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Number of 6-Matchings in Graphs

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Abstract

In this paper, a formula for the number of 6-matchings in graphs with the girth at least 6 based on the number of vertices, edges and the number of 6-cycles is derived.

1. Introduction

The graph theory is one of the most important theories in mathematics and because of several characteristics in graphs it has a lot of applications in other fields of mathematics and even other sciences as well. Finding any new characteristics in graphs and categorizing the graphs based on it is very important. A lot of works have been done by researchers that have had important role in finding such characteristics. So any research in this area that could study another characteristic of graphs will be of great importance. One of them is the concept of matching's. Graphs in this paper are assumed to be finite, loopless and contain no multiple edges. Let G be a graph with n vertices and m edges $V(G)$ and $E(G)$ are used as the sets of vertices and edges respectively. A matching in graph G is a spanning subgraph of G, whose components are only vertices and edges. A matching with k edges is called a k-matching. A matching in which all components are only edges is called a perfect matching. In graph G of order n the matching polynomial shown by $\mu(G,x)$ is defined by:

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$$\mu(G,x) = \sum_{k=0}^{[n/2]} (-1)^k p(G,k) x^{n-2k}$$

In which [ ] is the integer part of a real number and $p(G,k)$ denotes the number of k-matchings. The graphs that have the same matching polynomials are called co-matching graphs.

2. Preliminaries and related works

It is obvious that $p(G,0) = 1\,, p(G,1) = m$.

In the following lemma the number of 2-matchings was obtained by Cvetkovic et.al [4].

Lemma 2.1. If $d_1, d_2, \ldots . d_n$ denote the degrees of vertices of, then:

$$p(G,2) = \binom{m}{2} - \sum_{i=1}^{n} \binom{d_i}{2}$$

Using above lemma, Farrell and Guo [1] studied the graphs that are co-matching with regular graphs.

Lemma 2.2. Any graph that is co-matching with a regular graph is also regular of the same valency.

In the following theorem again by Farrell and Guo [1], a formula for the number of 3-matchings for graphs was obtained based on the vertex degrees, number of vertices and triangles in a graph.

Theorem 2.3.

$$p(G,3) = \binom{m}{3} - (m-2)\sum_{i} \binom{d_i}{2} + 2\sum_{i} \binom{d_i}{3} + 2\sum_{ij} (d_i - 1)(d_j - 1) - N_T$$

In which N_T denotes the number of triangles in G.

For regular graphs the following results are deduced as the direct consequences of above theorem and lemma 2.1.

Corollary 2.4. Let G be a d-regular graph with n vertices, then:

$p(G,2) = \frac{(n-4)d+2}{2^2(2!)}(nd)$

$p(G,3) = \frac{(n^2-12n+40)d^2+(6n-48)d+16}{2^3(3!)}(nd) - N_T$

Corollary 2.5. Let G and H be two co-matching regular graphs, then The number of triangles in G and H is equal.

In 2008, Behmaram [2] obtained a formula for the number of 4-matching in triangle-free graphs:

Theorem 2.6. Let G be a triangle-free graph with $V(G) = \{1,2,\dots,n\}$ and d_i is the degree of vertex i. Also let $N(i)$ be the set of neighbors of i in G. Hence, the number of 4-matchings is:

$$p(G,4) = \binom{m}{4} + (m-2)\sum_{ij}(d_i-1)(d_j-1) - \sum_i \binom{d_i}{4} - \sum_{\{i,j\}\subset v}\binom{d_i}{2}\binom{d_j}{2}$$
$$-\sum_i \sum_{\{k,t\}\subset N(i)}(d_k-1)(d_t-1) - \sum_i \binom{d_i}{2} p(G-i,2) - \sum_i \binom{d_i}{3}(m-d_i)$$
$$-\sum_i \sum_{\{k,s,t\}\subset N(i)}(d_k+d_s+d_t-3) + N_q$$

In which $N_q$is the number of 4-cycles in G.

Two following results are immediate consequences of above theorem.

Corollary 2.7. Let G be a triangle-free and d-regular graph with n vertices. Then:

$$p(G,4) = \frac{\sum_{k=0}^{3} p_k(n)d^k}{2^4(4!)}(nd) + N_q$$

In which

$$P_0(n) = 240\ , P_1(n) = 76n - 960\ \ , P_2(n) = 12n^2 - 240n + 1344$$

$$P_3(n) = n^3 - 24n^2 + 208n - 672$$

Corollary 2.8. Let G and H be two triangle-free regular graphs which are co-matching, then the number of 4-cycles in G and H is equal.

In 2013 Vesalian and Asgari [3] obtained a formula for the number of 5-matchings in graphs with the girth at least 5.

Theorem 2.9. Let G be a graph with girth at least 5 and $V(G) = \{1,2,\dots,n\}$. Denoted the degree of vertex i by d_i and the neighboring set of i by $N(i)$, then the number of 5-matchings is:

$$
\begin{aligned}
P(G,5) = \binom{m}{5} &- \sum_{i=1}^{n}\binom{d_i}{5} - \sum_{i=1}^{n}\binom{d_i}{4}(m-d_i) - \sum_{i=1}^{n}\binom{d_i}{3}\binom{m-d_i}{2} \\
&- \sum_{i=1}^{n}\binom{d_i}{2}P(G-i,3) - 3\sum_{ij}\binom{d_i-1}{2}\binom{d_i-1}{2} \\
&+ \sum_{ij}(d_i-1)(d_j-1)\binom{m-d_i-d_j+2}{2} + \sum_{ij}\sum_{\substack{K\in N(i)-\{j\}\\ t\in N(j)-\{i\}}}(d_k-1)(d_t-1) \\
&+ \sum_{i}\sum_{\{k,t\}\subset N(i)}\left[\binom{d_t-1}{2}(d_k-1) + \binom{d_k-1}{2}(d_t-1)\right] \\
&- (m-4)\sum_{i}\sum_{\{k,t\}\subset N(i)}(d_k-1)(d_t-1) \\
&+ 2\sum_{i}\sum_{\{k,s,t\}\subset N(i)}[(d_k-1)(d_s-1) + (d_k-1)(d_t-1) + (d_s-1)(d_t-1)] \\
&- \sum_{\{i,j\}\subset V}\binom{d_i}{2}\binom{d_j}{2}P(G-i-j,1) + (m-4)\sum_{i}\sum_{\{k,s,t\}\subset N(i)}(d_k+d_s+d_t-3) \\
&- 3\sum_{i}\sum_{\{k,s,t,r\}\subset N(i)}(d_k+d_s+d_t+d_r-4) - N_p
\end{aligned}
$$

In which N_p is the number of 5-cycles in graph G.

Immediate results of above theorem for regular graphs are:

Corollary 2.10. Let G be a d-regular graph with girth at least 5 with n vertices, then:

$$P(G,5) = \frac{\sum_{k=0}^{4} p_k(n)d^k}{2^5(5!)}(nd) - N_p$$

In which:

$$P_0(n) = 5376, P_1(n) = 1520n - 26880,$$

$$P_2(n) = 220n^2 - 6400n + 51840,$$

$$P_3(n) = 20n^3 - 720n^2 + 9440n - 46080,$$

$$P_4(n) = n^4 - 40n^3 + 640n^2 - 4960n + 16128$$

Corollary 2.11. Let G and H be two regular graphs with girth at least 5 that are co-matching, then the number of 5-cycle in them is equal.

3. Number of 6-matchings

In the following theorem a formula for the seventh coefficient of matching polynomial $p(G,6)$ for the graphs with girth at least 6 is derived.

Theorem 3.1. Let G be a graph with the girth at least 6 that $V(G)$ and $E(G)$ are the sets of vertices and edges in G, respectively and $V(G)=\{1,2,\dots,n\}$ and also d_i be the degree of vertex i and $N(i)$ be the nighboring set of i. Then the number of 6-matchings in graph G is:

$$p(G,6)=\binom{m}{6}-\sum_{\{i,j,k\}\subset V}\binom{d_i}{2}\binom{d_j}{2}\binom{d_k}{2}-\sum_i\binom{d_i}{2}p(G-i,4)-\sum_i\binom{d_i}{3}p(G-i,3)$$
$$-\sum_i\binom{d_i}{4}\binom{m-d_i}{2}-\sum_i\binom{d_i}{5}(m-d_i)-\sum_i\binom{d_i}{6}-\sum_{\{i,j\}\subset V}\binom{d_i}{3}\binom{d_j}{3}$$
$$-\sum_{\{i,j\}\subset V}\left[\binom{d_i}{3}\binom{d_j}{2}+\binom{d_i}{2}\binom{d_j}{3}\right]p(G-i-j,1)-\sum_{\{i,j\}\subset V}\binom{d_i}{2}\binom{d_j}{2}p(G-i-j,2)$$
$$+\sum_{ij}(d_i-1)(d_j-1)p(G-i-j,3)+\sum_{ij}\left[\binom{d_i-1}{2}(d_j-1)\right.$$
$$\left.+\binom{d_j-1}{2}(d_i-1)\right]\binom{m-d_i-d_j+2}{2}$$
$$+\sum_k\sum_{ij\in E(G-k)}\binom{d_k+1}{3}(d_i-1)(d_j-1)$$
$$+\sum_k\sum_{ij\in E(G-k)}\binom{d_k}{2}(d_i-1)(d_j-1)p(G-k-i-j,1)$$
$$-\sum_i\sum_{\{k,t\}\in N(i)}(d_k-1)(d_t-1)\binom{m-d_i-d_k-d_t+4}{2}$$
$$+(m-4)\sum_{ij}\binom{d_i-1}{2}\binom{d_j-1}{2}+(m-4)\sum_{ij}\sum_{\substack{k\in N(i)-\{j\}\\ t\in N(j)-\{i\}}}(d_k-1)(d_t-1)$$
$$-\sum_{ij}\sum_{\substack{k\in N(i)-\{j\}\\ t\in N(j)-\{i\}}}\left[\binom{d_k-1}{2}(d_t-1)+\binom{d_t-1}{2}(d_k-1)\right]$$
$$-\frac{1}{2}\sum_{ij}\sum_{kt\in E(G-i-j)}(d_i-1)(d_j-1)(d_k-1)(d_t-1)$$
$$-\sum_i\sum_{\{k,t\}\subset N(i)}\sum_{\substack{r\in N(t)-\{i\}\\ s\in N(k)-\{i\}}}(d_s-1)(d_r-1)$$

$$-\sum_{i}\sum_{j\in N(i)}\sum_{\{k,t\}\subset N(j)-\{i\}}\sum_{\substack{r\in N(t)-\{j\}\\ s\in N(k)-\{j\}}}(d_r+d_s-2)$$

$$-(m-4)\sum_{i}\sum_{\{k,t\}\subset N(i)}\left[\binom{d_k-1}{2}(d_t-1)+\binom{d_t-1}{2}(d_k-1)\right]$$

$$-(m-5)\sum_{i}\sum_{\{k,s,t\}\subset N(i)}[(d_k-1)(d_t-1)+(d_k-1)(d_s-1)$$

$$+(d_t-1)(d_s-1)]+(m-5)\sum_{i}\sum_{\{k,s,t,r\}\subset N(i)}(d_k+d_s+d_t+d_r-4)$$

$$+3\sum_{i}\sum_{\{k,t,\}\subset N(i)}\binom{d_k-1}{2}\binom{d_t-1}{2}+2\sum_{i}\sum_{\{k,t\}\subset N(i)}\left[\binom{d_k-1}{3}(d_t-1)\right.$$

$$\left.+\binom{d_t-1}{3}(d_k-1)\right]+3\sum_{i}\sum_{\{k,s,t\}\subset N(i)}\left[\binom{d_k-1}{2}(d_s-1)\right.$$

$$+\binom{d_s-1}{2}(d_k-1)+\binom{d_k-1}{2}(d_t-1)+\binom{d_t-1}{2}(d_k-1)$$

$$\left.+\binom{d_s-1}{2}(d_t-1)+\binom{d_t-1}{2}(d_s-1)\right]$$

$$-2\sum_{i}\sum_{\{k,s,t\}\subset N(i)}(d_k-1)(d_t-1)(d_s-1)$$

$$+3\sum_{i}\sum_{\{k,s,t,r\}\subset N(i)}[(d_k-1)(d_s-1)+(d_k-1)(d_t-1)$$

$$+(d_k-1)(d_r-1)+(d_s-1)(d_t-1)+(d_s-1)(d_r-1)$$

$$+(d_t-1)(d_r-1)]-4\sum_{i}\sum_{\{k,s,t,r\}\subset N(i)}\left[\binom{d_k-1}{2}+\binom{d_s-1}{2}+\binom{d_t-1}{2}\right.$$

$$\left.+\binom{d_r-1}{2}\right]-4\sum_{i}\sum_{\{k,s,t,r,q\}\subset N(i)}(d_k+d_s+d_t+d_r+d_q-5)+N_H$$

In which N_H is the number of 6-cycles.

Proof: If we show the number of unlabeled trees with n edges by $T(n)$ then [5]:

$$T(1)=1\,,T(2)=1\,,T(3)=2\,,T(4)=3\,,T(5)=6\,,T(6)=11$$

Now if $N(6)$ be the number of non-isomorphic jungles with 6 edges, using the partition of 6 and counting axiom we have:

$$N(6) = T(6) + T(5)T(1) + T(4)T(2) + T(4)T^2(1) + (T^2(3) - 1) + T(3)T(2)T(1) + T(3)T^3(1) + T^3(2) + T^2(2)T^2(1) + T(2)T^4(1) + T^6(1) = 34$$

Hence the number of non-isomorphic graphs with 6 edges that do not form a 6-matching by considering a 6-cycle is 34.

According to above these graphs are isomorphic with one of the graphs shown in Figure 1.

To obtain $p(G, 6)$, we subtract the number of subgraphs that do not form a 6-matching of the number of subsets of edges that have 6 edges i.e. , $\binom{m}{6}$. These subgraphs are isomorphic to one of the shown graphs in Figure 1.

Suppose $N_H, N_A, N_B, N_C, N_D, N_E, N_F, N_G, N_I, N_J, N_K, N_L, N_M, N_N, N_O, N_p, N_q, N_R, N_S, N_T, N_U, N_V, N_W, N_X, N_Y, N_Z, N_\varphi, N_\Theta, N_\Gamma, N_\mu, N_\Psi, N_\eta, N_\Lambda, N_\Delta$ in order be the number of subgraphs of G that are isomorphic to:

H, A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, q, R, S, T, U, V, W, X, Y, Z, φ, θ, Γ, μ, Ψ, η, Λ, Δ.

Now we calculate them all.

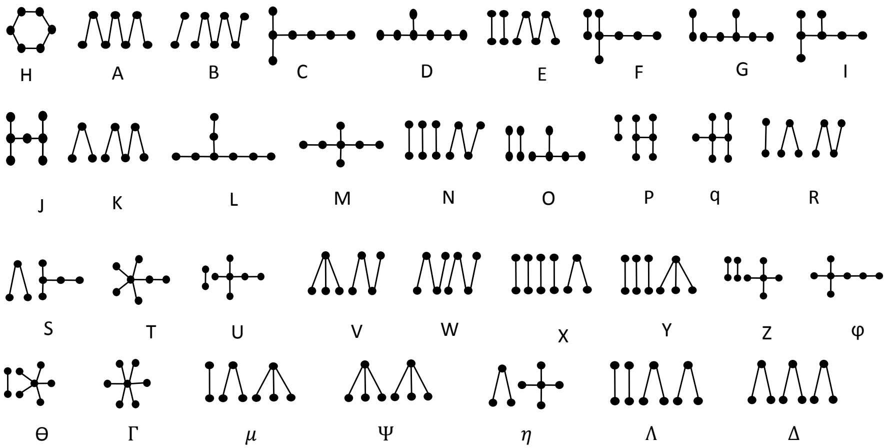

Figure 1

N_Γ : For counting the number of graphs that are isomorphic to Γ, we choose the vertex i and six edges adjacent to it. So we have:

$$N_\Gamma = \sum_{i=1}^{n} \binom{d_i}{6}$$

N_T : For counting the number of graphs that are isomorphic with T, first we choose a vertex like i of $V(G)$ and subset of $N(i)$ like $\{k, s, t, r, q\}$, then choose an edge adjacent to k, s, t, r, or q other than edges connecting k, s, t, r and q to i. Therefore:

$$N_T = \sum_i \sum_{\{k,s,t,r,q\} \subset N(i)} \left(d_q + d_r + d_s + d_t + d_k - 5\right)$$

N_M : For counting N_M, choose a vertex like i from $V(G)$ and a subset $\{k, s, t, r\}$ from $N(i)$. Now select two vertices from k, s, t, r with an edge connecting to any of them except the adjacent edges to i. So we have:

$$N_M = \sum_i \sum_{\{k,s,t,r\} \subset N(i)} [(d_k - 1)(d_s - 1) + (d_k - 1)(d_t - 1) + (d_k - 1)(d_r - 1) + (d_t - 1)(d_s - 1) + (d_t - 1)(d_r - 1) + (d_s - 1)(d_r - 1)]$$

N_J : For counting N_J , first we choose an vertex like i from $V(G)$ with a subset $\{k, t\}$ from $N(i)$, then two edges that are adjacent to k and two edges that are adjacent to t except the edges that connect k and t to i. Therefore:

$$N_J = \sum_i \sum_{\{k,t\} \subset N(i)} \binom{d_k - 1}{2} \binom{d_t - 1}{2}$$

N_q : For counting N_q , first we choose a vertex like i from $V(G)$ with a subset $\{k, s, t, r\}$ from $N(i)$, then select two edges that are adjacent to the vertices $k, s, t,$ or r except the edges that connect k, s, t or r to i. So:

$$N_q = \sum_i \sum_{\{k,s,t,r\} \subset N(i)} \left[\binom{d_k - 1}{2} + \binom{d_s - 1}{2} + \binom{d_t - 1}{2} + \binom{d_r - 1}{2}\right]$$

N_L : For counting N_L , first choose a vertex like i from $V(G)$ with a subset $\{k, s, t\}$ from $N(i)$, then select an edge adjacent to vertex k, an edge adjacent to vertex s and an edge adjacent to vertex t except the edges that connect k, s and t to i. Therefore:

$$N_L = \sum_i \sum_{\{k,s,t,\} \subset N(i)} (d_k - 1)(d_t - 1)(d_s - 1)$$

N_I : For counting N_I , first we select a vertex like i from $V(G)$ with a subset $\{k, s, t\}$ from $N(i)$, then two vertices from $\{k, s, t\}$ with two edges connected to one and an edge adjacent to another except the edges that are adjacent to i . Therefore:

$$N_I = \sum_i \sum_{\{k,s,t,\}\subset N(i)} \left[\binom{d_k - 1}{2}(d_s - 1) + \binom{d_s - 1}{2}(d_k - 1) + \binom{d_k - 1}{2}(d_t - 1) + \binom{d_t - 1}{2}(d_k - 1) + \binom{d_s - 1}{2}(d_t - 1) + \binom{d_t - 1}{2}(d_s - 1)\right]$$

N_C : For counting N_C , first we select an edge like ij from $E(G)$ with a vertex like t from $N(j) - \{i\}$ and a vertex like k from $N(i) - \{j\}$ and then select two edges adjacent to k and an edge adjacent to t or conversely, except ki and tj. Therefore:

$$N_C = \sum_{ij} \sum_{\substack{k\in N(i)-\{j\} \\ t\in N(j)-\{i]}} \left[\binom{d_k - 1}{2}(d_t - 1) + \binom{d_t - 1}{2}(d_k - 1)\right]$$

N_φ : For counting N_φ, we choose a vertex like i from $V(G)$ with a subset $\{k,t\}$ from i. Now select three edges adjacent to vertex k and an edge adjacent to vertex t or conversely, except the edges that are adjacent to vertex i. Then:

$$N_\varphi = \sum_i \sum_{\{k,t\}\subset N(i)} \left[\binom{d_k - 1}{3}(d_t - 1) + \binom{d_t - 1}{3}(d_k - 1)\right]$$

N_D : For counting N_D , we do the following steps in order:

1. Choose a vertex like i from $V(G)$
2. Choose a vertex like j from $N(i)$
3. Choose a subset like $\{k,t\}$ from $N(j) - \{i\}$
4. Choose a vertex like s from $N(k) - \{j\}$ and a vertex like r from $N(t) - \{j\}$
5. Choose an edge adjacent to r or s except the adjacent edges to k and t. Then we have:

$$N_D = \sum_i \sum_{j\in N(i)} \sum_{\{k,t\}\subset N(j)-\{i\}} \sum_{\substack{s\in N(k)-\{j\} \\ r\in N(t)-\{j\}}} (d_r + d_s - 2)$$

N_A : For counting N_A, first we select a vertex like i from $V(G)$ with a subset like $\{k,t\}$ from $N(i)$ then a vertex like r from $N(t) - \{i\}$ and a vertex like s from $N(k) - \{i\}$ with an edge adjacent to vertex r and an edge adjacent to vertex s except rt and sk. But it is possible

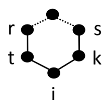

Figure 2

that the edge adjacent to r and the edge adjacent to s be adjacent together (Figure 2) that we will have a subgraph isomorphic to H and the number of such subgraphs must be subtracted.

According to Figure2, N_H is counted six times. Therefore:

$$N_A = \sum_{i} \sum_{\{k,t\}\subset N(i)} \sum_{\substack{r\in N(t)-\{i\} \\ s\in N(k)-\{i\}}} (d_s - 1)(d_r - 1) - 6N_H$$

N_F : For counting N_F, first we choose a vertex like i from $V(G)$ with a subset like $\{k,t\}$ from $N(i)$ then two edges adjacent to k and an edge adjacent to t or conversely , except the edges ki and ti. So far a graph with five edges is obtained (see Figure 3). If we show such graph with E_5 then the number of subgraphs of G that are isomorphic to E_5 is:

$$\sum_{i} \sum_{\{k,t\}\subset N(i)} \left[\binom{d_k - 1}{2}(d_t - 1) + \binom{d_t - 1}{2}(d_k - 1)\right]$$

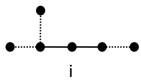

Figure 3

Finally choose an edge from graph G that does not belong to E_5. But this single edge must not be adjacent to any edge of E_5. Therefore subtract the number of graphs that this single edge is adjacent to E_5. This is shown in Figure 4:

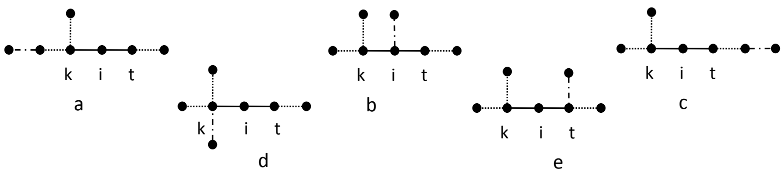

Figure 4

Now according to Figure $4(a,b,c), N_D, N_I$ and N_C are counted once and according to Figure $4(d), N_\varphi$ is counted three times and according to Figure $4(e), N_J$ is counted four times. Therefore:

$$N_F = (m-5)\sum_{i} \sum_{\{k,t\}\subset N(i)} \left[\binom{d_k - 1}{2}(d_t - 1) + \binom{d_t - 1}{2}(d_k - 1)\right] - 4N_J - 3N_\varphi - N_D - N_I - N_C$$

N_B : For counting N_B , first we select an edge like ij from $E(G)$ with a vertex like k from $N(i)-\{j\}$ and a vertex like t from $N(j)-\{i\}$ then choose an edge that is adjacent to vertex k and an edge that is adjacent to vertex t except the edges ki and tj. So we have a path of length six (P_6). Therefore, the number of subgraphs of G that are isomorphic to P_6 is:

$$\sum_{ij}\sum_{\substack{k\in N(i)-\{j\}\\ t\in N(j)-\{i\}}}(d_k-1)(d_t-1)$$

After choosing P_6 , we select an edge of graph G that does not belong to P_6. But this single edge must not be connected to any of edges of P_6. So we subtract the number of graphs that this single edge is adjacent to P_6 (see Figure 5).

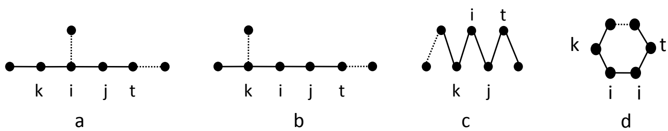

Figure 5

Now according to Figure 5(a), N_D once and according to Figure $5(b,c), N_C$ and N_A twice and according to Figure $5(d), N_H$ six times are counted. Therefore:

$$N_B=(m-5)\sum_{ij}\sum_{\substack{k\in N(i)-\{j\}\\ t\in N(j)-\{i\}}}(d_k-1)(d_t-1)-N_D-2N_A-2N_C-6N_H$$

N_G : For counting N_G , first we choose a vertex i from $V(G)$ with a subset from $N(i)$ like $\{k,s,t\}$ and two vertices of k,s or t with an edge adjacent to any of them except the edges that are adjacent to i (see Figure 6). We denote such graph with E_5' then the number of subgraphs of G that are isomorphic to E_5' is:

$$\sum_{i}\sum_{\{k,s,t\}\subset N(i)}[(d_k-1)(d_t-1)+(d_k-1)(d_s-1)+(d_t-1)(d_s-1)]$$

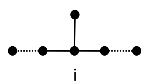

Figure 6

In continue we select an edge from G that does not belong to E_5' but this single edge must not be adjacent to any edges of E_5'. Therefore subtract number of graphs that this single edge is adjacent to E_5' (see Figure 7).

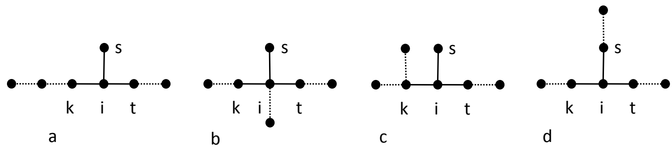

Figure 7

Now according to Figure $7(a), N_D$ once, according to Figure $7(b,c), N_M$ and N_I twice and according to Figure $7(d), N_L$ three times are counted. So N_G is:

$$N_G = (m-5)\sum_i \sum_{\{k,s,t\}\subset N(i)} [(d_k-1)(d_t-1)+(d_k-1)(d_s-1)+(d_t-1)(d_s-1)] - N_D - 2N_M - 2N_I - 3N_L$$

N_P : For counting N_P, first we select an edge like ij from $E(G)$ and then two edges that are adjacent to vertex i and two edges that are adjacent to vertex j except the edge ij. So far a graph with five edges is obtained (see Figure 8).

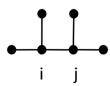

Figure 8

If we denote such graph with E_5'' then the number of subgraphs of G that are isomorphic to E_5'' is equal to:

$$\sum_{ij} \binom{d_i - 1}{2}\binom{d_j - 1}{2}$$

Finally we choose an edge from graph G that does not belong to E_5'' that is not adjacent to any of edges of E_5''. So subtract the number of graphs that this single edge is adjacent to E_5'' (see Figure 9).

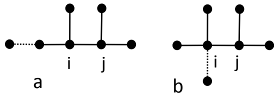

Figure 9

Now according to Figure $9(a), N_I$ once and according to Figure $9(b), N_q$ three times are counted. Therefore:

$$N_p = (m-5)\sum_{ij}\binom{d_i-1}{2}\binom{d_j-1}{2} - N_I - 3N_q$$

N_U : For countingN_U, first we select a vertex like i from $V(G)$ with a subset $\{k,s,t,r\}$ from $N(i)$ and then an edge that is adjacent to k , s, t or r except the edges that connect k, s, t and r to i. So far a graph with five edges is obtained (see Figure 10).

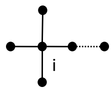

Figure 10

We donate such graph with E_5''', the number of subgraphs of G that are isomorphic to E_5''' is:

$$\sum_i \sum_{\{k,s,t,r\}\subset N(i)} (d_k + d_s + d_t + d_r - 4)$$

Finally we select an edge from graph G that does not belong to E_5''' that is adjacent to any edges of E_5'''. Therefore subtract the number of graphs that this single edge is adjacent to E_5''' (see Figure 11).

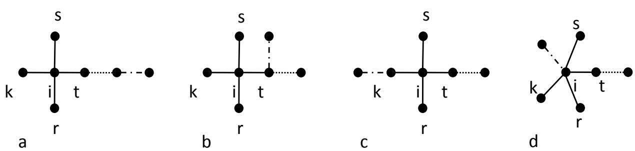

Figure 11

According to Figure $11(a), N_\varphi$ once, according to Figure $11(b,c), N_q$ and N_M twice and according to Figure $11(d), N_T$ four times are counted. Therefore:

$$N_U = (m-5)\sum_i \sum_{\{k,s,t,r\}\subset N(i)} (d_k + d_s + d_t + d_r - 4) - N_\varphi - 2N_q - 2N_M - 4N_T$$

N_O : For counting N_O, first we choose an edge like ij from $E(G)$ and two edges that are adjacent to vertex i and an edge that is adjacent to vertex j or conversely, except the edge ij and at the end, select a 2-matching from graph $G-i-j$. Now subtract the number of the cases in which the edges of 2-matching that are adjacent to these three edges that are adjacent to i and j (see Figure 12).

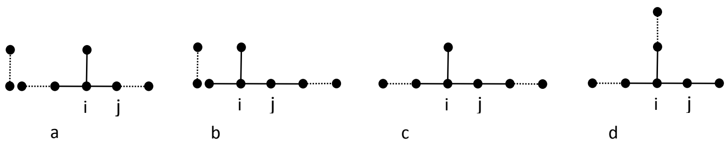

Figure 12

According to Figure 12(a), N_G twice, according to Figure 12(b, c), N_F and N_D once and according to Figure 12(d), N_L three times are counted. Therefore:

$$N_O = \sum_{ij} \left[\binom{d_i - 1}{2}(d_j - 1) + \binom{d_j - 1}{2}(d_i - 1)\right] P(G-i-j, 2) - N_D - N_F - 2N_G - 3N_L$$

N_θ : For counting N_θ, we choose an edge like i and five edges that are adjacent to it and then select another edge that is not adjacent to i. This single edge is not adjacent to any edge that is adjacent to i. Therefore subtract the number of graphs that single edge is only adjacent to the edges that are adjacent to i (see Figure 13).

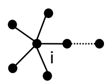

Figure 13

So:

$$N_\theta = \sum_i \binom{d_i}{5}(m - d_i) - N_T$$

N_E : For counting N_E, first choose a vertex like i from $V(G)$ with a subset $\{k,t\}$ from $N(i)$ and then an edge that is adjacent to vertex k and an edge that is adjacent to vertex t except the edges ki and ti and finally select a 2-matching from graph $G-i-k-t$. Subtract the cases that the edges of 2-matching that are adjacent to these two edges are adjacent to vertices k and t (see Figure 14).

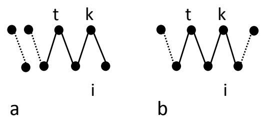

Figure 14

According to Figure $14(a), N_B$ twice and according to Figure $14(b), N_A$ once are counted. Therefore:

$$N_E = \sum_i \sum_{\{k,t\}\subset N(i)} (d_k - 1)\,(d_t - 1)p(G - i - k - t, 2) - N_A - 2N_B$$

N_K: for counting N_K, choose a vertex like i from $V(G)$ with a subset $\{k, t\}$ from $N(i)$ and then an edge that is adjacent to vertex k and an edge that is adjacent to vertex t except the edges ki and ti. So a path with length four (p_5) is obtained. At the end select two edges adjacent to each other except the edges that pass the vertices i, k and t. Now subtract the number of graphs in which these two adjacent edges are possibly connected to edges of p_5.

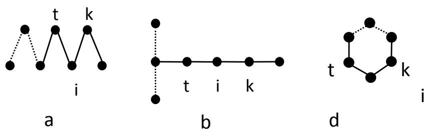

Figure 15

But according to Figure $15(a), N_A$ twice, according to Figure $15(b), N_C$ once and according to Figure $15(c), N_H$ six times are counted. So:

$$N_K = \sum_i \sum_{\{k,t\}\subset N(i)} (d_k - 1)(d_t - 1)\left[\binom{m - d_i - d_k - d_t + 2}{2} - P(G - i - k - t, 2)\right]$$
$$- 2N_A - N_C - 6N_H$$

N_N : For counting N_N, we choose an edge like ij from $E(G)$ with two edges adjacent to it , then a 3-matching from graph $G - i - j$. Now subtract the cases in which the edges of 3-matching may be connected to edges adjacent to ij (see Figure 16).

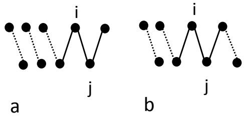

Figure 16

In mentioned process, N_E twice (Figure 16-a) and N_B once (Figure 16-b) are counted. Therefore:

$$N_N = \sum_{ij} (d_i - 1)(d_j - 1) P(G - i - j, 3) - 2N_E - N_B$$

N_X : For counting N_X, first we choose a vertex like i from $V(G)$ with two edges adjacent to it and then select a 4-matching of graph $G - i$. Now subtract the number of cases in the edges of 4-matching are connected to two edges adjacent to i (see Figure 17).

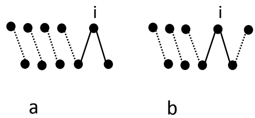

Figure 17

According to Figure 17(a), N_N twice and according to Figure 17(b), N_E are counted once. Therefore N_X is:

$$N_X = \sum_{i} \binom{d_i}{2} P(G - i, 4) - 2N_N - N_E$$

N_S : For counting N_S, first we select an edge like ij from $E(G)$ and the two edges adjacent to vertex i and edge adjacent to vertex j or conversely, except the edges passing vertices i and at the end select two edges adjacent to each other except the edges that pass the vertices i and k. Now subtract the number of the graphs in which these adjacent edges may be adjacent to the edges that are adjacent to vertices i and j (see Figure 18).

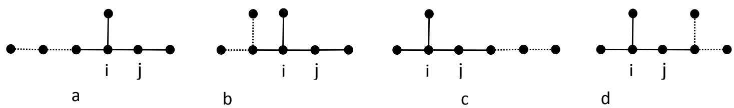

Figure 18

But in the above calculations, N_D, N_I and N_C (Figure 18-a-b-c) once and N_J (Figure 18-d) twice are counted. Therefore:

$$N_S = \sum_{ij} \left[\binom{d_i - 1}{2} (d_j - 1) + \binom{d_j - 1}{2} (d_i - 1) \right] \left[\binom{m - d_i - d_j + 1}{2} - P(G - i - j, 2) \right] - N_D - N_I - N_C - 2N_J$$

N_Y : For counting N_Y, first we choose a vertex like i form $V(G)$ with three edges adjacent to it and then select a 3-matching from graph $G - i$. Now subtract the number of cases in which the edges of 3-matching are connected to three edges adjacent to vertex i (see Figure 19).

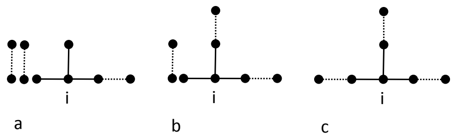

Figure 19

According to Figure 19(a, b, c), N_O, N_G and N_L are counted once each. So:

$$N_Y = \sum_i \binom{d_i}{3} P(G - i, 3) - N_O - N_G - N_L$$

N_Z : For counting N_Z, first we choose a vertex like i from $V(G)$ with four edges adjacent to it then select a 2-matching of graph $G - i$ and subtract the cases that may be in them the edges of 2-matching are connected to four edges adjacent to vertex i (see Figure 20).

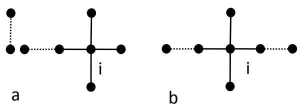

Figure 20

According to Figure 20(a-b), N_U and N_M are counted once. Therefore:

$$N_Z = \sum_i \binom{d_i}{4} P(G - i, 2) - N_U - N_M$$

N_η : For counting N_η, first we choose a vertex like i from $V(G)$ with four edges adjacent to it then select two adjacent edges that are not adjacent to vertex i. Now subtract the number of graphs in which these two adjacent edges may be connected to four edges adjacent to vertex i (see Figure 21).

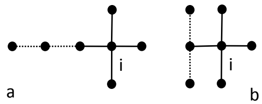

Figure 21

According to Figure 21(a-b), $N_{\varphi}$and N_q are counted once each. So:

$$N_{\eta} = \sum_i \binom{d_i}{4}\left[\binom{m-d_i}{2} - P(G-i,2)\right] - N_{\varphi} - N_q$$

N_{Ψ} : For counting N_{Ψ}, first we choose a subset $\{i,j\}$ from $V(G)$ and then select three edges adjacent to vertices i and j. But subtract the number of graphs in which the passing edges from vertices i and j be connected together or i and j have common edge or there be an edge from i to j or conversely (see Figure 22).

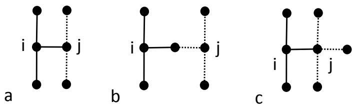

Figure 22

In above process, the number of graphs isomorphic with Figure 22(a) is counted once and on the other side number of such graphs is $\sum_{ij}\binom{d_i-1}{2}\binom{d_j-1}{2}$. Also according to Figure 22 (b-c), N_J and $N_q$are counted once each. So:

$$N_{\Psi} = \sum_{\{i,j\}\subset V}\binom{d_i}{3}\binom{d_j}{3} - \sum_{ij}\binom{d_i-1}{2}\binom{d_j-1}{2} - N_J - N_q$$

N_W : For counting N_W, first we choose an edge like ij from $E(G)$ with two edges adjacent to it and then select an edge like kt from $\mathrm{E}(G-i-j)$ with to edges adjacent to it. Now subtract the number of graphs in which two edges adjacent to ij and kt may be connected or ij and kt have a common adjacent edge (see Figure 23).

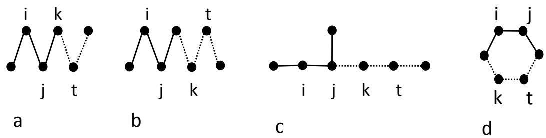

Figure 23

In above calculations, the number of graphs isomorphic with Figure 23(a) is counted once and on the other side the number of these graphs is:

$$\sum_{jk} \sum_{\substack{i \in N(j)-\{k\} \\ t \in N(k)-\{j\}}} (d_i - 1)(d_t - 1)$$

Also according to Figure 23(b-c), N_A and N_D once and according to Figure 23(d), N_H three times are counted. Therefore:

$$N_W = \frac{1}{2} \sum_{ij} \sum_{kt \in E(G-i-j)} (d_i - 1)(d_j - 1)(d_k - 1)(d_t - 1) - \sum_{jk} \sum_{\substack{i \in N(j)-\{k] \\ t \in N(k)-\{j\}}} (d_i - 1)(d_t - 1) - N_A - N_D - 3N_H$$

N_V : For counting N_V, first we choose a vertex like k from $V(G)$ with three edges adjacent to it then select an edge like ij from E$(G - k)$ with two edges adjacent to it. But subtract the number of graphs in which two edges adjacent to ij are adjacent to three adjacent edge to k or ij and k have a common adjacent edge (see Figure 24).

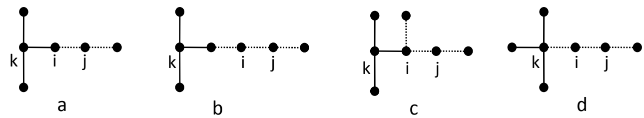

Figure 24

In above calculation the number of graphs isomorphic with Figure 24(a) that is counted once is:

$$\sum_{i} \sum_{\{j,k\} \subset N(i)} \left[\binom{d_j - 1}{2} (d_k - 1) + \binom{d_k - 1}{2} (d_j - 1) \right]$$

Also according to Figure 24(b-c-d), N_C, N_I and N_φ are counted once each. So:

$$N_V = \sum_k \sum_{ij \in E(G-k)} (d_i - 1)(d_j - 1)\binom{d_k}{3}$$
$$- \sum_i \sum_{\{j,k\} \subset N(i)} \left[\binom{d_j - 1}{2}(d_k - 1) + \binom{d_k - 1}{2}(d_j - 1)\right] - N_C - N_I - N_\varphi$$

N_R : For counting N_R, first we choose a vertex like k from $V(G)$ with two edges adjacent to it and then select an edge like ij from $\mathrm{E}(G-k)$ with two edges adjacent to it, then at the end select a 1-matching from graph $G-k-i-j$. Subtract the number of cases in which two edges adjacent to vertex k, two edges adjacent to edge ij or this single edges either are adjacent together or ij and k have a same edge (see Figure 25).

According to Figure 25(a- b-c), the number of graphs isomorphic with (a), (b) or (c) considering the fact that are counted twice is:

$$2\sum_i \sum_{\{k,j\} \subset N(i)} (d_k - 1)(d_j - 1)(m - d_i - d_j - d_k + 2)$$

Also N_B twice (Figure 25-d), N_A twice (Figure 25-e), N_A twice (Figure 25-f), N_D once (Figure 25-g), N_H twelve times (Figure 25-h), N_G twice (Figure 25-i), N_D once (Figure 25-j), N_D once (Figure 25-k), N_L six times (Figure 25-l), N_F once (Figure 25-m), N_C once (Figure 25-n), N_D once (Figure 25-o), N_W four times (Figure 25-p), N_A twice (Figure 25-q) and N_k twice (Figure 25-r) are counted. So:

$$N_R = \sum_k \sum_{ij \in E(G-K)} \binom{d_k}{2}(d_i - 1)(d_j - 1)P(G-k-i-j, 1)$$
$$- 2\sum_i \sum_{\{k,j\} \subset N(i)} (d_k - 1)(d_j - 1)(m - d_i - d_j - d_k + 2) - 2N_B - 6N_A$$
$$- 4N_D - 6N_L - 2N_G - 4N_W - 2N_K - N_C - N_F - 12N_H$$

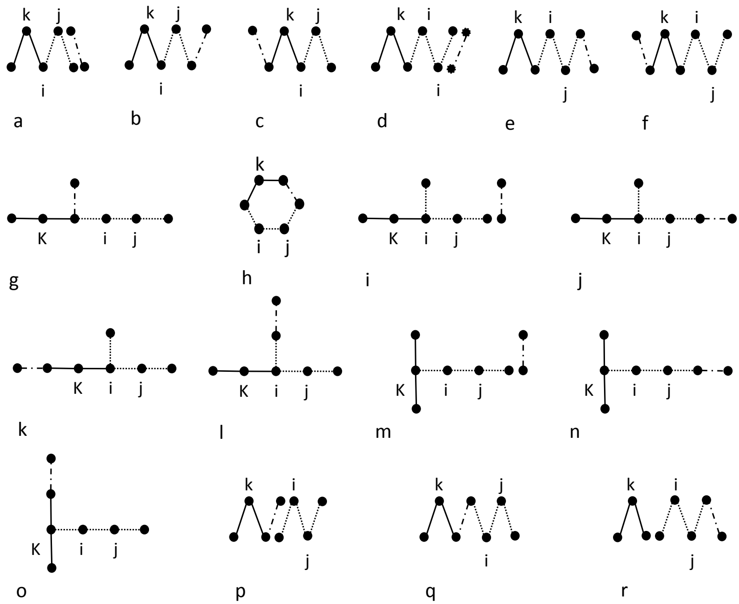

Figure 25

N_μ : For counting N_μ , first we choose a subset $\{i,j\}$ from $V(G)$ with three edges adjacent to i and two edges adjacent to j or conversely, then select an edge from graph G that is not adjacent to vertices i and j. But the number of graphs in which the passing edges form i and j are connected or i and j have same edge or the mentioned single edge is not connected to edges of $i$and j, must be subtracted (see Figure 26).

According to Figure 26(a-b-c) the number of graphs isomorphic to (a) once, isomorphic to (b) once and isomorphic to (c) twice are counted. So the number of graphs isomorphic to (a) plus the number of graphs isomorphic to (b) plus two times the number of graph isomorphic to (c) is:

$$\sum_{ij}\left[\binom{d_i}{2}(d_j-1)+\binom{d_j}{2}(d_i-1)\right](m-d_i-d_j+1)$$

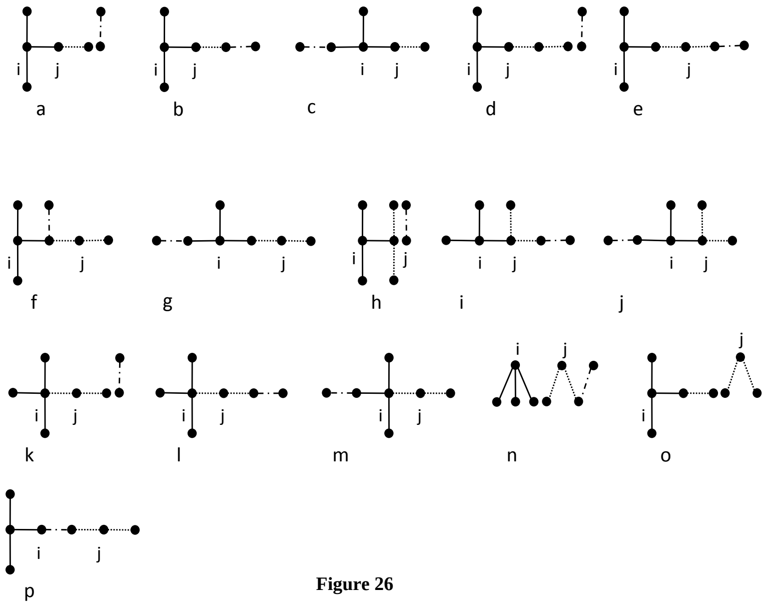

Figure 26

Also N_F once (Figure 26-d), N_C once (Figure 26-e), N_I once (Figure 26-f), N_D once (Figure 26-g), N_p twice (Figure 26-h), N_I once (Figure 26- i), N_I once (Figure 26-j), N_U once (Figure 26-k), N_φ once (Figure 26-l), N_M twice (Figure 26-m), N_V twice (Figure 26-n), N_S once (Figure 26-o) and N_C once (Figure 26-p) are counted. Therefore:

$$N_\mu = \sum_{\{i,j\}\subset V} \left[\binom{d_i}{3}\binom{d_j}{2} + \binom{d_i}{2}\binom{d_j}{3}\right] P(G-i-j,1)$$

$$- \sum_{ij} \left[\binom{d_i-1}{2}(d_j-1) + \binom{d_j-1}{2}(d_i-1)\right](m-d_i-d_j+1) - N_D - N_F$$

$$- N_U - N_\varphi - N_S - 2N_P - 2N_C - 2N_M - 2N_V - 3N_I$$

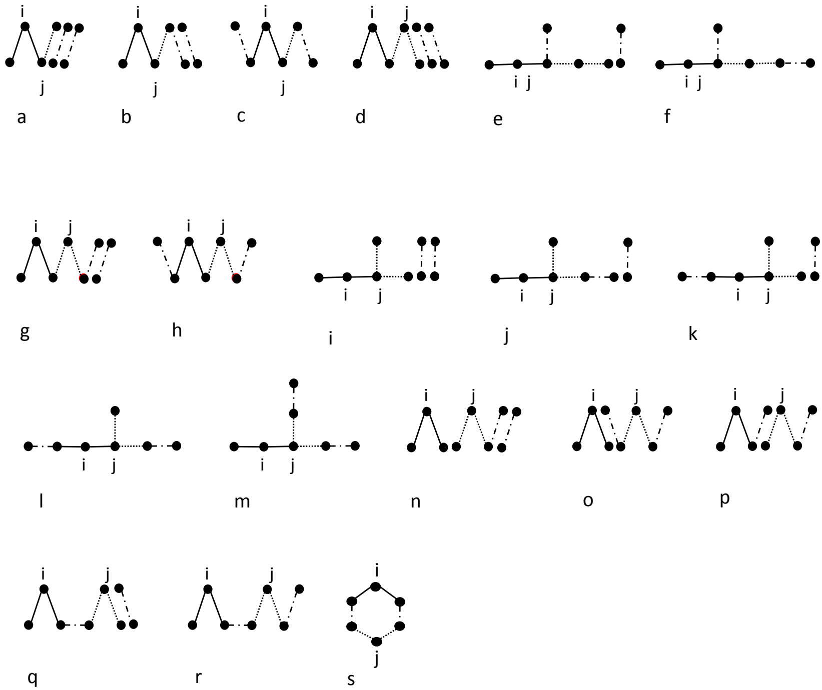

Figure 27

N_Λ : For counting N_Λ , first we choose a subset $\{i, j\}$ form $V(G)$ with two edges adjacent to i and two adjacent to j and then select a 2-matching from graph $G - i - j$. Now the number of graphs in which the passing edges from i and j are adjacent or i and j have no common edge or the edges of 2-matching are connected to edges of $i$and j, must be subtracted (see Figure 27).

In above calculation, the number graphs isomorphic with Figure $27(a)$ once, the number of graphs isomorphic to Figure $27(b)$ twice, and the number of graphs isomorphic to Figure $27(c)$ once, are counted. So the graphs isomorphic to (a), plus the two times of the graphs isomorphic to (b) and plus the graphs isomorphic to (c) is:

$$\sum_{ij}(d_i - 1)(d_j - 1)P(G - i - j, 2)$$

Also N_E once (Figure 27-d), N_G once (Figure 27-e), N_D once (Figure 27-f), N_B twice (Figure 27-g), N_A once (Figure 27-h), N_O once (Figure 27-i), N_G twice (Figure 27-j), N_F once (Figure 27-k), N_D once (Figure 27-l), N_L three times (Figure 27-m), N_R twice (Figure 27-n), N_K once (Figure 27-o), N_W four times (Figure 27-p), N_B once (Figure 27-q), N_A twice (Figure 27-r) and N_H three times (Figure 27-s) are counted. So:

$$N_\Lambda = \sum_{\{i,j\}\subset V} \binom{d_i}{2}\binom{d_j}{2} P(G-i-j,2) - \sum_{ij} (d_i-1)(d_j-1)\, P(G-i-j,2) - N_E - N_F - N_o - N_K - 2N_R - 2N_D - 3N_A - 3N_B - 3N_L - 3N_G - 4N_W - 3N_H$$

N_Δ : In this case we choose a subset $\{i,j,k\}$ from $V(G)$ with two edges adjacent to i, two edges adjacent to k and number of the cases in which the edges adjacent to i,j or k are connected or vertices i,j or k have common edge, must be subtracted (see Figure 28).

In above process, the number of graphs isomorphic to (a), (b) or (c) once and the number of graphs isomorphic to (d) or (e) twice are counted. It is easy to show that the number of graphs isomorphic to (a), (b) or (c) plus two times the number of graphs isomorphic to (d) or (e) is equal to:

$$\sum_i \sum_{jk\in E(G-i)} \binom{d_i}{2}(d_j-1)(d_k-1) - \sum_i \sum_{\{j,k\}\subset N(i)} (d_j-1)(d_k-1)$$

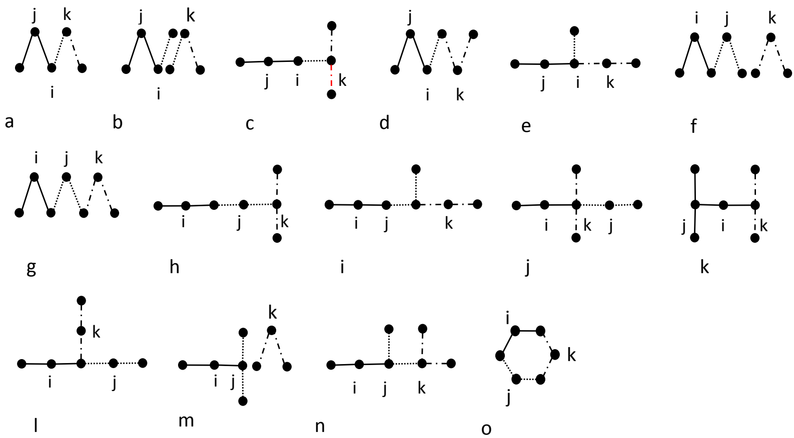

Figure 28

Also N_K once (Figure 28- f), N_A once (Figure 28- g), N_C once (Figure 28- h), N_D once (Figure 28- i),N_M once (Figure 28- j), N_J once (Figure 28- k), N_L once (Figure 28- l), N_S once (Figure 28- m), N_I once (Figure 28- n) and N_H twice (Figure 28- o) are counted. So:

$$N_\Delta = \sum_{\{i,j,k\}\subset V} \binom{d_i}{2}\binom{d_j}{2}\binom{d_k}{2} + \sum_{i}\sum_{\{j,k\}\subset N(i)} (d_j-1)(d_k-1) - \sum_{i}\sum_{jk\in E(G-i)} \binom{d_i}{2}(d_j-1)(d_k-1) - N_K - N_A - N_C - N_D - N_M - N_J - N_L - N_S - N_I - 2N_H$$

From all above and direct substitution in the formula:

$$\mathrm{P}\,(\mathrm{G},6) = \binom{m}{6} - N_A - N_B - N_C - N_D - N_E - N_F - N_G - N_H - N_I - N_J - N_K - N_L - N_M - N_N - N_O - N_P - N_q - N_R - N_S - N_T - N_U - N_V - N_W - N_X - N_Y - N_Z - N_\varphi - N_\theta - N_\Gamma - N_\mu - N_\Psi - N_\eta - N_\Lambda - N_\Delta.$$

This completes the proof.

Corollary 3.2. Let G be a d-regular graph with the girth at least 6 and suppose i is a vertex of $V(G)$, then:

$$P(G,6) = \binom{m}{6} - \binom{n}{3}\binom{d}{2}^3 - n\binom{m-d}{2}\binom{d}{4} - n\binom{d}{5}(m-d) - n\binom{d}{6} - \binom{n}{2}\binom{d}{3}^2 + 2\mathrm{m}(\mathrm{d}-1)\binom{d-1}{2}\binom{m-2d+2}{2} - m(d-1)^3\binom{m-3d+4}{2} + \mathrm{m}(\mathrm{m}-4)\binom{d-1}{2}^2 + \mathrm{m}(\mathrm{m}-4)(\mathrm{d}-1)^4 - 6\mathrm{m}(\mathrm{d}-1)^3\binom{d-1}{2} - \mathrm{n}(\mathrm{d}-1)^4\binom{d}{2} - 2n(\mathrm{d}-1)^3\binom{d}{3} + 4n(\mathrm{d}-1)\binom{d-1}{3}\binom{d}{2} + 18n(\mathrm{d}-1)\binom{d-1}{2}\binom{d}{3} + 18n(\mathrm{d}-1)^2\binom{d}{4} - 16n\binom{d-1}{2}\binom{d}{4} - 20n(d-1)\binom{d}{5} - 2n(m-4)(\mathrm{d}-1)\binom{d-1}{2}\binom{d}{2} - 3n(m-5)(\mathrm{d}-1)^2\binom{d}{3} + 4n(m-5)(\mathrm{d}-1)\binom{d}{4} + 3n\binom{d}{2}\binom{d-1}{2}^2 - n\binom{d}{3}\{\binom{m-d}{3} - [m-d-2][d\binom{d-1}{2} + (n-d-1)\binom{d}{2}] + (m-2d)(d-1)^2 + 2\left[\binom{d-1}{3} + (n-d-1)\binom{d}{3}\right]\} - 2\binom{d}{2}\binom{d}{3}\left[(m-2d)\binom{n}{2} + m\right] - n\binom{d}{2}^3\left[\binom{m-2d}{2} - 2(d-1)\binom{d-1}{2} - \binom{d-2}{2} - (n-2d-1)\binom{d}{2}\right] + n(d-1)^2(m-d)\binom{d+1}{3} - \binom{d}{2}^2\left[\binom{n}{2} - n\binom{d}{2} - m\right]\left[\binom{m-2d}{2}\right.$$

$$-2d\binom{d-1}{2}-(m-2d-2)\binom{d}{2}\Big]+m(d-1)^2\left[\binom{m-2d+1}{2}-2(d-1)\binom{d-1}{2}\right.$$
$$\left.-(n-2d)\binom{d}{2}\right]-m\binom{d}{2}^2\left[\binom{m-2d+1}{2}-2(d-1)\binom{d-1}{2}-(n-2d)\binom{d}{2}\right]$$
$$+(d-1)^2\binom{d}{2}[n(m-d^2)(m-3d+1)+nd(d-1)(m-3d+2)]$$
$$+m(d-1)^2\left\{\binom{m-2d+1}{3}-(m-2d-1)[2(d-1)\binom{d-1}{2}+\binom{d}{2}(n-2d)]\right.$$
$$\left.+\frac{1}{2}(m-2d+1)(d-1)^2+4(d-1)\binom{d-1}{3}+2(n-2d)\binom{d}{3}-2(d-1)^3\right\}$$
$$-n\binom{d}{2}P(G-i,4)+N_H$$

Proof: We have

$$\sum_i\binom{d_i}{2}p(G-i,4)=n\binom{d}{2}p(G-i,4)\qquad,\qquad \big(\text{For an arbitrary } i \text{ of } V(G)\big)$$

$$\sum_{\{i,j\}\subset V}\left[\binom{d_i}{3}\binom{d_j}{2}+\binom{d_i}{2}\binom{d_j}{3}\right]p(G-i-j,1)=2\binom{d}{2}\binom{d}{3}\left[\binom{n}{2}(m-2d)+m\right]$$

$$\sum_{ij}(d_i-1)(d_j-1)p(G-i-j,2)=m(d-1)^2\left[\binom{m-2d+1}{2}-2(d-1)\binom{d-1}{2}-(n-2d)\binom{d}{2}\right]$$

$$\sum_i\binom{d_i}{3}p(G-i,3)=n\binom{d}{3}\left\{\binom{m-d}{3}-(m-d-2)[d\binom{d-1}{2}+(n-d-1)\binom{d}{2}]+2\left[d\binom{d-1}{3}\right.\right.$$
$$\left.\left.+(n-d-1)\binom{d}{3}\right]+(m-2d)(d-1)^2\right\}$$

$$\sum_k\sum_{ij\in E(G-k)}\binom{d_k}{2}(d_i-1)(d_j-1)p(G-k-i-j,1)$$
$$=(d-1)^2\binom{d}{2}[n(m-d^2)(m-3d+1)+nd(d-1)(m-3d+2)]$$

$$\sum_k\sum_{ij\in E(G-k)}\binom{d_k+1}{3}(d_i-1)(d_j-1)=n(d-1)^2(m-d)\binom{d+1}{3}$$

$$\sum_{ij}\sum_{kt\in E(G-i-j)}(d_i-1)(d_j-1)(d_k-1)(d_t-1)=m(m-2d+1)(d-1)^4$$

$$\sum_{\{i,j\}\subset V}\binom{d_i}{2}\binom{d_j}{2}p(G-i-j,2)=\binom{d}{2}^2\left\{n\binom{d}{2}\left[\binom{m-2d}{2}-2(d-1)\binom{d-1}{2}-\binom{d-2}{2}\right.\right.$$
$$\left.-(n-2d-1)\binom{d}{2}\right]+m\left[\binom{m-2d+1}{2}-2(d-1)\binom{d-1}{2}-(n-2d)\binom{d}{2}\right]$$
$$\left.+\left[\binom{n}{2}-n\binom{d}{2}-m\right]\left[\binom{m-2d}{2}-2d\binom{d-1}{2}-(m-2d-2)\binom{d}{2}\right]\right\}$$

$$\sum_{ij}(d_i-1)(d_j-1)p(G-i-j,3)=m(d-1)^2\left\{\binom{m-2d+1}{3}-(m-2d-1)[2(d-1)\binom{d-1}{2}\right.$$
$$+\binom{d}{2}(n-2d)]$$
$$\left.+4(d-1)\binom{d-1}{3}+2(n-2d)\binom{d}{3}+(m-2d+1)(d-1)^2-2(d-1)^3\right\}$$

Now using the theorem 3.1 and direct substitution in above formulas, the result is obtained.

Corollary 3.3. Let G and H be two regular graphs with the girth at least 6 that are co matching. Then the number of 6-cycles in G and H is equal.

Proof: Considering the fact that $p(G,6) = p(H,6)$, $p(G-i,4) = p(H-i,4)$ and using the corollary 3.2 the result is proved.

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