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Comments on "Some New Results on Energy of Graphs"☆

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Abstract

Vaidya and Popat have discussed energy of two particular graph compositions in MATCH Commun. Math. Comput. Chem. 77 (2017), 589–594. We establish a link to a paper of Stevanović from 2005 from which their results follow as special cases.

The *non-complete extended p-sum* (shortly NEPS) of graphs is a very general graph operation. Many graph operations are special cases of NEPS, to name just the Cartesian product and the direct product of graphs. It was defined for the first time in [1], while the following definition is taken from [2, p. 66], with a minor modification.

Definition 1 *Let $\mathcal{B}$ be a set of binary n-tuples, i.e. $\mathcal{B} \subseteq \{0,1\}^n \setminus \{(0,\dots,0)\}$ such that for every $i = 1,\dots,n$ there exists $\beta \in \mathcal{B}$ with $\beta_i = 1$. The* non-complete extended p-sum (NEPS) *of graphs $G_1,\dots,G_n$ with basis $\mathcal{B}$, denoted by* $\mathrm{NEPS}(G_1,\dots,G_n;\mathcal{B})$, *is the graph with the vertex set $V(G_1)\times\cdots\times V(G_n)$, in which two vertices $(u_1,\dots,u_n)$ and $(v_1,\dots,v_n)$ are adjacent if and only if there exists $(\beta_1,\dots,\beta_n) \in \mathcal{B}$ such that u_i is adjacent to v_i in G_i whenever $\beta_i = 1$, and $u_i = v_i$ whenever $\beta_i = 0$.*

Graphs $G_1,\dots,G_n$ are called the *factors* of NEPS. The simplest case of NEPS is obtained when its basis consists of a single n-tuple $\mathcal{B} = \{(1,1,\dots,1)\}$. In such case $\mathrm{NEPS}(G_1,\dots,G_n;\{(1,1,\dots,1)\})$ is denoted simply as $G_1 \times G_2 \times \cdots \times G_n$ and called

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the *product* of $G_1, \ldots, G_n$. Two vertices $(u_1, \ldots, u_n)$ and $(v_1, \ldots, v_n)$ are thus adjacent in $G_1 \times G_2 \times \cdots \times G_n$ if and only if u_i is adjacent to v_i in G_i for each $i = 1, \ldots, n$.

One of the most important properties of NEPS of graphs is that its spectrum can be represented by the spectra of its factors.

Theorem 1 **[2, Theorem 2.23]** *The spectrum of* $\mathrm{NEPS}(G_1, \ldots, G_n; \mathcal{B})$ *consists of all possible values* Λ *given by*

$$\Lambda = \sum_{\beta \in \mathcal{B}} \lambda_1^{\beta_1} \cdots \lambda_n^{\beta_n}, \tag{1}$$

where λ_i *is an arbitrary eigenvalue of* G_i, $i = 1, \ldots, n$.

Let $E(G)$ denotes the energy of a graph G, i.e., the sum of absolute values of eigenvalues of adjacency matrix of G [3, 4]. Based on previous result, it was proved in 2005 that

Theorem 2 [5] *For graphs* $G_1, G_2, \ldots, G_n$ *holds*

$$E(G_1 \times G_2 \times \cdots \times G_n) = E(G_1) \cdot E(G_2) \cdot \cdots \cdot E(G_n). \tag{2}$$

In the same paper it was further proved that the energy of NEPS is representable as a function of the energy of its factors solely if NEPS is actually the product of graphs.

Theorem 3 [5] *Let* $\mathcal{B}$ *be a basis of* NEPS. *Then there exists a function* $f_{\mathcal{B}} \colon \mathbb{R}^n \mapsto \mathbb{R}$ *such that*

$$E(\mathrm{NEPS}(G_1, G_2, \ldots, G_n; \mathcal{B})) = f_{\mathcal{B}}(E(G_1), E(G_2), \ldots, E(G_n))$$

holds for any graphs G_1, G_2, $\ldots$, G_n *if and only if* $\mathcal{B} = \{(1, 1, \ldots, 1)\}$.

In their recent paper [6], Vaidya and Popat introduced two graph compositions:

- the *splitting graph* $S'(G)$ of a graph G is obtained by adding to each vertex v a new vertex v' such that v' is adjacent to every vertex that is adjacent to v in G;
- the *shadow graph* $D_2(G)$ of a graph G is obtained by taking two copies of G, say G' and G'', and then by joining each vertex v' in G' to the neighbors of the corresponding vertex u'' in G''.

Let us reinterpret these compositions in terms of NEPS as follows. In the splitting graph $S'(G)$ denote each vertex v of the original graph G as $(v, 0)$ and a corresponding new vertex v' as $(v, 1)$. We see from the above definition that any two adjacent vertices

u and v in G give rise to the following pairs of adjacent vertices in $S'(G)$: $((u,0),(v,0))$, $((u,0),(v,1))$ and $((u,1),(v,0))$. Hence $S'(G)$ is the product of G and a two-vertex graph H that consists of vertices 0 and 1 with a loop at 0 and an edge between 0 and 1. Then Theorem 2.1 from [6] that $E(S'(G)) = E(G)\sqrt{5}$ follows directly from Theorem 2 and the fact that the adjacency matrix of H has eigenvalues $\frac{1+\sqrt{5}}{2}$ and $\frac{1-\sqrt{5}}{2}$, and hence H has energy $\sqrt{5}$. Analogously one can see that the shadow graph $D_2(G)$ is just a product of G and a complete graph K_2, so that Theorem 3.1 from [6] that $E(D_2(G)) = 2E(G)$ follows again from Theorem 2 and the fact that the complete graph K_2 has energy 2.

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