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Correcting the Number of L-Borderenergetic Graphs of Order 9 and 10

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Abstract

In the paper "A Computer Search for the L-Borderenergetic Graphs", *MATCH Commun. Math. Comput. Chem.* 77 (2017) 595–606, 120 non-isomorphic non-complete L-borderenergetic graphs of order 10 are reported. By computer search, we find that their total number is 232. The missing graphs are now determined.

1 New L-borderenergetic graphs on 10 vertices

A graph of order n is said to be L-borderenergetic [5] if its Laplacian energy is equal to the Laplacian energy of the complete graph of order n, i.e., if it is equal to $2n - 2$. For more notation consult [4]. For the recent results on the L-Bordereneregetic graphs refer [1–3, 5, 6].

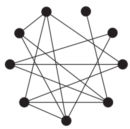
In [4], it was reported that there are exactly 65 non-isomorphic non-complete L-borderenergetic graphs upto 9 vertices. In [1], at the same period of time, it was reported that there exits exactly 75 such graphs respectively. (See Table 1).

n	4	5	6	7	8	9
number	2	1	11	4	31	16

n	4	5	6	7	8	9
number	2	1	11	5	33	23

Table 1. The numbers of connected non-complete and non-isomorphic L-borderenergetic graphs on n vertices for $4 \le n \le 9$.

Based on the software package GraphTea, we developed a program to search for L-borderenergetic graphs on 9 and 10 vertices. As a result, we find 1,112 additional L-borderenergetic graph on 9 and 10 vertices respectively. Figure 1 depicts the respective missed L-borderenergetic graph of order 9.



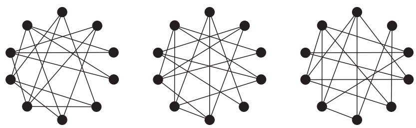
H

Figure 1. Graph H.

The Laplacian spectrum of H is $S_p(H) = \{0, 3, x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$,
where $x_1, x_2, x_3, x_4, x_5, x_6, x_7$ are the roots of the eqaution

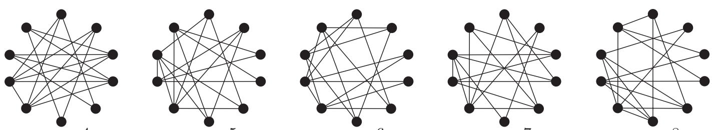
$$x^7 - 25x^6 + 255x^5 - 1363x^4 + 4076x^3 - 6728x^2 + 5595x - 1773 = 0.$$

It can be verified that H is not isomorphic to any graph reported in [1,4], and its Laplacian energy is equal to 16.



H_{10}^1 H_{10}^2 H_{10}^3

Figure 2. Cospectral L-borderenergetic graphs with 16 edges



H_{10}^4 H_{10}^5 H_{10}^6 H_{10}^7 H_{10}^8

Figure 3. Cospectral L-borderenergetic graphs with 17 edges

In [4], the number of L-borderenergetic graphs on 10 vertices have been presented. For $n = 10$, the number of L-borderenergetic graphs are given as 120, but actually it is 232. We now present all the 112 missing L-borderenergetic graphs on 10 vertices. In [4], the graphs G_6^{10} and G_9^{10} depicted in the Figure 4, has Lapalcian energy 16.37281 and 18.10018 respectively. At first, for G_6^{10} we give the three non-isomorphic non-complete and Laplacian cospectral graphs (See Figure 2) with Laplacian spectrum $S_p\left(H_{10}^i\right) = \{0, 1, 2, 2, 2, 4, 5, 5, 5, 6\}$ for $i = 1, 2, 3$. Next, for G_9^{10} we have four non-isomorphic non-complete equidegreed Laplacian cospectral graphs $H_{10}^4, H_{10}^5, H_{10}^6, H_{10}^7$ and a non equidegreed graph H_{10}^8 with Laplacian spectrum $S_p\left(H_{10}^i\right) = \{0, 1, 2, 2, 3, 4, 4, 5, 6, 7\}$ for $i = 4, 5, \ldots, 8$ (See Figure 3).

In Table 2, we give the Missed L-borderenergetic graphs H_{10}^i $(i = 9, 10, \ldots, 87)$ which are cospectral to some $G_{10}^j (j = 1, 2, \ldots, 120)$ on 10 vertices (See Figure 5, 6 and 7).

G_{10}^4	H_{10}^9	G_{10}^{35}	H_{10}^{32}	G_{10}^{62}	$H_{10}^{62,63,64}$
G_{10}^8	H_{10}^{10}	G_{10}^{38}	H_{10}^{33}	G_{10}^{64}	$H_{10}^{65,66}$
G_{10}^{10}	H_{10}^{11}	G_{10}^{44}	H_{10}^{34}	G_{10}^{65}	H_{10}^{67}
G_{10}^{11}	$H_{10}^{12,13}$	G_{10}^{46}	H_{10}^{35}	G_{10}^{71}	$H_{10}^{68,69,70,71,72,73}$
G_{10}^{14}	H_{10}^{14}	G_{10}^{48}	H_{10}^{36}	G_{10}^{73}	H_{10}^{74}
G_{10}^{16}	$H_{10}^{15,16}$	G_{10}^{49}	$H_{10}^{37,38,39,40,41,42}$	G_{10}^{74}	H_{10}^{75}
G_{10}^{21}	H_{10}^{17}	G_{10}^{50}	$H_{10}^{43,44}$	G_{10}^{75}	H_{10}^{76}
G_{10}^{22}	H_{10}^{18}	G_{10}^{51}	H_{10}^{45}	G_{10}^{76}	H_{10}^{77}
G_{10}^{23}	H_{10}^{19}	G_{10}^{52}	$H_{10}^{46,47}$	G_{10}^{80}	$H_{10}^{78,79}$
G_{10}^{25}	H_{10}^{20}	G_{10}^{53}	H_{10}^{48}	G_{10}^{82}	H_{10}^{80}
G_{10}^{27}	H_{10}^{21}	G_{10}^{54}	H_{10}^{49}	G_{10}^{88}	$H_{10}^{81,82}$
G_{10}^{29}	$H_{10}^{22,23,24,25,26,27}$	G_{10}^{55}	H_{10}^{50}	G_{10}^{92}	H_{10}^{83}
G_{10}^{31}	H_{10}^{28}	G_{10}^{57}	$H_{10}^{51,52,53,54}$	G_{10}^{95}	H_{10}^{84}
G_{10}^{32}	H_{10}^{29}	G_{10}^{58}	$H_{10}^{55,56,57}$	G_{10}^{108}	H_{10}^{85}
G_{10}^{33}	H_{10}^{30}	G_{10}^{59}	$H_{10}^{58,59}$	G_{10}^{110}	H_{10}^{86}
G_{10}^{34}	H_{10}^{31}	G_{10}^{61}	$H_{10}^{60,61}$	G_{10}^{115}	H_{10}^{87}

Table 2. Missed L-borderenergetic graphs H_{10}^i $(i = 9, 10, \ldots, 87)$ on 10 vertices, cospectral to some graph $G_{10}^j (j = 1, 2, \ldots, 120)$.

A graph is Laplacian integral if the spectrum of its Laplacian matrix consists entirely of integers. We say that a polynomial with integer coefficients is quintic (resp. octic, nonic) if the highest degree of all its irreducible factors in rational field is five (resp. eight, nine). In accordance with this, a L-borderenergetic graph is said to be a quintic (resp. octic, nonic) if the characteristic polynomial of Laplacian matrix is quintic (resp. octic, nonic). Next we present 27 non-isomorphic non-complete L-borderenergetic graphs (1 quintic, 2 octic, 2 nonic and 22 integral) with their Laplacian spectrum, which are totally missed out in [4] (See Figure 4 and 8).

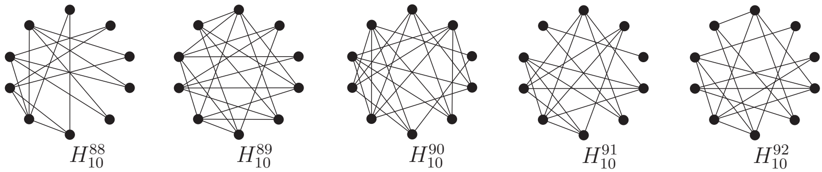


Figure 4. Non integral L-borderenergetic graphs on 10 vertices

The Laplacian spectrum of the graphs depicted in Figure 4 are

$$S_p\left(H_{10}^{88}\right) = \left\{0, 3-\sqrt{2}, 2, 3+\sqrt{2}, 5, x_1, x_2, x_3, x_4, x_5\right\}$$

where $x_i (i = 1, 2, \ldots, 5)$ are the distinct roots of the equation

$$x^5 - 17x^4 + 104x^3 - 282x^2 + 332x - 136 = 0.$$

$$S_p\left(H_{10}^{89}\right) = \{0, 4, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\},$$
$$S_p\left(H_{10}^{90}\right) = \{0, 5, y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8\}$$

where $x_i, y_i (i = 1, 2, \ldots, 8)$ are the distinct roots of the equations respectively.

$$x^8 - 36x^7 + 552x^6 - 4696x^5 + 24171x^4 - 76826x^3 + 146723x^2 - 153342x + 66880 = 0,$$
$$y^8 - 37y^7 + 584y^6 - 5126y^5 + 27296y^4 - 89968y^3 + 178254y^2 - 192354y + 85162 = 0.$$

$$S_p\left(H_{10}^{91}\right) = \{0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$$
$$S_p\left(H_{10}^{92}\right) = \{0, y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9\}$$

where $x_i, y_i (i = 1, 2, \ldots, 9)$ are the distinct roots of the equations respectively.

$$x^9 - 34x^8 + 498x^7 - 4109x^6 + 20941x^5 - 67862x^4 + 138260x^3 - 167506x^2 + 105431x - 23865 = 0$$
$$y^9 - 34y^8 + 499y^7 - 4138y^6 + 21300y^5 - 70316y^4 + 148221y^3 - 191382y^2 + 136537y - 40770 = 0$$

The Laplacian spectrum of the 22 Laplacian integral graphs missed out in [4] (See Figure 8) are

$S_p(H_{10}^{93}) = \{0,1,1,1,3,3,3,3,5,10\}$	$S_p(H_{10}^{104}) = \{0,3,6,6,7,7,7,7,7,10\}$
$S_p(H_{10}^{94}) = \{0,1,1,1,3,3,3,4,4,10\}$	$S_p(H_{10}^{105}) = \{0,4,4,4,4,6,6,6,6,10\}$
$S_p(H_{10}^{95}) = \{0,1,2,4,5,5,5,5,5,8\}$	$S_p(H_{10}^{106}) = \{0,4,4,4,5,6,6,6,7,10\}$
$S_p(H_{10}^{96}) = \{0,1,3,3,3,4,5,5,6,8\}$	$S_p(H_{10}^{107}) = \{0,4,4,5,5,6,6,6,8,10\}$
$S_p(H_{10}^{97}) = \{0,1,3,4,4,5,5,5,7,8\}$	$S_p(H_{10}^{108}) = \{0,4,5,5,5,6,6,6,9,10\}$
$S_p(H_{10}^{98}) = \{0,1,5,5,5,5,5,5,9,10\}$	$S_p(H_{10}^{109}) = \{0,4,5,6,6,6,7,7,9,10\}$
$S_p(H_{10}^{99}) = \{0,1,5,5,5,6,6,6,6,10\}$	$S_p(H_{10}^{110}) = \{0,5,5,5,5,6,6,6,10,10\}$
$S_p(H_{10}^{100}) = \{0,2,5,5,5,6,6,7,7,9\}$	$S_p(H_{10}^{111}) = \{0,5,5,5,5,6,6,7,9,10\}$
$S_p(H_{10}^{101}) = \{0,2,5,5,5,6,6,7,7,9\}$	$S_p(H_{10}^{112}) = \{0,5,5,5,5,7,7,7,7,10\}$
$S_p(H_{10}^{102}) = \{0,3,3,5,5,5,6,6,8,9\}$	$S_p(H_{10}^{113}) = \{0,5,5,5,6,6,6,7,10,10\}$
$S_p(H_{10}^{103}) = \{0,3,6,6,6,6,7,7,9,10\}$	$S_p(H_{10}^{114}) = \{0,5,5,5,6,6,7,7,9,10\}$

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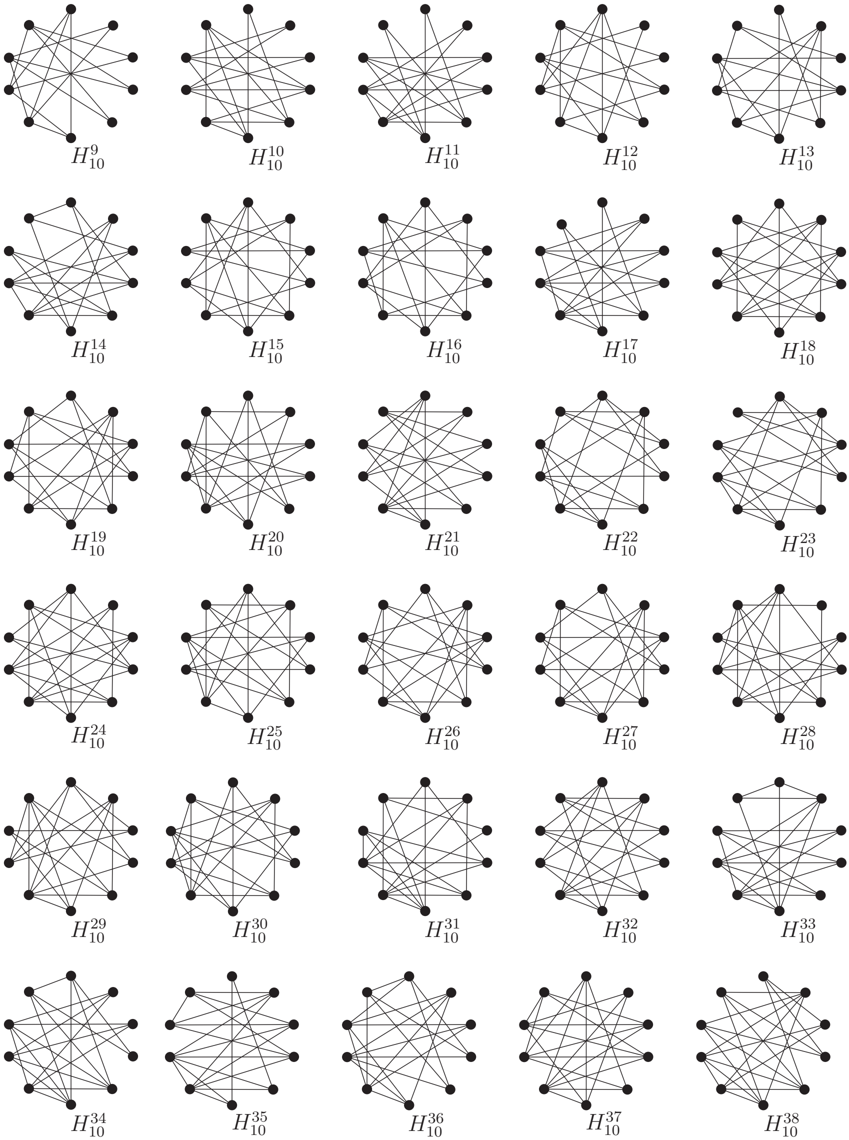


Figure 5. Missed L-borderenergetic graphs on 10 vertices.

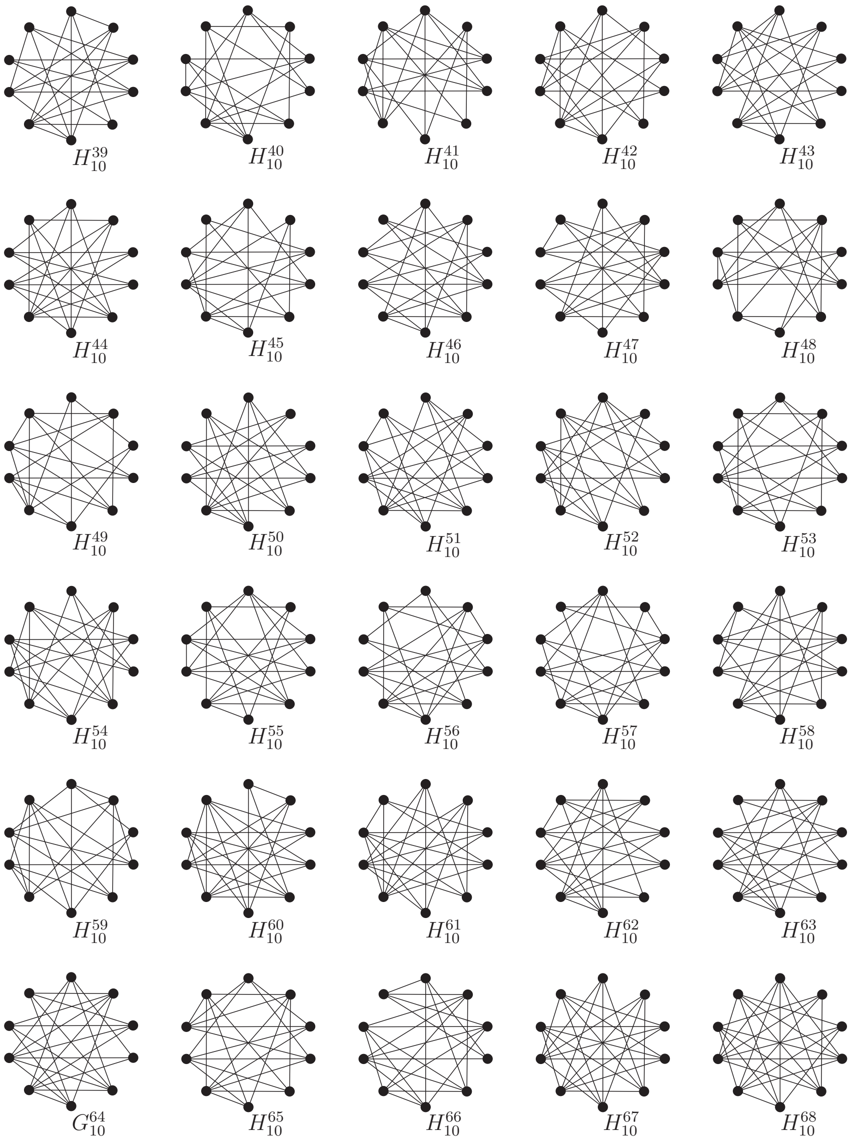
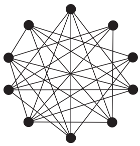
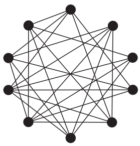
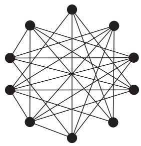


Figure 6. Missed L-borderenergetic graphs on 10 vertices (Continued).


H_{10}^{69}


H_{10}^{70}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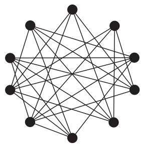
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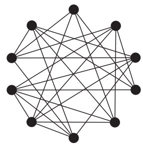
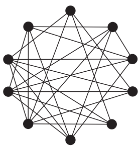

H_{10}^{73}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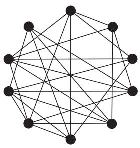
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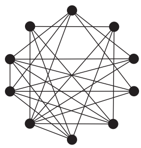
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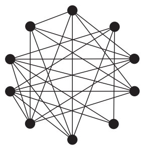
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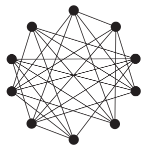
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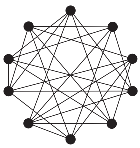
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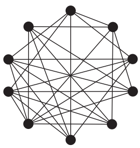
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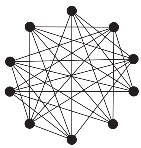
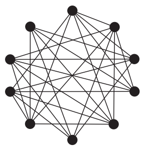
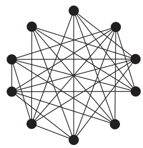
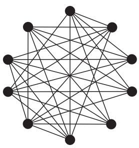

H_{10}^{81}


H_{10}^{82}


H_{10}^{83}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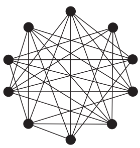
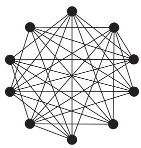

H_{10}^{85}


H_{10}^{86}

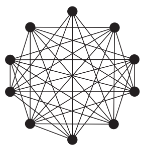

H_{10}^{87}

Figure 7. Missed L-borderenergetic graphs on 10 vertices (Continued).

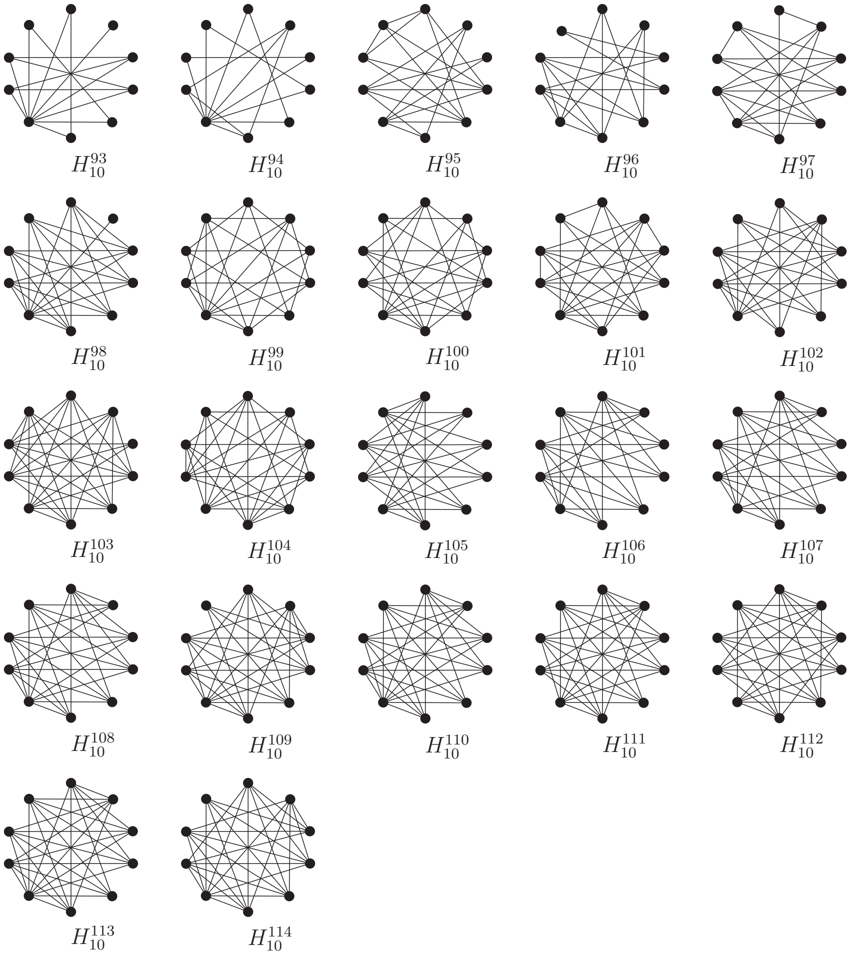


Figure 8. Integral L-borderenergetic on 10 vertices