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Correcting a Paper on First Zagreb Index

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Abstract

This short note is to point out that Theorems 2 and 4 and Corollary 3 from "New Upper Bounds for the First Zagreb Index" [S. M. Hosamani, B. Basavanagoud, New upper bounds for the first Zagreb index, *MATCH Commun. Math. Comput. Chem* , **74** (1) (2015) 97–101] are not completely correct and/or have already been published elsewhere.

1 Introduction

Let G be a simple graph with n vertices and m edges, with the sequence of vertex degrees $\Delta = d_1 \geq d_2 \geq \cdots \geq d_n = \delta$. S. M. Hasamani and B. Basavanagoud [1] considered inequalities that determine upper bound for the First Zagreb index $M_1(G) = \sum_{i=1}^{n} d_i^2$, introduced in [2]. The authors proved two theorems, namely Theorem 2 and Theorem 4, which determine upper bounds for the invariant $M_1(G)$ in terms on parameters n, m, Δ and δ. Unfortunately, in Theorem 2, the integer function $\alpha(n)$ which takes part in the upper bound for the invariant $M_1(G)$ is wrongly defined. Also, the conclusion of Theorem 4, which relates to the case when equality occurs is wrong. Moreover, the inequality is well known and already proved in [3]. The assertion given in Corollary 3 is not quite correct.

2 Main errors and comments

The main contribution of the paper [1] is contained in Theorems 2 and 4. In what follows we point out to the main errors in [1] and give our comments and corrections.

1. In Theorem 2 of [1] the following inequality was proved
$$M_1(G) \leq \frac{\alpha(n)(\Delta-\delta)^2+4m^2}{n}. \tag{1}$$
The integer function $\alpha(n)$ is defined as
$$\alpha(n) = n\left\lceil\frac{n}{2}\right\rceil\left(1-\frac{1}{n}\left\lceil\frac{n}{2}\right\rceil\right),$$
where $\lceil x\rceil$ is the largest integer greater than or equal to x.

Comment: Function $\alpha(n)$ is wrongly defined. It should be defined as
$$\alpha(n) = n\left\lfloor\frac{n}{2}\right\rfloor\left(1-\frac{1}{n}\left\lfloor\frac{n}{2}\right\rfloor\right)$$
where $\lfloor x\rfloor$ is the largest integer equal to or less than x.

2. In Theorem 4 the following inequality was proved
$$M_1 \leq (\delta+\Delta)2m - n\delta\Delta \tag{2}$$
with equality in (2) if and only if G is a regular graph.

Comment: Firstly, the conclusion which relates to the equality case is wrong. Namely, the equality in (2) occurs if and only if G is regular or bidegreed graph. Second, the inequality (2) was proved in [3] with correct conclusion of equality case.

3. In Corollary 3 the author claim that since $\alpha(n) \leq \frac{n^2}{4}$, therefore
$$M_1(G) \leq \frac{n^2(\Delta-\delta)^2+16m^2}{4n}. \tag{3}$$

Comment: Firstly, the authors didn't point out that the inequality (3) is well-known and proved in [4]. Second, according to (1) and (3) it might be concluded that inequality (1) is stronger than (3) for each $n, n \in N$. Since $\alpha(n)$ is explicitly given by
$$\alpha(n) = \frac{n^2}{4}\left(1-\frac{(-1)^{n+1}+1}{2n^2}\right) = \begin{cases} \frac{n^2}{4}, & \text{if } n \text{ is even} \\ \frac{(n-1)(n+1)}{4}, & \text{if } n \text{ is odd} \end{cases},$$
for even n the inequality (1) coincides with inequality (3). Since $\frac{(n-1)(n+1)}{4} < \frac{n^2}{4}$, the inequality (1) is stronger than (3) for odd n.

References

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