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A Computer Search for the Borderenergetic Graphs of Order 10*

Xueliang Li[1], Meiqin Wei[1], Shicai Gong[2]

[1]*Center for Combinatorics and LPMC-TJKLC*
Nankai University, Tianjin 300071, China
lxl@nankai.edu.cn; weimeiqin@163.com

[2]*Zhejiang A & F University, Hangzhou, 311300, China*
scgong@zafu.edu.cn

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Abstract

For a given simple graph G, the energy of G, denoted by $\mathcal{E}(G)$, is defined as the sum of the absolute values of all eigenvalues of its adjacency matrix, which was introduced by Gutman. A graph G of order n is said to be borderenergetic if its energy equals the energy of the complete graph K_n, i.e., if $\mathcal{E}(G) = 2(n-1)$. The problem of finding all the borderenergetic graphs on n vertices tends to be very complicated. Recently, Gong et al. gave complete lists of non-complete borderenergetic graphs on $n = 7, 8, 9$ vertices, and there is no non-complete borderenergetic graph on $n \leq 6$ vertices. However, the problem becomes very difficult when $n > 9$. In this paper, we use a computer search to find out all the borderenergetic graphs on $n = 10$ vertices. The number of such graphs is 47, among which 37 are non-cospectral. This could provide experience for further study on the borderenergetic graphs with large number of vertices.

1 Introduction

Let G be a graph of order n and $A(G)$ the adjacency matrix of G. The characteristic polynomial [2] of $A(G)$ is defined as

$$\phi(G,\lambda) = det(\lambda I - A(G)) = \sum_{i=0}^{n} a_i \lambda^{n-i},$$

which is called the *characteristic polynomial* of G. The n roots of the equation $\phi(G,\lambda) = 0$, denoted by $\lambda_1, \lambda_2, \cdots, \lambda_n$, are the *eigenvalues* of G. These eigenvalues form the *spectrum*

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of the graph G, denoted by $Sp(G)$. The *energy* of G, denoted by $\mathcal{E}(G)$, is defined as

$$\mathcal{E}(G) = \sum_{i=1}^{n} |\lambda_i|,$$

which was proposed by Gutman in 1977; see [4]. For more knowledge on the graph energy we refer to [5,6].

It is well known that the complete graph K_n has energy $2(n-1)$. A graph of order n is said to be *hyperenergetic* if $\mathcal{E}(G) > 2(n-1)$ and *non-hyperenergetic* if $\mathcal{E}(G) > 2(n-1)$. Recently, Gong et al. [3] proposed the definition of borderenergetic graphs. A graph of order n with energy $2(n-1)$ is called a *borderenergetic graph*. And a borderenergetic graph is said to be *non-complete* if it is different from the complete graph K_n. In [3], Gong et al. performed a computer-aided calculation for graphs on $n \leq 9$ vertices and gave complete lists of non-complete borderenergetic graphs on $n = 7, 8, 9$ vertices, the number of which is 1, 6, 17, respectively, and there is no non-complete borderenergetic graphs on $n \leq 6$ vertices. However, the problem of finding all the borderenergetic graphs on n vertices becomes very difficult when $n > 9$. Actually, when running our program by our computer, it took about 10 seconds for $n = 8$ and 2 minutes for $n = 9$. But, it took dramatically long time, about 5 days, to find out all 47 such graphs for $n = 10$.

In this paper, we give all the borderenergetic graphs on $n = 10$ vertices by some computer-aided calculations. In Section 2, we divide all the borderenergetic graphs on $n = 10$ vertices into 3 classes and check if the equality $\mathcal{E}(G) = 2(n-1)$ holds, respectively. Throughout this paper, all graphs are assumed to be simple, undirected and finite.

2 Main results

A graph G is said to be *integral* if all its eigenvalues are integers. In addition, we say an integral coefficients polynomial is *quadratic (cubic)* if the highest degree of all its irreducible factors in rational field is two (three). And a borderenergetic graph is said to be a *quadratic (cubic) borderenergetic graph* if its characteristic polynomial is quadratic (cubic).

We perform a computer-aided calculation for $n = 10$ vertices and obtain that there exist exactly 47 non-isomorphic borderenergetic graphs of order 10, among which 37 are non-cospectral. In order to check the equality $\mathcal{E}(G) = 2(n-1)$ clearly, we divide all the borderenergetic graphs on $n = 10$ vertices into 3 classes: integral borderenergetic graphs, quadratic borderenergetic graphs and cubic borderenergetic graphs.

At first, we focus on the integral borderenergetic graphs of order 10.

2.1 Integral borderenergetic graphs of order 10

There are exactly 14 non-isomorphic integral borderenergetic graphs of order $n = 10$. These graphs are depicted in Figure 2.1. The spectra of the graphs shown in Figure 2.1 are listed as follows:

$$Sp(G_1^1) = \{6, 2, 1, 0, -1, -1, -1, -2, -2, -2\}$$
$$Sp(G_2^1) = \{7, 1, 1, 0, 0, -1, -1, -2, -2, -3\}$$
$$Sp(G_3^1) = \{7, 1, 1, 0, -1, -1, -1, -2, -2, -2\}$$
$$Sp(G_4^1) = \{5, 2, 1, 1, -1, -1, -1, -2, -2, -2\}$$
$$Sp(G_5^1) = \{5, 3, 1, -1, -1, -1, -1, -1, -2, -2\}$$
$$Sp(G_6^1) = \{5, 2, 1, 1, -1, -1, -1, -2, -2, -2\}$$
$$Sp(G_7^1) = \{6, 2, 1, 0, 0, -1, -2, -2, -2, -2\}$$
$$Sp(G_8^1) = \{6, 1, 1, 1, 0, 0, -2, -2, -2, -3\}$$
$$Sp(G_9^1) = \{7, 1, 1, -1, -1, -1, -1, -1, -1, -3\}$$
$$Sp(G_{10}^1) = \{5, 1, 1, 1, 1, -1, -2, -2, -2, -2\}$$
$$Sp(G_{11}^1) = \{6, 1, 1, 1, -1, -1, -1, -2, -2, -2\}$$
$$Sp(G_{12}^1) = \{5, 1, 1, 1, 1, -1, -2, -2, -2, -2\}$$
$$Sp(G_{13}^1) = \{6, 2, 1, 0, 0, -1, -1, -2, -2, -3\}$$
$$Sp(G_{14}^1) = \{7, 1, 1, 0, -1, -1, -1, -2, -2, -2\}$$

It is easy to verify that the energies of the integral graphs depicted in Figure2.1 are all equal to $18 = 2 \times (10 - 1)$. In other words, they are all integral borderenergetic graphs of order 10. Additionally, Balińska et al. [1] listed are all the integral graphs on 10 vertices (displayed in List 3 of [1]) and their spectra (given in List 6 of [1]). Among all these integral graphs, there are exactly 14 graphs having energy 18. And also, these 14 graphs are exactly the graphs shown in Figure 2.1. It means that the graphs depicted in Figure 2.1 are exactly all the integral borderenergetic graphs on 10 vertices. There is no doubt that this gives strong support to the validity of our computer-aided calculation.

Here we should point out that there are 3 pairs of cospectral but non-isomorphic integral borderenergetic graphs G_3^1 and G_{14}^1, G_4^1 and G_6^1, G_{10}^1 and G_{12}^1.

For $n = 10$, the number of integral borderenergetic graphs is 14. However, the integral borderenergetic graphs are only a small part of all borderenergetic graphs. We will give all the other borderenergetic graphs in the following.

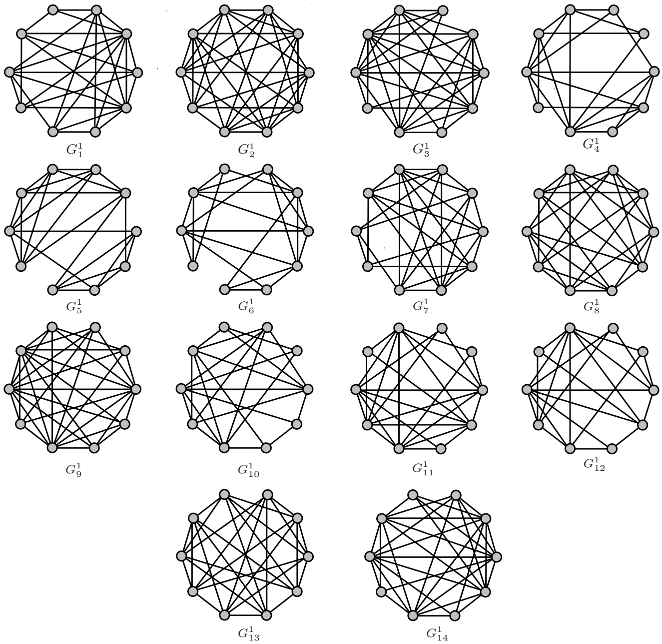

Figure 2.1. The 14 integral borderenergetic graphs of order 10

2.2 Quadratic borderenergetic graphs of order 10

Among all borderenergetic graphs of order 10, there are exactly 25 quadratic borderenergetic graphs. In this part, we will show all the quadratic borderenergetic graphs of order 10 in Figures 2.2 and 2.3. At the same time, we give the spectra of all these graphs. One can check that the energies of all these graphs are also equal to $18 = 2 \times (10 - 1)$, which implies that they are all borderenergetic graphs of order 10.

Next, we give the spectra of the graphs shown in Figures 2.2 and 2.3:

$$Sp(G_1^2) = \left\{4 + \sqrt{14}, 1, 4 - \sqrt{14}, -1, -1, -1, -1, -1, -2, -2\right\}$$

$$Sp(G_2^2) = \left\{\frac{7 + \sqrt{33}}{2}, 1, 1, \frac{7 - \sqrt{33}}{2}, -1, -1, -1, -1, -2, -3\right\}$$

$$Sp(G_3^2) = \left\{3 + \sqrt{6}, 2, 1, 3 - \sqrt{6}, -1, -1, -1, -2, -2, -2\right\}$$

$$Sp(G_4^2) = \left\{\frac{7 + \sqrt{41}}{2}, 2, \frac{7 - \sqrt{41}}{2}, 0, -1, -1, -1, -2, -2, -2\right\}$$

$$Sp(G_5^2) = \left\{4 + \sqrt{11}, 1, 4 - \sqrt{11}, -1, -1, -1, -1, -1, -2, -2\right\}$$

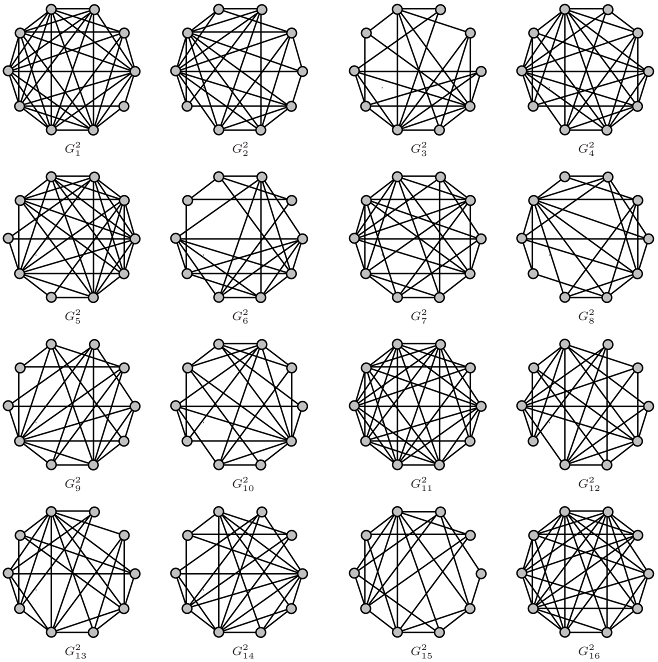

Figure 2.2. The 25 quadratic borderenergetic graphs of order 10 (G_1^2–G_{16}^2)

$$Sp(G_6^2) = \left\{3+\sqrt{6}, 2, 1, 3-\sqrt{6}, -1, -1, -1, -2, -2, -2\right\}$$

$$Sp(G_7^2) = \left\{\frac{7+\sqrt{33}}{2}, 1, 1, \frac{7-\sqrt{33}}{2}, 0, -1, -1, -2, -2, -3\right\}$$

$$Sp(G_8^2) = \left\{3+\sqrt{6}, 2, 1, 3-\sqrt{6}, -1, -1, -1, -2, -2, -2\right\}$$

$$Sp(G_9^2) = \left\{3+\sqrt{6}, 2, 1, 3-\sqrt{6}, -1, -1, -1, -2, -2, -2\right\}$$

$$Sp(G_{10}^2) = \left\{3+\sqrt{6}, 1, 1, 1, 3-\sqrt{6}, -1, -2, -2, -2, -2\right\}$$

$$Sp(G_{11}^2) = \left\{4+\sqrt{13}, 1, 4-\sqrt{13}, -1, -1, -1, -1, -1, -1, -3\right\}$$

$$Sp(G_{12}^2) = \left\{\frac{7+\sqrt{17}}{2}, \frac{7-\sqrt{17}}{2}, 1, 1, 0, -1, -2, -2, -2, -2\right\}$$

$$Sp(G_{13}^2) = \left\{\frac{7+\sqrt{17}}{2}, 2, \frac{7-\sqrt{17}}{2}, 0, -1, -1, -1, -2, -2, -2\right\}$$

$$Sp(G_{14}^2) = \left\{\frac{7+\sqrt{17}}{2}, 2, \frac{7-\sqrt{17}}{2}, 0, -1, -1, -1, -2, -2, -2\right\}$$

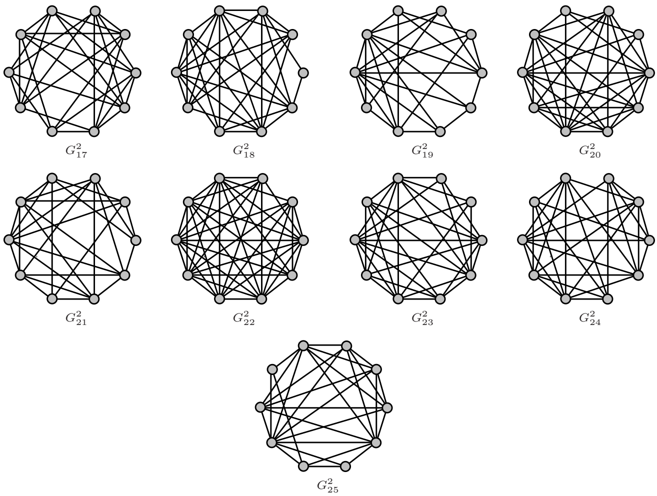

Figure 2.3. The 25 quadratic borderenergetic graphs of order 10 (G_{17}^2–G_{25}^2)

$$Sp(G_{15}^2) = \left\{3+\sqrt{5}, 2, 1, 3-\sqrt{5}, -1, -1, -1, -2, -2, -2\right\}$$

$$Sp(G_{16}^2) = \left\{\frac{9+\sqrt{33}}{2}, \frac{9-\sqrt{33}}{2}, 0, 0, -1, -1, -1, -2, -2, -2\right\}$$

$$Sp(G_{17}^2) = \left\{3+2\sqrt{2}, 1, 1, 1, 3-2\sqrt{2}, -1, -1, -1, -3, -3\right\}$$

$$Sp(G_{18}^2) = \left\{4+2\sqrt{2}, 4-2\sqrt{2}, 1, 0, -1, -1, -1, -2, -2, -2\right\}$$

$$Sp(G_{19}^2) = \left\{\frac{7+\sqrt{17}}{2}, \frac{7-\sqrt{17}}{2}, 1, 1, 0, -1, -2, -2, -2, -2\right\}$$

$$Sp(G_{20}^2) = \left\{\frac{9+\sqrt{33}}{2}, \frac{9-\sqrt{33}}{2}, 0, -1, -1, -1, -1, -1, -2, -2\right\}$$

$$Sp(G_{21}^2) = \left\{3+2\sqrt{2}, 2, 1, 3-2\sqrt{2}, -1, -1, -1, -1, -2, -3\right\}$$

$$Sp(G_{22}^2) = \left\{\frac{9+\sqrt{57}}{2}, \frac{9-\sqrt{57}}{2}, 0, -1, -1, -1, -1, -1, -2, -2\right\}$$

$$Sp(G_{23}^2) = \left\{4+\sqrt{7}, 4-\sqrt{7}, 1, 0, 0, -1, -2, -2, -2, -2\right\}$$

$$Sp(G_{24}^2) = \left\{\frac{7+\sqrt{33}}{2}, 2, \frac{7-\sqrt{33}}{2}, 0, -1, -1, -1, -1, -2, -3\right\}$$

$$Sp(G_{25}^2) = \left\{\frac{7+\sqrt{33}}{2}, 2, \frac{7-\sqrt{33}}{2}, -1, -1, -1, -1, -1, -2, -2\right\}$$

Here we also point out that there are 3 groups of cospectral but non-isomorphic borderenergetic graphs $\{G_3^2, G_6^2, G_8^2, G_9^2\}$, $\{G_{12}^2, G_{19}^2\}$, $\{G_{13}^2, G_{14}^2\}$. A new phenomenon appears that the first group has 4 cospectral but non-isomorphic borderenergetic graphs.

2.3 Cubic borderenergetic graphs of order 10

All cubic borderenergetic graphs of order 10 are displayed in Figure 2.4.

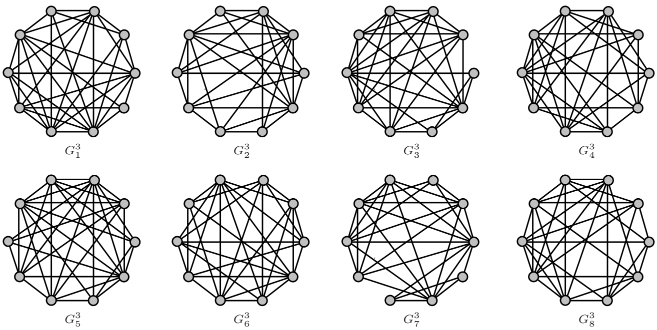

Figure 2.4. The 8 cubic borderenergetic graphs of order 10

Similarly, we give the spectra of these graphs as follows:

$$Sp(G_1^3) = \{0, -1, -1, -1, -2, -2, -2, x_1, x_2, x_3\}$$
$$Sp(G_2^3) = \{1, 0, -1, -2, -2, -2, -2, y_1, y_2, y_3\}$$
$$Sp(G_3^3) = \{-1, -1, -1, -1, -1, -2, -2, z_1, z_2, z_3\}$$
$$Sp(G_4^3) = \{1, 0, -1, -1, -2, -2, -3, u_1, u_2, u_3\}$$
$$Sp(G_5^3) = \{0, -1, -1, -1, -2, -2, -2, x_1, x_2, x_3\}$$
$$Sp(G_6^3) = \{1, 0, -1, -1, -2, -2, -3, u_1, u_2, u_3\}$$
$$Sp(G_7^3) = \{-1, -1, -1, -1, -1, -2, -2, p_1, p_2, p_3\}$$
$$Sp(G_8^3) = \left\{1, \frac{\sqrt{5}-3}{2}, -1, -1, -1, -\frac{\sqrt{5}+3}{2}, -3, q_1, q_2, q_3\right\},$$

where

$$x_1 = \frac{1}{3}\left(216 + 3i\sqrt{1407}\right)^{\frac{1}{3}} + \frac{13}{\left(216 + 3i\sqrt{1407}\right)^{\frac{1}{3}}} + 3$$

$$x_2 = -\frac{1}{6}\left(216 + 3i\sqrt{1407}\right)^{\frac{1}{3}} - \frac{13}{2\left(216 + 3i\sqrt{1407}\right)^{\frac{1}{3}}} + 3 + \frac{1}{2}i\sqrt{3}\left(\frac{1}{3}\left(216 + 3i\sqrt{1407}\right)^{\frac{1}{3}} - \frac{13}{\left(216 + 3i\sqrt{1407}\right)^{\frac{1}{3}}}\right)$$

$$x_3 = -\frac{1}{6}\left(216+3i\sqrt{1407}\right)^{\frac{1}{3}} - \frac{13}{2\left(216+3i\sqrt{1407}\right)^{\frac{1}{3}}} + 3 - \frac{1}{2}i\sqrt{3}\left(\frac{1}{3}\left(216+3i\sqrt{1407}\right)^{\frac{1}{3}} - \frac{13}{\left(216+3i\sqrt{1407}\right)^{\frac{1}{3}}}\right)$$

$$y_1 = \frac{1}{3}\left(143+3i\sqrt{1038}\right)^{\frac{1}{3}} + \frac{31}{3\left(143+3i\sqrt{1038}\right)^{\frac{1}{3}}} + \frac{8}{3}$$

$$y_2 = -\frac{1}{6}\left(143+3i\sqrt{1038}\right)^{\frac{1}{3}} - \frac{31}{6\left(143+3i\sqrt{1038}\right)^{\frac{1}{3}}} + \frac{8}{3} + \frac{1}{2}i\sqrt{3}\left(\frac{1}{3}\left(143+3i\sqrt{1038}\right)^{\frac{1}{3}} - \frac{31}{3\left(143+3i\sqrt{1038}\right)^{\frac{1}{3}}}\right)$$

$$y_3 = -\frac{1}{6}\left(143+3i\sqrt{1038}\right)^{\frac{1}{3}} - \frac{31}{6\left(143+3i\sqrt{1038}\right)^{\frac{1}{3}}} + \frac{8}{3} - \frac{1}{2}i\sqrt{3}\left(\frac{1}{3}\left(143+3i\sqrt{1038}\right)^{\frac{1}{3}} - \frac{31}{3\left(143+3i\sqrt{1038}\right)^{\frac{1}{3}}}\right)$$

$$z_1 = \frac{1}{3}\left(189+3i\sqrt{2622}\right)^{\frac{1}{3}} + \frac{13}{\left(189+3i\sqrt{2622}\right)^{\frac{1}{3}}} + 3$$

$$z_2 = -\frac{1}{6}\left(189+3i\sqrt{2622}\right)^{\frac{1}{3}} - \frac{13}{2\left(189+3i\sqrt{2622}\right)^{\frac{1}{3}}} + 3 + \frac{1}{2}i\sqrt{3}\left(\frac{1}{3}\left(189+3i\sqrt{2622}\right)^{\frac{1}{3}} - \frac{13}{\left(189+3i\sqrt{2622}\right)^{\frac{1}{3}}}\right)$$

$$z_3 = -\frac{1}{6}\left(189+3i\sqrt{2622}\right)^{\frac{1}{3}} - \frac{13}{2\left(189+3i\sqrt{2622}\right)^{\frac{1}{3}}} + 3 - \frac{1}{2}i\sqrt{3}\left(\frac{1}{3}\left(189+3i\sqrt{2622}\right)^{\frac{1}{3}} - \frac{13}{\left(189+3i\sqrt{2622}\right)^{\frac{1}{3}}}\right)$$

$$u_1 = \frac{1}{3}\left(179+3i\sqrt{807}\right)^{\frac{1}{3}} + \frac{34}{3\left(179+3i\sqrt{807}\right)^{\frac{1}{3}}} + \frac{8}{3}$$

$$u_2 = -\frac{1}{6}\left(179+3i\sqrt{807}\right)^{\frac{1}{3}} - \frac{17}{3\left(179+3i\sqrt{807}\right)^{\frac{1}{3}}} + \frac{8}{3} + \frac{1}{2}i\sqrt{3}\left(\frac{1}{3}\left(179+3i\sqrt{807}\right)^{\frac{1}{3}} - \frac{34}{3\left(179+3i\sqrt{807}\right)^{\frac{1}{3}}}\right)$$

$$u_3 = -\frac{1}{6}\left(179+3i\sqrt{807}\right)^{\frac{1}{3}} - \frac{17}{3\left(179+3i\sqrt{807}\right)^{\frac{1}{3}}} + \frac{8}{3} - \frac{1}{2}i\sqrt{3}\left(\frac{1}{3}\left(179+3i\sqrt{807}\right)^{\frac{1}{3}} - \frac{34}{3\left(179+3i\sqrt{807}\right)^{\frac{1}{3}}}\right)$$

$$p_1 = \frac{1}{3}\left(270+30\sqrt{51}\right)^{\frac{1}{3}} + \frac{10}{\left(270+30\sqrt{51}\right)^{\frac{1}{3}}} + 3$$

$$p_2 = -\frac{1}{6}\left(270+30\sqrt{51}\right)^{\frac{1}{3}} - \frac{5}{\left(270+30\sqrt{51}\right)^{\frac{1}{3}}} + 3 + \frac{1}{2}i\sqrt{3}\left(\frac{1}{3}\left(270+30\sqrt{51}\right)^{\frac{1}{3}} - \frac{10}{\left(270+30\sqrt{51}\right)^{\frac{1}{3}}}\right)$$

$$p_3 = -\frac{1}{6}\left(270+30\sqrt{51}\right)^{\frac{1}{3}} - \frac{5}{\left(270+30\sqrt{51}\right)^{\frac{1}{3}}} + 3 - \frac{1}{2}i\sqrt{3}\left(\frac{1}{3}\left(270+30\sqrt{51}\right)^{\frac{1}{3}} - \frac{10}{\left(270+30\sqrt{51}\right)^{\frac{1}{3}}}\right)$$

$$q_1 = \frac{1}{6}\left(1324+12i\sqrt{5295}\right)^{\frac{1}{3}} + \frac{68}{3\left(1324+12i\sqrt{5295}\right)^{\frac{1}{3}}} + \frac{8}{3}$$

$$q_2 = -\frac{1}{12}\left(1324+12i\sqrt{5295}\right)^{\frac{1}{3}} - \frac{34}{3\left(1324+12i\sqrt{5295}\right)^{\frac{1}{3}}} + \frac{8}{3} + \frac{1}{2}i\sqrt{3}\left(\frac{1}{6}\left(1324+12i\sqrt{5295}\right)^{\frac{1}{3}} - \frac{68}{3\left(1324+12i\sqrt{5295}\right)^{\frac{1}{3}}}\right)$$

$$q_3 = -\frac{1}{12}\left(1324+12i\sqrt{5295}\right)^{\frac{1}{3}} - \frac{34}{3\left(1324+12i\sqrt{5295}\right)^{\frac{1}{3}}} + \frac{8}{3} - \frac{1}{2}i\sqrt{3}\left(\frac{1}{6}\left(1324+12i\sqrt{5295}\right)^{\frac{1}{3}} - \frac{68}{3\left(1324+12i\sqrt{5295}\right)^{\frac{1}{3}}}\right)$$

The above expressions of $x_k, y_k, z_k, u_k, p_k, q_k$ $(k = 1, 2, 3)$ are very complicated, and it seems that they are complex numbers since the imaginary number i is a part of them. But, by common knowledge (every eigenvalue of an undirected graph is a real number), they all are real numbers. For convenience, we give the numerical values of them as follows:

$$x_k \in \{0.37019,\ 1.51972,\ 7.11009\}$$
$$y_k \in \{0.21433,\ 1.47977,\ 6.30590\}$$
$$z_k \in \{0.15878,\ 1.78516,\ 7.05606\}$$
$$u_k \in \{0.24748,\ 1.24111,\ 6.51140\}$$
$$p_k \in \{0.57638,\ 1.84653,\ 6.57709\}$$
$$q_k \in \{0.10945,\ 1.40976,\ 6.48079\}$$

Because the expressions of $x_k, y_k, z_k, u_k, p_k, q_k$ $(k = 1, 2, 3)$ are very complicated, it becomes very hard to directly calculate the energy, i.e., the sum of the absolute values of these eigenvalues. So we turn to considering the characteristic polynomial to find helps for the calculation of the energy. Since the analysis for the energies of the 8 graphs depicted in Figure 2.4 are similar, we only give the details for the calculation of energy of graph G_1^3.

The characteristic polynomial of graph G_1^3 is

$$\phi(G_1^3, \lambda) = \lambda(\lambda+1)^3(\lambda+2)^3(\lambda^3 - 9\lambda^2 + 14\lambda - 4).$$

Let x_1, x_2, x_3 be the roots of the cubic equation $\lambda^3 - 9\lambda^2 + 14\lambda - 4 = 0$. It follows that $x_1+x_2+x_3 = 9$. Fortunately, from the curve of $f(\lambda) = \lambda^3 - 9\lambda^2 + 14\lambda - 4$, we can see that x_1, x_2, x_3 are three positive real roots of $\lambda^3 - 9\lambda^2 + 14\lambda - 4 = 0$. Therefore, $|x_1|+|x_2|+|x_3| = x_1+x_2+x_3 = 9$ and the energy of the graph G_1^3 is $\mathcal{E}(G_1^3) = 1\times3+2\times3+9 = 18 = 2\times(10-1)$.

In addition, we point out that in this class there are 2 pairs of cospectral but non-isomorphic borderenergetic graphs G_1^3 and G_5^3, G_4^3 and G_6^3.

Up to now, all the 47 non-complete borderenergetic graphs of order 10 have been found out, among which 37 are non-cospectral.

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