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A New Method for Solution of Fuzzy Reaction Equation

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Abstract

In this work we will try to solve fuzzy reaction equation undergeneralized differentiability by using noniterative method. There are four different solutions of the problem when the fuzzy derivative is considered as generalization of the H-derivative [1]. Numerical solution of each system is obtained by applying superposition method. The applicability of presented algorithm is shown by solving an examples of for fuzzy reaction equation.

1 Introduction

Fuzzy differential equations (FDEs) are utilized for the purpose of the modeling problems in science and engineering. Different approaches are used for solving fuzzy differential equations. The first approach is based on Zadeh's extension principle [2].In this approach the associated crisp problem is solved and in the obtained solution the boundary values are substituted instead of the real constant. The second approach the problem is solved by writing in the form of a family of differential inclusion [3–6]. In the last few years, many works have been performed by several authors in numerical solutions of fuzzy differential equations [1,7–9]. Most of the problems in science and engineering require the solutions of a fuzzy differential equation (FDE) which are satisfied in two-point boundary values. There are theoretical difficulties that are making the problem difficult, namely, the fact that there are large classes of two-point boundary value problems that have no solution under Hukuhara differentiability [5,10,11]. On the other hand, interpreting fuzzy differential equations [1, 7–9] through differential inclusions makes the problem difficult from the numerical point of view. In the third approach it is assumed that the derivatives in the equation are generalized in H derivative form or in the strongly generalized H-derivative form. In the present paper we use generalized differentiability concept [12–14] to numerically solve fuzzy two-point boundary value problems. In this paper, we consider a two-point boundary value problem for a fuzzy reaction differential equation

$$f''(t) + A(t)f'(t) + B(t)f(t) = r(t) \tag{1}$$

where $A(t), B(t)$ are known continuous functions on $[0, l]$. The boundary conditions are

$$f(a) = \gamma \tag{2}$$

$$f(b) = \lambda \tag{3}$$

where $\gamma, \lambda \in \mathcal{F}$ and $F : [a, b] \to \mathcal{F}$ is fuzzy function.
This paper is organized as follows:
In Section 2, we present the basic definition and necessary preliminary information. In Section 3, boundary value problem for second-order FDEs under generalized differentiability is studied. Numerical algorithm for solving considered problem is introduced in Section 4 and finally In Section 5 we show an application of numerical solutions to illustrate our algorithm.

2 Preliminaries

We give some definitions and introduce the necessary notation which will be used this article.

Definition 1. Let X be a nonempty set. A fuzzy set u in X is characterized by its membership function $u : X \to [0, 1]$. Thus, $u(x)$ is interpreted as the degree of membership of an element x in the fuzzy set u for each $x \in X$.
Let us denote by $\mathcal{F}$ the class of fuzzy subsets of the real axis (i.e.,$u : \mathbb{R} \to [0, 1]$) satisfying the following properties:

(i) u is normal, that is, there exists $x_0 \in \mathbb{R}$ such that $u(x_0) = 1$.
(ii) u is convex fuzzy set (i.e. $u(\lambda x+(1-\lambda)y) \geq \min\{u(x), u(y)\}$, for all $\lambda \in [0, 1], x, y \in \mathbb{R}$).
(iii) u is upper semicontinuous on $\mathbb{R}$.

(iv) $cl\{x \in \mathbb{R} | u(x) > 0\}$ is compact where cl denotes the closure of a subset.

Then $\mathcal{F}$ is called the space of fuzzy numbers. For each $\alpha \in (0,1]$ the α-level set $[u]^\alpha$ of a fuzzy set u is the subset of points $x \in \mathbb{R}$ with

$$[u]^\alpha = [\underline{u}^\alpha, \overline{u}^\alpha] \tag{4}$$

where $\underline{u}$ and $\overline{u}$ are called lower and upper branch of u respectively.
For $u \in \mathbb{E}$ we define the length of u as:

$$len(u) = \sup_\alpha (\overline{u}^\alpha - \underline{u}^\alpha) \ . \tag{5}$$

A fuzzy number in parametric form is presented by an ordered pair of functions $(\underline{u}^\alpha, \overline{u}^\alpha)$, $0 \leq \alpha \leq 1$, satisfying the following properties:

(i) $\underline{u}^\alpha$ is a bounded nondecreasing left-continuous function of α over $(0,1]$ and right continuous for $\alpha = 0$.

(ii) $\overline{u}^\alpha$ is a bounded nonincreasing left-continuous function of α over $(0,1]$ and right continuous for $\alpha = 0$.

(iii) $\underline{u}^\alpha \leq \overline{u}^\alpha, \ 0 \leq \alpha \leq 1$.

For $u, v \in \mathcal{F}$ and $\lambda \in \mathbb{R}$, the sum $u + v$ and the product λu are defined by $[u+v]^\alpha = [u]^\alpha + [v]^\alpha$, $[\lambda u]^\alpha = \lambda [u]^\alpha$, for all $\alpha \in [0,1]$, where $[u]^\alpha + [v]^\alpha$ means the usual addition of two intervals (subsets) of $\mathbb{R}$ and $\lambda[u]^\alpha$ means the usual product between a scalar and a subset of $\mathbb{R}$. The metric on $\mathbb{E}$ is defined by the equation

$$D(u,v) = \sup_{\alpha \in [0,1]} \max\{|\underline{u}^\alpha - \underline{u}^\alpha|, |\overline{u}^\alpha - \overline{u}^\alpha|\} \tag{6}$$

is a Haussdorf distance of two interval $[u]^\alpha$ and $[v]^\alpha$.

Definition 2. Let $x, y \in \mathcal{F}$. If there exists $z \in \mathcal{F}$ such that $x = y + z$, then z is called the H- difference of x, y and it is denoted $x \ominus y$.

Definition 3. (Differentiability in the sense of Hukuhara) Let $I = (0, l)$ and $f : I \to \mathcal{F}$ is a fuzzy function. We say that f is differentiable at $t_0 \in I$ if there exists an element $f'(t_0) \in \mathcal{F}$ such that the limits

$$\lim_{h \to 0^+} \frac{f(t_0 + h) \ominus f(t_0)}{h}, \quad \lim_{h \to 0^+} \frac{f(t_0) \ominus f(t_0 - h)}{h}$$

exist and are equal to $f'(t_0)$. Here, the limits are taken in the metric space $(\mathcal{F}, D)$.

Definition 4. Let $f : I \to \mathcal{F}$ and fix to $t_0 \in I$. We say that

$(i) f$ is (1) differentiable at t_0, if there exists an element $f'(t_0) \in \mathcal{F}$ such that for all $h > 0$ sufficiently near to 0, there exist $f(t_0 + h) \ominus f(t_0), f(t_0) \ominus f(t_0 - h)$, and the limits (in the metric D)

$$\lim_{h \to 0^+} \frac{f(t_0 + h) \ominus f(t_0)}{h} = \lim_{h \to 0^+} \frac{f(t_0) \ominus f(t_0 - h)}{h} = F'(t_0) \tag{7}$$

(ii) f is (1) differentiable at t_0, if there exists an element $f'(t_0) \in \mathcal{F}$ such that for all $h > 0$ sufficiently near to 0, there exist $f(t_0 + h) \ominus f(t_0), f(t_0) \ominus f(t_0 - h)$, and the limits (in the metric D)

$$\lim_{h \to 0^-} \frac{f(t_0 + h) \ominus f(t_0)}{h} = \lim_{h \to 0^-} \frac{f(t_0) \ominus f(t_0 - h)}{h} = F'(t_0) \tag{8}$$

If f is (n)-differentiable at t_0, we denote its first derivatives by $D_n^{(1)} f(t_0)$, for $n = 1, 2$. It is obviously that Hukuhara differentiable function has increasing length of support. If the function doesn't has this properties then this function is not H-differentiable. To avoid this difficulty the authors in [15], introduced a more general definition of derivative for fuzzy number valued function in the following form.

Theorem 5. Let $f : I \to \mathcal{F}$ be fuzzy function, where $[f(t)]^\alpha = [\underline{f}_\alpha(t), \overline{f}_\alpha(t)]$ for each $\alpha \in [0, 1]$.

(i) If f is (1) differentiable in the first form, then $\underline{f}_\alpha$ and $\overline{f}_\alpha$ are differentiable functions and $[D_1^1 f(t)]^\alpha = [\underline{f}'_\alpha(t), \overline{f}'_\alpha(t)]$.

(ii) If f is (2) differentiable in the first form, then $\underline{f}_\alpha$ and $\overline{f}_\alpha$ are differentiable functions and $[D_2^1 f(t)]^\alpha = [\overline{f}'_\alpha(t), \underline{f}'_\alpha(t)]$.

Proof. See [4].

Now let fuzzy function f is (1) or (2) differentiable, then the first derivative $D_1^1 f$ for $D_2^1 f$ might be (n)-differentiable $(n = 1, 2)$ and there are four possibilities $D_1^1(D_1^1 f(t))$, $D_2^1(D_1^1 f(t)), D_1^1(D_2^1 f(t))$ and $D_2^1(D_2^1 f(t))$. The second derivatives $D_n^1(D_m^1 f(t))$ are denoted by $D_{n,m}^2 f(t)$ for $n, m = 1, 2$. Similar to Theorem 5, in [14] authors get following results for the second derivatives.

Theorem 6. Let $D_1^1 f : I \to \mathcal{F}$ or $D_2^1 f : I \to \mathcal{F}$ be fuzzy functions, where $[f(t)]^\alpha = [\underline{f}^\alpha(t), \overline{f}^\alpha(t)]$ for $\forall \alpha \in [0, 1]$. Then

(i) If $D_1^1 f$ is (1) differentiable, then $\underline{f}'^\alpha$ and $\overline{f}'^\alpha$ are differentiable functions and $[D_{1,1}^2 f(t)]^\alpha = [\underline{f}''_\alpha(t), \overline{f}''_\alpha(t)]$.

(ii) If $D_1^1 f$ is (2) differentiable, then $\underline{f}'^{\alpha}$ and $\overline{f}'^{\alpha}$ are differentiable functions and $[D_{1,2}^2 f(t)]^{\alpha} = [\overline{f}''_{\alpha}(t), \underline{f}''_{\alpha}(t)]$.

(iii) If $D_2^1 f$ is (1) differentiable, then $\underline{f}'^{\alpha}$ and $\overline{f}'^{\alpha}$ are differentiable functions and $[D_{2,1}^2 f(t)]^{\alpha} = [\overline{f}''_{\alpha}(t), \underline{f}''_{\alpha}(t)]$.

(iv) If $D_2^1 f$ is (2) differentiable, then $\underline{f}'^{\alpha}$ and $\overline{f}'^{\alpha}$ are differentiable functions and $[D_{2,2}^2 f(t)]^{\alpha} = [\underline{f}''_{\alpha}(t), \overline{f}''_{\alpha}(t)]$.

Proof. See [15].

3 Fuzzy Boundary Value Problem (FBVP)

We solve the FBVP under generalized Hukuhara derivative [13]. Four two-point boundary value problems system are possible for problem $(1) - (2)$, as follows:
$(1, 1)$ system

$$\underline{f}''(t,\alpha) + A(t)\underline{f}'(t,\alpha) + B(t)\underline{f}(t,\alpha) = \underline{r}(t,\alpha)$$
$$\overline{f}''(t,\alpha) + A(t)\overline{f}'(t,\alpha) + B(t)\overline{f}(t,\alpha) = \overline{r}(t,\alpha)$$
$$\underline{f}'(a,\alpha) = \underline{\gamma}_{\alpha} \ ; \ \overline{f}'(a,\alpha) = \overline{\gamma}_{\alpha}$$
$$\underline{f}'(b,\alpha) = \underline{\lambda}_{\alpha} \ ; \ \overline{f}'(b,\alpha) = \overline{\lambda}_{\alpha}$$

$(1, 2)$ system :

$$\overline{f}''(t,\alpha) + A(t)\underline{f}'(t,\alpha) + B(t)\underline{f}(t,\alpha) = \overline{r}(t,\alpha)$$
$$\underline{f}''(t,\alpha) + A(t)\overline{f}'(t,\alpha) + B(t)\overline{f}(t,\alpha) = \underline{r}(t,\alpha)$$
$$\underline{f}'(a,\alpha) = \underline{\gamma}_{\alpha} \ ; \ \overline{f}'(a,\alpha) = \overline{\gamma}_{\alpha}$$
$$\underline{f}'(b,\alpha) = \underline{\lambda}_{\alpha} \ ; \ \overline{f}'(b,\alpha) = \overline{\lambda}_{\alpha}$$

$(2, 1)$ system:

$$\overline{f}''(t,\alpha) + A(t)\overline{f}'(t,\alpha) + B(t)\underline{f}(t,\alpha) = \overline{r}(t,\alpha)$$
$$\underline{f}''(t,\alpha) + A(t)\underline{f}'(t,\alpha) + B(t)\overline{f}(t,\alpha) = \underline{r}(t,\alpha)$$
$$\underline{f}'(a,\alpha) = \underline{\gamma}_{\alpha} \ ; \ \overline{f}'(a,\alpha) = \overline{\gamma}_{\alpha}$$
$$\underline{f}'(b,\alpha) = \underline{\lambda}_{\alpha} \ ; \ \overline{f}'(b,\alpha) = \overline{\lambda}_{\alpha}$$

$(2,2)$ system is as follows:

$$\underline{f}''(t,\alpha) + A(t)\overline{f}'(t,\alpha) + B(t)\underline{f}(t,\alpha) = \underline{r}(t,\alpha)$$
$$\overline{f}''(t,\alpha) + A(t)\overline{f}'(t,\alpha) + B(t)\overline{f}(t,\alpha) = \overline{r}(t,\alpha)$$
$$\underline{f}'(a,\alpha) = \underline{\gamma}_\alpha \ ; \ \overline{f}'(a,\alpha) = \overline{\gamma}_\alpha$$
$$\underline{f}'(b,\alpha) = \underline{\lambda}_\alpha \ ; \ \overline{f}'(b,\alpha) = \overline{\lambda}_\alpha$$

We choose the type of solution and translate problem $(1)-(3)$ to the corresponding system of boundary value problem. We can construct solution of the fuzzy boundary value problem $(1)-(3)$ [16]. We look for a domain in which the solution and its derivatives have valid level sets according to the type of differentiability. For example, for finding $(1,2)$ solution, we solve system $(1,2)$ system and look for a domain where the solution is $(1,2)$-differentiable.

4 Noniterative Method

The techniques based on transforming linear ordinary differential equations from boundary value to initial value problems. For linear ordinary differential equations, it is in general possible to reduce the boundary value problem to two or more initial value problems [17]. Consider, the linear fuzzy second order differential equation

$$f''(t,\alpha) + A(t)f'(t,\alpha) + B(t)f(t,\alpha) = r(t,\alpha) \tag{9}$$

subject to the boundary conditions

$$f(a,\alpha) = \gamma_\alpha \tag{10}$$

$$f(b,\alpha) = \lambda_\alpha \tag{11}$$

Where $A(t)$ and $B(t)$ are continuous functions of t, the continuity of $A(t)$ and $B(t)$ assure the existence and uniqueness of the solution of equation (9). To transform equation $(9)-(11)$ into an initial value problem, we assume:

$$f(t,\alpha) = f_1(t,\alpha) + \mu f_2(t,\alpha) \tag{12}$$

where μ is a constant to be determined. Substituting $f(t,\alpha)$ defined Eq.(12) into Eq. (9), two equations are obtained:

$$f_1''(t,\alpha) + A(t)f_1'(t,\alpha) + B(t)f_1(t,\alpha) = r(t,\alpha) \tag{13}$$

$$f_2''(t,\alpha) + A(t)f_2'(t,\alpha) + B(t)f_2(t,\alpha) = 0 \tag{14}$$

The first boundary condition in Eq. (10) is next transformed to

$$f_1(a,\alpha) + \mu f_2(a,\alpha) = f_a(\alpha) \tag{15}$$

from which

$$f_1(a,\alpha) = \gamma(\alpha), \qquad f_2(a,\alpha) = 0. \tag{16}$$

If the two unknown boundary conditions are equal to

$$f_1'(a,\alpha) = 0, \qquad f_2'(a,\alpha) = 1 \tag{17}$$

then the unknown constant μ is identified as the missing initial slope. As a final step, the boundary condition at the second point is transformed to:

$$f_1(b,\alpha) + \mu f_2(b,\alpha) = f_b(\alpha) \tag{18}$$

from which:

$$\mu = \frac{f_b(\alpha) - f_1(b,\alpha)}{f_2(b,\alpha)} \tag{19}$$

As a result, Eq. (9) is transformed into an fuzzy initial value problem, since now Eq. (13) and Eq. (14) can be integrated backward from $x = a$ to $x = b$ by using the fuzzy initial condition given by Eqs. (16) and (17). From Eq. (19)we calculate μ which according to equation Eq. (14) the missing initial slope is. The solution of the original differential equation can be calculated from equation Eqs.(9) [17]. By the following theorem, the solution of the fuzzy initial value problem is equivalent to the system of ODEs.

Theorem 7. Let us consider FIVP where $F : [t_0, t_0 + a] \times \mathcal{F} \times \mathcal{F} \to \mathcal{F}$ is such that

(i) $[F(t, x(t), x'(t))]^\alpha = [\underline{f}(t, x, x')(\alpha), \overline{f}(t, x, x')(\alpha)]$ for each $\alpha \in [0, 1]$.

(ii) The functions $\underline{f}(t, x, x')(\alpha)$ and $\overline{f}(t, x, x')(\alpha)$ are equicontinuous, i.e. for any $\varepsilon > 0$ there exists $\delta > 0$ such that for any $(t, y, z), (t_1, y_1, z_1) \in [t_0, t_0 + a, t_0 + b] \times \mathbb{R}^2$ we have $|\underline{f}(t, y, z)(\alpha) - \underline{f}(t_1, y_1, z_1)(\alpha)| < \varepsilon$ and $|\overline{f}(t, y, z)(\alpha) - \overline{f}(t_1, y_1, z_1)(\alpha)| < \varepsilon$ whenever $\|(t_1, y_1, z_1) - (t, y, z)\| < \delta$.

(iii) $[F(t, x(t), x'(t))]^\alpha$ is named uniformly bounded on any bounded set, if exist $L > 0$ such that $|\underline{f}(t, y_1, z_1)(\alpha) - \underline{f}(t, y_2, z_2)(\alpha)| < L \max\{|t_1 - t_2|, |y_1 - y_2|, |z_1 - z_2|\}$ and also $|\overline{f}(t, y_1, z_1)(\alpha) - \overline{f}(t, y_2, z_2)(\alpha)| < L \max\{|t_1 - t_2|, |y_1 - y_2|, |z_1 - z_2|\}$

Now, problem is to solve the systems for $\forall(m, n), m, n = 1, 2$. Then for (n, m)-differentiability, the FIVP and the corresponding (n, m) system are equivalent.

5 Numerical Examples

In this section we report some results of our numerical calculations using the noniterative method described in the previous section.

Example: Let us consider fuzzy solution of the dimensionless an isothermal first-order reaction. The tubular reactor is a conduit through which a mixture of chemically reacting fluid flows [18]. Such reactors are of fundamental importance in the design of chemical plants. Consider an isothermal first-order reaction $A \to B$, the rate of which, according to the theory of chemical kinetics, is given by kC_A, where k and C_A are the rate of reaction and the concentration of species A, respectively. In the analysis of such systems, it is customarily assumed that the axial dispersion can be described by an effective diffusion coefficient E_a giving a diffusion flux of

$$N = E_a(-dC_A/dx) \tag{20}$$

A balance of the fluxes of species A in and out of an infinitesimal volume will give total flux into left-side surface $= vC_A + N$

total flux out of right-side surface $= [vC_A + d(vC_A)] + (N + dN)$

rate of disappearance of species $A = kC_A dx$

where v is the axial velocity of species A. Both k and v are assumed to be constants. Conservation of species A requires that

$$(vC_A + N) = [vC_A + d(vC_A] + (N + dN) + kC_A dx \tag{21}$$

Upon simplification and applying Eq.(20), we get

$$E_a \frac{d^2C_A}{dx^2} - v\frac{dC_A}{dx} - kC_A = 0 \tag{22}$$

If we define

$$y = \frac{C_A}{C_{A0}}, \qquad z = \frac{x}{L}, \quad N_{pe} = \frac{vL}{Ea}, \qquad R = \frac{kL}{v} \tag{23}$$

Eq.(22) then becomes

$$\frac{d^2y}{dz^2} - N_{\rho e}\frac{dy}{dz} - N_{\rho e}Ry = 0 \tag{24}$$

If C_{A0} is the concentration of the fluid entering the tube, the flux at the entrance of the tube is vC_{A0}, from whence it is transported downstream by convection and diffusion,i.e.,

$$vC_{A0} = vC_A(0) + N(0) \tag{25}$$

or

$$vC_{A0} = vC_A(0) - Ea\frac{dC_A(0)}{dx} \tag{26}$$

$$1 = y(0) - \frac{1}{N_{\rho e}}\frac{dy(0)}{dz} \tag{27}$$

If the length of the tube is denoted by L, then at $x = L$, the boundary condition is $\frac{dC_A(L)}{dx} = 0$ or, in dimensionless form, $\frac{dy(1)}{dz} = 0$
which means physically that the length of the tube is long enough for A to react to form B. Eq.(24), subject to the boundary conditions represents a boundary value problem. To solve this equation noniteratively, let us shift the variables z and y by
$z = 1 - s$, $y = 1 - f$. After the rearranging we obtain the following:

$$\begin{aligned} f''(t,\alpha) &= RN_{\rho e}f(t,\alpha) - N_{\rho e}f'(t,\alpha) - RN_{\rho e} \\ f'(0,\alpha) &= (0,\alpha) \\ f(1,\alpha) + \frac{1}{N_{\rho e}}f'(1,\alpha) &= (0,\alpha) \end{aligned}$$

To apply the method of superposition method, we write:

$$f(t,\alpha) = \theta(t,\alpha) + \beta\phi(t,\alpha)$$

where β is a constant to be determined. The two initial value problems can be written as

$$\begin{aligned} &\theta''(t,\alpha) = RN_{\rho e}\theta(t,\alpha) - N_{\rho e}\theta'(t,\alpha) - RN_{\rho e} \\ &\theta'(0,\alpha) = 0 \\ &\theta'(0,\alpha) = 0 \end{aligned}$$

and

$$\begin{aligned} &\phi''(t,\alpha) = RN_{\rho e}\phi(t,\alpha) - N_{\rho e}\phi'(t,\alpha) \\ &\phi'(0,\alpha) = 0 \\ &\phi(0,\alpha) = 1 \end{aligned}$$

The constant β can be determined by the boundary conditions at $t = 1$, where $\beta = -(N_{\rho e}\theta(1,\alpha) + \theta'(1,\alpha))/(N_{\rho e}\phi(1,\alpha) + \phi'(1,\alpha))$, $N_{\rho e} = 1$ and $R = 1$
First example is done four different solution of the problem $(1 - 3)$. As seen from the Figure 1, the results of numerical solution and exact solution $t \in [0,1]$ are presented.

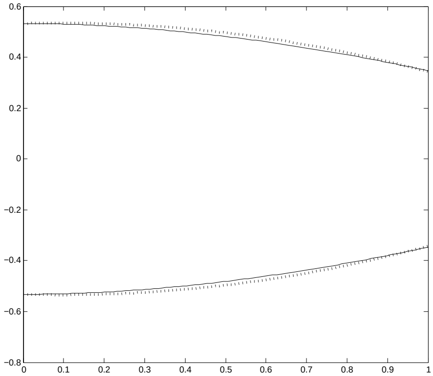

Figure 1. Solution of the reaction equation by using generalized differentiability $\underline{f}(t,\alpha)$ and $\overline{f}(t,\alpha)$.

We see $\underline{f}(t,\alpha)$ and $\overline{f}(t,\alpha)$ show a valid fuzzy number, that is for $t \leq 0.85$ we have $(2,2)$ solution and for $t \geq 0.86$ we have $(1,1)$ solution.

In the the Figure 2, we tested for different α-cut. The straight line showed $\underline{f}(t,\alpha)$ and $\overline{f}(t,\alpha)$ for $\alpha = 0$. If we consider $(1,2)$ and $(2,1)$ system, $(1,2)$ solution is for $t \leq 0.404$ and for $t \geq 0.414$.

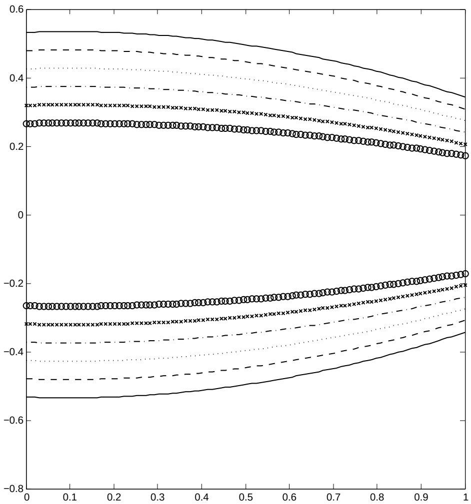

Figure 2. For different values α-cut the results of $f(t,\alpha)$.

This solution is shown in Figure 3.

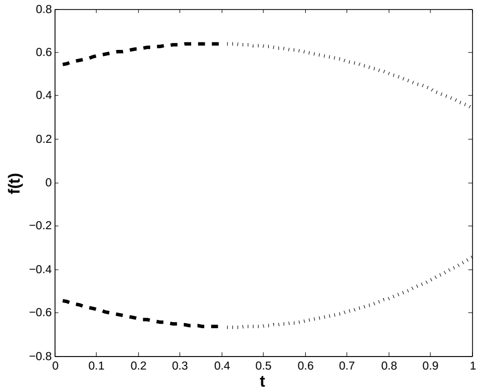

Figure 3. The fuzzy solution of (1,2)-derivative :(1,2)-solution(dash),(2,1)-solution(dot).

Second example is given values of absolute errors between numerical results and exact solution in Table 1.

Table 1. Absolute error of $(1, 1)$ solution.

α	$\underline{f}$	$\overline{f}$
0	0.0028664	0.0028951
0.1	0.0025798	0.0026056
0.2	0.0022932	0.0023161
0.3	0.0020065	0.0020266
0.4	0.0017199	0.0017371
0.5	0.0014332	0.0014476
0.6	0.0011466	0.0011580
0.7	0.0008599	0.0008685
0.8	0.0005733	0.0005790
0.9	0.0002866	0.0002895
1	0.0000000	0.0000000

6 Conclusion

In the paper noniterative method has been applied to fuzzy reaction equation via the strongly generalized differentiability concept. The disadvantage of strongly generalized differentiability of a function with respect to H-differentiability seems to be that a fuzzy reaction equation has not a unique solution. The advantage of the existence of these solutions is that we can choose that reflects better the behaviour of the modelled real-world system. The proposed method was tested and illustrated in figures.

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