid 0 \leq x, y \leq 1\}$, otherwise, the first equation in Eq.(1) will have $\frac{s}{y} \frac{\partial c}{\partial y} \rightarrow \infty\left(y \rightarrow 0^{+}\right)$. Similarly, the condition (9) must also be satisfied. Moreover, the variation of $c(x, y)$ and $\theta(x, y)$ is greatly affected by the heat transfer at the boundary, so condition (7) should be satisfied. From the above analysis, we can know that (7)-(9) are exactly satisfied. To find the solutions of $c(x, y)$ and $\theta(x, y),(10)$ and (11) should also be exactly satisfied. The conditions (2)-(6) for describing the boundary changes of $c(x, y)$ and $\theta(x, y)$ in the natural state can be regarded as reference conditions.

In this paper, we use ADM to solve the model of the coupled homogeneous-catalytic reaction. The main framework of this article is as follows: in Section 2, we devote ourselves to deduce the approximate analytic solutions of the model by ADM; in Section 3, we give the concrete expressions of approximate analytical solutions and draw the graphs of approximate analytic solutions with MATLAB; in Section 4, we summarize and comment our paper.

## 2 Derivation process for approximate analytic solutions by ADM

In this section, the ADM is used to find the approximate analytic solutions for Eq.(1).
Define the following linear operators $L_{x}=\frac{\partial}{\partial x}, L_{y}=\frac{\partial}{\partial y}, L_{x x}=\frac{\partial^{2}}{\partial x^{2}}$ and $L_{y y}=\frac{\partial^{2}}{\partial y^{2}}$, and $N(\theta)$ denotes the strong non-linear function $\exp \left[\frac{\gamma_{h} \theta}{1+\theta}\right]$, Eq.(1) can be written as

$$
\left\{\begin{array}{l}
f(y) L_{x} c=\frac{1}{P e} L_{x x} c+\frac{1}{p}\left(L_{y y} c+\frac{s}{y} L_{y} c\right)-\frac{\phi^{2}}{p} c N(\theta)  \tag{12}\\
f(y) L_{x} \theta=\lambda \frac{L e_{f}}{P e} L_{x x} \theta+\frac{L e_{f}}{p}\left(L_{y y} \theta+\frac{s}{y} L_{y} \theta\right)+\beta \frac{\phi^{2}}{p} c N(\theta)
\end{array}\right.
$$

In order to obtain the solutions satisfying (7), (8), and (9), we choose the $y$-direction as the search direction of solution. Furthermore, the inverse operator $L_{y y}^{-1}$ is defined as below

$$
\begin{equation*}
L_{y y}^{-1}(\cdot)=\int_{1}^{y}\left[\int_{0}^{u}(\cdot) d t\right] d u \tag{13}
\end{equation*}
$$

According to [26], let $f(y)=1$. By applying the inverse operator $L_{y y}^{-1}$ to Eq.(12), the following results are obtained

$$
\begin{align*}
c & =p L_{y y}^{-1} L_{x} c-\frac{p}{P e} L_{y y}^{-1} L_{x x} c-s L_{y y}^{-1}\left(\frac{1}{y} L_{y} c\right)+p D a L_{y y}^{-1}[c N(\theta)]+A(x) y \\
& +B(x)  \tag{14}\\
\theta & =\frac{p}{L e_{f}} L_{y y}^{-1} L_{x} \theta-\lambda \frac{p}{P e} L_{y y}^{-1} L_{x x} \theta-s L_{y y}^{-1}\left(\frac{1}{y} L_{y} \theta\right)-\frac{p}{L e_{f}} \beta D a L_{y y}^{-1}[c N(\theta)] \\
& +C(x) y+D(x), \tag{15}
\end{align*}
$$

where $D a=\frac{\phi^{2}}{p}$, and $A(x), B(x), C(x), D(x)$ are the undetermined functions. Implementing the first-order partial derivative operations on both sides of (14) about $y$ and combining with the boundary condition $y=0, \frac{\partial c}{\partial y}=0$, we can figure out $A(x)=0$. Then, letting $y=1,0 \leq x \leq 1$ on both sides of (14), we have $c(x, 1)=B(x), 0 \leq x \leq 1$. Similarly, on the basis of (15) and the boundary condition $y=0, \frac{\partial \theta}{\partial y}=0$, we can get $C(x)=0,0 \leq x \leq 1$. Letting $y=1,0 \leq x \leq 1$ on both sides of (15), $\theta(x, 1)=D(x), 0 \leq x \leq 1$ can be obtained. Now Eqs.(14) and (15) can be rewritten as

$$
\begin{align*}
c & =p L_{y y}^{-1} L_{x} c-\frac{p}{P e} L_{y y}^{-1} L_{x x} c-s L_{y y}^{-1}\left(\frac{1}{y} L_{y} c\right)+p D a L_{y y}^{-1}[c N(\theta)]+B(x)  \tag{16}\\
\theta & =\frac{p}{L e_{f}} L_{y y}^{-1} L_{x} \theta-\lambda \frac{p}{P e} L_{y y}^{-1} L_{x x} \theta-s L_{y y}^{-1}\left(\frac{1}{y} L_{y} \theta\right)-\frac{p}{L e_{f}} \beta D a L_{y y}^{-1}[c N(\theta)] \\
& +D(x) \tag{17}
\end{align*}
$$

The ADM decomposes the unknown functions $c(x, y)$ and $\theta(x, y)$ as

$$
\begin{align*}
& c(x, y)=\sum_{n=0}^{\infty} c_{n}(x, y)  \tag{18}\\
& \theta(x, y)=\sum_{n=0}^{\infty} \theta_{n}(x, y) . \tag{19}
\end{align*}
$$

To execute the recursive algorithm that will be involved below in the ADM and make use of the double decomposition method [30], $B(x)$ and $D(x)$ can also be decomposed into

$$
\begin{align*}
& B(x)=\sum_{n=0}^{\infty} b_{n} x^{n}  \tag{20}\\
& D(x)=\sum_{n=0}^{\infty} d_{n} x^{n} \tag{21}
\end{align*}
$$

where

$$
\begin{align*}
& b_{0}=B(0)=c(0,1)  \tag{22}\\
& d_{0}=D(0)=\theta(0,1) \tag{23}
\end{align*}
$$

The nonlinear function $N(\theta)$ is decomposed as

$$
\begin{equation*}
N(\theta)=\sum_{n=0}^{\infty} \overline{A_{n}}(n=0,1,2, \cdots), \tag{24}
\end{equation*}
$$

where $\overline{A_{n}}(n=0,1,2, \cdots)$ are known as Adomian polynomials. In this case, they are presented by the following formula $[8,31]$.

$$
\begin{equation*}
\overline{A_{n}}=\frac{1}{n!}\left[\frac{d^{n}}{d \lambda^{n}} N\left(\sum_{k=0}^{\infty} \lambda^{k} \theta_{k}\right)\right]_{\lambda=0}=\frac{1}{n!}\left[\frac{d^{n}}{d \lambda^{n}} N\left(\sum_{k=0}^{n} \lambda^{k} \theta_{k}\right)\right]_{\lambda=0}(n=0,1,2, \cdots) \tag{25}
\end{equation*}
$$

Hence, we obtain

$$
\begin{align*}
& \overline{A_{0}}\left(\theta_{0}\right)=N\left(\theta_{0}\right)=\exp \left(\gamma_{h} \theta_{0} /\left(\theta_{0}+1\right)\right)  \tag{26}\\
& \left.\overline{A_{1}}\left(\theta_{0}, \theta_{1}\right)=\gamma_{h} \exp \left(\gamma_{h} \theta_{0} / \theta_{0}+1\right)\right)\left(\theta_{1} /\left(\theta_{0}+1\right)-\left(\theta_{0} \theta_{1}\right) /\left(\theta_{0}+1\right)^{2}\right) \tag{27}
\end{align*}
$$

Then $c N(\theta)$ can be expressed as an infinite series form of Adomian's polynomials

$$
\begin{equation*}
c N(\theta)=\sum_{n=0}^{\infty} A_{n}(n=0,1,2, \cdots), \tag{28}
\end{equation*}
$$

where the $A_{n}(n=0,1,2, \cdots)$ are calculated by the following formula

$$
\begin{equation*}
A_{n}\left(c_{0}, c_{1}, \ldots, c_{n} ; \theta_{0}, \theta_{1}, \ldots, \theta_{n}\right)=\sum_{k=0}^{n} c_{k} \overline{A_{n-k}}(n=0,1,2, \cdots) \tag{29}
\end{equation*}
$$

According to (16), (17), (22), and (23), it is easy to choose the values of $c_{0}$ and $\theta_{0}$ and write the recursive relations as follows

$$
\begin{align*}
& c_{0}= b_{0},  \tag{30}\\
& \theta_{0}=d_{0},  \tag{31}\\
& \vdots
\end{aligned} \quad \begin{aligned}
c_{n} & =p L_{y y}^{-1} L_{x} c_{n-1}-\frac{p}{P e} L_{y y}^{-1} L_{x x} c_{n-1}-s L_{y y}^{-1}\left(\frac{1}{y} L_{y} c_{n-1}\right)+p D a L_{y y}^{-1} A_{n-1}+b_{n} x^{n},  \tag{33}\\
\theta_{n} & =\frac{p}{L e_{f}} L_{y y}^{-1} L_{x} \theta_{n-1}-\lambda \frac{p}{P e} L_{y y}^{-1} L_{x x} \theta_{n-1}-s L_{y y}^{-1}\left(\frac{1}{y} L_{y} \theta_{n-1}\right)-\frac{p}{L e_{f}} \beta D a L_{y y}^{-1} A_{n-1} \\
& +d_{n} x^{n} .
\end{align*}
$$

Let $u=\frac{p}{P e}, v=\frac{p}{L e_{f}}$, then (33) and (34) can be changed into

$$
\begin{align*}
c_{n} & =p L_{y y}^{-1} L_{x} c_{n-1}-u L_{y y}^{-1} L_{x x} c_{n-1}-s L_{y y}^{-1}\left(\frac{1}{y} L_{y} c_{n-1}\right)+p D a L_{y y}^{-1} A_{n-1}+b_{n} x^{n}  \tag{35}\\
\theta_{n} & =v L_{y y}^{-1} L_{x} \theta_{n-1}-\lambda u L_{y y}^{-1} L_{x x} \theta_{n-1}-s L_{y y}^{-1}\left(\frac{1}{y} L_{y} \theta_{n-1}\right)-v \beta D a L_{y y}^{-1} A_{n-1} \\
& +d_{n} x^{n} \tag{36}
\end{align*}
$$

where $n \geq 1$. Generally speaking, from many examples [24,32], we can know that a very few terms are enough. On the other hand, since the convergence speed is fast [33], several terms can substantially represent an accurate solution. Moreover, the more the number of terms, the higher the accuracy. At this point, in view of the simplicity of later numerical computation, we take six terms to approximate the exact solution and suppose the expressions

$$
\begin{align*}
\bar{c} & =\bar{c}\left(x, y ; p, s, \lambda, u, v, D a, \beta, \gamma_{h}, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) \\
& =b_{0}+\sum_{n=1}^{5} c_{n}\left(x, y ; p, s, \lambda, u, v, D a, \beta, \gamma_{h}, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) \tag{37}
\end{align*}
$$

and

$$
\begin{align*}
\bar{\theta} & =\bar{\theta}\left(x, y ; p, s, \lambda, u, v, D a, \beta, \gamma_{h}, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) \\
& =d_{0}+\sum_{n=1}^{5} \theta_{n}\left(x, y ; p, s, \lambda, u, v, D a, \beta, \gamma_{h}, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) \tag{38}
\end{align*}
$$

refer to the six-term approximation to $c$ and $\theta$ separately. A set of values of dimensionless parameters $p, s, \lambda, u, v, D a, \beta, \gamma_{h}, \phi_{c}, \gamma_{c}$ are given, then Eqs.(37) and (38) can be transformed into

$$
\begin{align*}
\bar{c} & =\bar{c}\left(x, y ; p, s, \lambda, u, v, D a, \beta, \gamma_{h}, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) \\
& =\bar{c}\left(x, y ; b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) \\
& =b_{0}+\sum_{n=1}^{5} c_{n}\left(x, y ; b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right)  \tag{39}\\
\bar{\theta} & =\bar{\theta}\left(x, y ; p, s, \lambda, u, v, D a, \beta, \gamma_{h}, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) \\
& =\bar{\theta}\left(x, y ; b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) \\
& =d_{0}+\sum_{n=1}^{5} \theta_{n}\left(x, y ; b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) . \tag{40}
\end{align*}
$$

Substituting (39) and (40) into (7) yields

$$
\begin{align*}
& \sum_{n=1}^{5} \frac{\partial \theta_{n}}{\partial y}\left(x, 1, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) \\
& -\alpha\left[\sum_{n=1}^{5} c_{n}\left(x, 1, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right)+b_{0}\right] \\
& \exp \left[\frac{\gamma_{c}\left(\sum_{n=1}^{5} \theta_{n}\left(x, 1, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right)+d_{0}\right)}{1+d_{0}+\sum_{n=1}^{5} \theta_{n}\left(x, 1, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right)}\right]=0 \tag{41}
\end{align*}
$$

where $\alpha=\frac{\beta}{L e_{f}} \frac{\phi_{c}^{2}}{(1+s)}$. The left side of (41) is a polynomial of order 5 in $x$, so

$$
\begin{align*}
& P\left(x, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right)=\sum_{n=1}^{5} \frac{\partial \theta_{n}}{\partial y}\left(x, 1, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) \\
& -\alpha\left(\sum_{n=1}^{5} b_{n} x^{n}+b_{0}\right) \exp \left[\frac{\gamma_{c}\left(\sum_{n=1}^{5} d_{n} x^{n}+d_{0}\right)}{1+d_{0}+\sum_{n=1}^{5} d_{n} x^{n}}\right]=0 \tag{42}
\end{align*}
$$

By comparing the coefficients of $x$ on both sides of (42), a set of nonlinear algebraic equations can be obtained

$$
\left\{\begin{align*}
P\left(0, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) & =0  \tag{43}\\
\frac{1}{n!} \frac{\partial^{n} P}{\partial x^{n}}\left(0, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) & =0(n=1,2, \ldots, 5) .
\end{align*}\right.
$$

In order to meet the boundary conditions (10) and (11), let

$$
\begin{align*}
& b_{0}+\sum_{n=1}^{5} c_{n}\left(0, y, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right)=b_{0}  \tag{44}\\
& d_{0}+\sum_{n=1}^{5} \theta_{n}\left(0, y, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right)=d_{0} \tag{45}
\end{align*}
$$

The left sides of (44) and (45) are the 10 order polynomials about $y$, so comparing the coefficients of $y$ on both sides of (44) and (45) respectively yields

$$
\begin{align*}
& \left\{\begin{aligned}
\sum_{n=1}^{5} c_{n}\left(0,0, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) & =0 \\
\frac{1}{k!} \sum_{n=1}^{5} \frac{\partial^{k} c_{n}}{\partial y^{k}}\left(0,0, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) & =0(k=1,2, \ldots, 10)
\end{aligned}\right.  \tag{46}\\
& \left\{\begin{aligned}
\sum_{n=1}^{5} \theta_{n}\left(0,0, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) & =0 \\
\frac{1}{k!} \sum_{n=1}^{5} \frac{\partial^{k} \theta_{n}}{\partial y^{k}}\left(0,0, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) & =0(k=1,2, \ldots, 10)
\end{aligned}\right. \tag{47}
\end{align*}
$$

Solving (43), (46), and (47), we can gain all values of $b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}$. For all the values, they satisfy (43), (46) and (47). At last, supposing

$$
\begin{equation*}
\bar{c}(x, y)=\bar{c}\left(x, y, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) \tag{48}
\end{equation*}
$$

and

$$
\begin{equation*}
\bar{\theta}(x, y)=\bar{\theta}\left(x, y, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) \tag{49}
\end{equation*}
$$

are used as approximate analytic solutions of $c(x, y)$ and $\theta(x, y)$.

## 3 Detailed expressions and graphs for approximate analytic solutions

In order to solve the system of Eq.(1), a large number of mathematical operations are needed. We choose MATLAB as the calculation tool and refer to the relevant datas of the coupled homogeneous-catalytic reaction [26] to determine the dimensionless parameter values of the model.

Take the values appropriately and assign the set of values to the dimensionless parameters $p, s, \lambda, u, v, D a, \alpha, \beta, \gamma_{h}, \phi_{c}, \gamma_{c}$, e.g. $p=0.02, s=0, \lambda=0.9, u=0.0004, v=$ $0.04, D a=5, \alpha=0.3, \beta=0.15, \gamma_{h}=26.47, \phi_{c}^{2}=1.0, \gamma_{c}=15$, and these values are substituted into (39) and (40) respectively, then the following equations are generated

$$
\begin{align*}
\bar{c}\left(x, y, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) & =b_{0}+b_{1} x \\
& +\left(b_{0} \exp \left(\left(2647 d_{0}\right) /\left(100\left(d_{0}+1\right)\right)\right)\left(y^{2}-1\right)\right) / 20 \\
& +\sum_{n=2}^{5} c_{n}\left(x, y, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right)  \tag{50}\\
\bar{\theta}\left(x, y, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right)= & d_{0}+d_{1} x \\
& -\left(3 b_{0} \exp \left(\left(2647 d_{0}\right) /\left(100\left(d_{0}+1\right)\right)\right)\left(y^{2}-1\right)\right) / 200 \\
& +\sum_{n=2}^{5} \theta_{n}\left(x, y, b_{0}, b_{1}, \ldots, b_{5}, d_{0}, d_{1}, \ldots, d_{5}\right) . \tag{51}
\end{align*}
$$

After all the parameter values have been confirmed, the solution of Eq.(43) is searched near the initial point

$$
\begin{equation*}
\left(b_{0}, b_{1}, b_{2}, b_{3}, b_{4}, b_{5}, d_{0}, d_{1}, d_{2}, d_{3}, d_{4}, d_{5}\right)=(0,0,0,0,0,0,0,0,0,0.1,0.1,0.1) \tag{52}
\end{equation*}
$$

by applying the MATLAB function fsolve (), the solution is

$$
\begin{array}{r}
\left(b_{0}, b_{1}, b_{2}, b_{3}, b_{4}, b_{5}, d_{0}, d_{1}, d_{2}, d_{3}, d_{4}, d_{5}\right)=(0.0004,-0.0027,0.0268,0.0366,0.0037, \\
-0.0246,0.0021,0.0033,-0.0067,0.0777,0.0827,0.1000) \tag{53}
\end{array}
$$

When fsolve () is used to solve nonlinear equations, what initial values are set will lead to what kind of solution is obtained. Furthermore, the algorithm starts with initial values and looks for values on both sides (iterative process). Once certain precision is satisfied, the result will be outputted. In this article, by trying different initial values many times, we find that (52) is a set of very good initial values. Through this set of values, we use fsolve () to calculate the solution of (43).

The maximum absolute value of 0.000069 is generated by substituting (53) into the left side of each equation of (46) and (47), which can therefore be educed that (53) is not only the solution of (43), but also an approximate solution of (46) and (47). Substituting (53) into (50) and (51) respectively obtains expressions of the approximate analytic solutions $c(x, y)$ and $\theta(x, y)$. These expressions are so long that we don't actually give them here, and their graphs are given in Figure 1 and Figure 2.


Figure 1. Distribution of dimensionless fluid concentration $c(x, y)$


Figure 3. Distribution of dimensionless fluid concentration $c(x, y)$


Figure 2. Distribution of dimensionless temperature $\theta(x, y)$


Figure 4. Distribution of dimensionless temperature $\theta(x, y)$

Similarly, when assigning a set of values $p=0.02, s=0, \lambda=0.9, u=0.0004, v=$ $0.04, D a=5, \alpha=0.3, \beta=0.15, \gamma_{h}=100, \phi_{c}^{2}=1.0, \gamma_{c}=15$ to dimensionless parameters, we can use MATLAB to get Figures 3 and 4 ; When assigning a set of values $p=0.02, s=$ $0, \lambda=0.9, u=0.0004, v=0.04, D a=5, \alpha=0.3, \beta=0.15, \gamma_{h}=26.47, \phi_{c}^{2}=1.0, \gamma_{c}=100$ to dimensionless parameters, Figures 5 and 6 can be obtained.


Figure 5. Distribution of dimensionless fluid concentration $c(x, y)$


Figure 6. Distribution of dimensionless temperature $\theta(x, y)$

From Figures 1, 2, 3 and 4 above, we can see that when $\gamma_{h}$ increases from 26.47 to 100, that is, when dimensionless activation energy for the homogeneous reaction increases, the concentration of the fluid decreases, and the temperature basically does not change. That is to say, the fluid concentration decreases with the increase of the activation energy for the homogeneous reaction, and the temperature is not essentially affected by the activation energy for the homogeneous reaction. In the same way, from Figures 1, 2, 5 and 6 above, we can see that when $\gamma_{c}$ increases from 15 to 100 , that is, when dimensionless activation energy for the catalytic reaction increases, the concentration of the fluid decreases, and the temperature also drops. That is to say, the fluid concentration and temperature decrease with the increase of dimensionless activation energy for the catalytic reaction, but the effect on temperature is slight; At the same time, from these graphs above, we can know that the effect of dimensionless activation energy for the catalytic reaction on concentration and temperature is greater than that of dimensionless activation energy for the homogeneous reaction on concentration and temperature.

The purpose of the coupled homogeneous-catalytic reaction model is to determine the coupling effects of various transport and kinetic parameters, and to illustrate their effects on the reactor performance. The prediction of dimensionless temperature and concentration distribution will lay a good foundation for the design and optimization of the coupled homogeneous-catalytic reaction.

## 4 Conclusion

In this work, the coupled homogeneous-catalytic reaction model composed by a system of strongly nonlinear second-order partial differential equations has been analyzed by ADM. In engineering problems, there are often systems of second-order partial differential equations with multiple boundary conditions. The exact solutions of these equations may not exist. At this time, it is necessary to analyze the boundary conditions and use ADM to obtain approximate analytical solutions satisfying the given boundary conditions in varying degrees. The ADM is a powerful method to solve many problems in some scientific applications modeled by nonlinear differential equations and are well worth studying further.

Acknowledgments: This research was supported by Scientific and Technological Innovation Programs of Higher Education Institutions in Shanxi (No. 2017116).

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