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Chemical Hyperalgebras for Three Consecutive Oxidation States of Elements

Sang-Cho Chung[1], **Kang Moon Chun***[2], **Nak Jung Kim**[1],
Soo Yong Jeong[3], **Hanna Sim**[4], **Jongwook Lee**[1],
Hyunkyung Maeng[1]

[1] *Department of Mathematics Education, Mokwon University, Daejeon 302-729, Korea*
`math888@naver.com, window0711@naver.com, whddnr2ek@naver.com,`
`aodgusrud@naver.com`

[2] *Department of Chemistry, Chungnam National University, Daejeon 305-764, Korea*
`kmchun255@hanmail.net`

[3] *Department of Biomedicinal Chemistry, Mokwon University, Daejeon 302-729, Korea*
`jsteven123321@hanmail.net`

[4] *Department of Mathematics, Chungnam National University, Daejeon 305-764, Korea*
`tlagkssk@hanmail.net`

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Abstract

In this paper, we investigate mathematical structures of chemical reactions for three consecutive oxidation states of elements.

1 Introduction

In 1934 F. Marty introduced that algebraic hyperstructures represent a natural extension of classical algebraic structures[14], and to this day, the study have been studied by many mathematicians[3, 4]. In 2013, B. Davvaz[6] had to summarize the basic algebraic hyperstructures in his book 'Polygroup Theory and Related Systems.'

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*Corresponding author: kmchun255@hanmail.net

We can see the results of research that B. Davvaz and A. D. Nezad studied for n-ary hypergroups and chemical examples in hypergroups[7, 12]. Also B. Davvaz, A. D. Nezad and A. Benvidi especially showed ternary algebraic hyperstructures in chain reactions and chemical hyperalgebra in dismutation reactions[8, 9]. In 2012[9], they had a set of tin(Sn) ions $\{Sn, Sn^{2+}, Sn^{4+}\}$, a set of indium(In) ions $\{In, In^{+}, In^{3+}\}$ and a set of vanadium(V) ions $\{V, V^{2+}, V^{3+}, VO^{2+}, VO_2^{+}\}$ studied algebraic hyperstructures and also K. M. Chun[1] and S. C. Chung[2] did for the same subjects. In 2014, B. Davvaz, A. D. Nezad and M. Mazloum-Ardakani [10, 11] give some algebraic hyperstructures for redox reactions and radiolytic processes in cement medium.

In this paper, we present examples of hypersructures for mathematical operations of standard reduction potentials for three consecutive oxidation states of elements.

2 Algebraic structures

Let H be a non-empty set and a function $\cdot : H \times H \longrightarrow \wp^*(H)$ be a *hyperoperation*, where $\wp^*(H)$ is the set of all non-empty subset of H. The couple $(H, \cdot)$ is called a *hypergroupoid*. For the non-empty subset A, B of H, we define $A \cdot B = \cup_{a\in A, b\in B} a \cdot b$, and for a singleton $\{a\}$ we denote $\{a\} \cdot B = a \cdot B$ and $B \cdot \{a\} = B \cdot a$.

Definition 2.1 *[6] The hypergroupoid $(H, \cdot)$ is called a semihypergroup if*

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z, \ \textit{for all } x, y, z \in H.$$

The hypergroupoid $(H, \cdot)$ is called an H_v-semigroup if

$$x \cdot (y \cdot z) \cap (x \cdot y) \cdot z \neq \emptyset, \ \textit{for all } x, y, z \in H.$$

The hypergroupoid $(H, \cdot)$ is called a quasihypergroup if

$$x \cdot H = H \cdot x = H, \ \textit{for all } x \in H.$$

The hyperoperation $(\cdot)$ is called commutative if

$$x \cdot y = y \cdot x, \ \textit{for all } x, y \in H.$$

The hypergroupoid $(H, \cdot)$ is called a hypergroup if it is a semihypergroup and a quasihypergroup.

The hypergroupoid $(H, \cdot)$ *is called an* H_v*-group if it is an* H_v*-semigroup and a quasi-hypergroup.*

The hypergroupoid $(H, \cdot)$ *is called a commutative hypergroup if it is a hypergroup with a commutative hyperoperation* $(\cdot)$*.*

The hypergroupoid $(H, \cdot)$ *is called a commutative* H_v*-group if it is an* H_v*-group with a commutative hyperoperation* $(\cdot)$*.*

3 Oxidation-reduction reactions

Luder and Zuffanti have been classified primarily overall of chemical reactions into acid-base reaction, oxidation-reduction reaction and free radical reaction. Whereafter Pearson regarded free radical reaction as oxidation-reduction reaction and then all of chemical reactions was a simplification to acid-base reaction and oxidation-reduction reaction.

We know that most elements have more than one positive oxidation state; almost all can be prepared as the free elements with oxidation state 0 but some have negative oxidation states. Reactions in which elements change their oxidation numbers are known as oxidation-reduction or redox reactions. Redox reactions normally require the presence of two reactants: in one reactant, the oxidation number of an element is reduced, and in the other, an oxidation number of an element increases.

Electrochemistry is defined as the study of the interchange of chemical and electrical energy. It is primarily concerned with two processes that involve oxidation-reduction reactions; the generation of an electrical current from a spontaneous chemical reaction and the opposite process, the use of a current to produce chemical change.

Thermodynamically, redox reactions under the standard conditions go if the free energy change($\triangle G^\circ$) is negative. Instead of measuring thermodynamic parameters through the experiments, we measure the electromotive force(EMF; symbolized as E° and measured in volts) generated when the reaction occurs spontaneously in a electrochemical cell. The EMF(E°) can be related to the free energy change($\triangle G^\circ$) by the equation

$$\triangle G^\circ = -nFE^\circ$$

where n is the number of equivalents oxidized or reduced, and F is the the Faraday constant($9.64853 \times 10^4 C/mol$).

Disproportionation is a chemical reaction, typically a redox reaction, where a molecule is transformed into two or more dissimilar products. Disproportionation reactions follow the form:

$$Hg_2Cl_2 \longrightarrow Hg + HgCl_2$$

And because of the oxidation states of the oxygen and the hydrogen don't change in water(H_2O) dissociating into $H_3O^+ + OH^-$, it is an example of a disproportionation reaction that is not a redox reaction. The reverse reaction of disproportionation, when a compound in an intermediate oxidation state is formed is called *comproportionation*:

$$\overset{+2}{\underline{N}O} + \overset{+4}{\underline{N}_2O_4} \longrightarrow \overset{+3}{\underline{N}O_2^-} \text{ (not balanced)}$$

4 Standard reduction potentials for three consecutive oxidation states of elements

Through the Latimer diagrams of all elements, we selected a lot of chemical species that were recorded three consecutive standard reduction potentials in acidic and/or basic solution, e.g.[17].

$$Sn^{4+} \xrightarrow{0.15} Sn^{2+} \xrightarrow{-0.137} Sn$$
$$In^{3+} \xrightarrow{-0.444} In^{+} \xrightarrow{-0.126} In$$

The reverse of disproportionation is called *comproportionation*. It will form a product with an oxidation number intermediate of the two reactants.

For example,

$$Sn + Sn^{4+} \longrightarrow 2Sn^{2+}$$

an element tin(Sn) in the oxidation states 0 and 4+ can comproportionate to the state 2+, and the comproportionation reaction is spontaneous, since EMF$(0.15 + 0.137)$ is positive. But an element indium(In) in oxidation states 0 and 3+ can not comproportionate to the state +1, since EMF$(-0.444 + 0.126)$ is negative. The products from the comproportionation of two each other elements are different. From now, we can know the fact that major products of reaction depend on the stability of thermodynamics in all possible combinations.

Let A_1, A_2 and A_3 be chemical species and let x, y and z be the number of equivalents oxidized or reduced of A_1, A_2 and A_3 respectively. Let $m = x - y$ and $n = y - z$ be the electronic differences and let α, β and γ be potential differences between A_i's such as the following Figure 1:

Figure 1: The standard reduction potentials of three chemical species

$$\begin{tikzcd}
x \arrow[r, dash, dotted, bend left, "m"] & y \arrow[r, dash, dotted, bend left, "n"] & z \\
A_1 \arrow[r, "\alpha"] \arrow[rr, bend right, "\gamma"'] & A_2 \arrow[r, "\beta"] & A_3
\end{tikzcd}$$

Then γ relates to α and β such as following Lemma.

Lemma 4.1 *[13] Let α and β be potential differences in the Figure 1. Then we have a value of γ such that*

$$\gamma = \frac{m\alpha + n\beta}{m+n}$$

Proof. Consider the following diagram:

$$A_1{}^x \longrightarrow A_2{}^y, \quad \Delta G_1{}^\circ = -m\alpha F \tag{1}$$

$$A_2{}^y \longrightarrow A_3{}^z, \quad \Delta G_2{}^\circ = -n\beta F \tag{2}$$

where F is the Faraday constant. Let's find γ such that

$$A_1{}^x \longrightarrow A_3{}^z, \quad \Delta G_3{}^\circ = -(m+n)\gamma F$$

Using (1) and (2), we can find

$$\begin{aligned}
\gamma &= -\frac{\Delta G_3{}^\circ}{(m+n)F} \\
&= -\frac{-m\alpha F - n\beta F}{(m+n)F} \\
&= \frac{m\alpha + n\beta}{m+n}
\end{aligned}$$

Example 4.2 *Gold(Au) forms two of different ions including Au^{3+}, Au^+ and Au. The oxidation states of these species are $+3, +1$ and 0 respectively. The standard reduction potentials of Au^{3+}, Au^+ and Au reactions are:*

$$\begin{tikzcd}
Au^{3+} \arrow[r, "\alpha"] \arrow[rr, bend right, "\gamma"'] & Au^+ \arrow[r, "\beta"] & Au
\end{tikzcd}$$

$$\begin{aligned}
Au^{3+} &\longrightarrow Au^+, & \Delta G_1{}^\circ &= -m\alpha F = -2\alpha F \\
Au^+ &\longrightarrow Au, & \Delta G_2{}^\circ &= -n\beta F = -\beta F,
\end{aligned}$$

where $\alpha = 1.36$ *and* $\beta = 1.83$ *([17]). Using Lemma 4.1, we can find* $\gamma = (2{\cdot}1.36{+}1.83)/3 = 1.52$.

The three consecutive oxidation states of elements can be generalized as the following expression:

Figure 2: The diagram of three consecutive oxidation states

$$\begin{tikzcd} a \arrow[r, "\alpha"] \arrow[r, dotted, no head, bend left=60, "m"] \arrow[rr, bend right, "\gamma"'] & b \arrow[r, "\beta"] \arrow[r, dotted, no head, bend left=60, "n"] & c \end{tikzcd}$$

where a, b and c are chemical species, real numbers α, β and γ are potential differences, and positive integers m and n are electronic differences.

As explained above, the following Table 1 obtained from the thermodynamic results in all possible combinations of oxidation reduction reactions.

Table 1: All possible oxidation-reduction reactions of three species

$+$	a	b	c
a	$a+a(0)$	$b+a(\alpha-\alpha=0)$ $c+a(-\alpha+\gamma)$	$b+a(\alpha-\gamma)$ $b+b(\alpha-\beta)$ $c+a(\gamma-\gamma=0)$ $c+b(\gamma-\beta)$
b		$a+c(-\alpha+\beta)$ $b+b(0)$	$c+a(\beta-\gamma)$ $c+b(\beta-\beta=0)$
c			$c+c(0)$

Definition 4.3 *Let* G *be a set of two or more chemical species and a hyperoperation* $\oplus$ *on* G *is defined as follows;*

$$\oplus : G \times G \to \wp^*(G)$$

where $\wp^*(G)$ *is the set of all non-empty subset of* G*. For all* $x, y \in G$*,* $x \oplus y$ *is defined the major product, that is, it is selected from the largest combination among all spontaneous redox reactions.*

Theorem 4.4 *If G is a set of two chemical species a and b, then $(G, \oplus)$ is a commutative hypergroup.*

Proof. Since $(G, \oplus)$ has the following hyperoperation table,

$\oplus$	a	b
a	a	a, b
b	a, b	b

we can easily show that it is a commutative hypergroup.

In this case, $(G, \oplus)$ is called a *type* ${}_2H$.

Example 4.5 *Let $G = \{a, b, c\}$ be a set of three chemical species and α, β and γ be positive real numbers, and m, n be positive integers.*

Consider the following diagram with potential differences $\alpha, -\beta$ and $-\gamma$, and electronic differences m and n.

$$\begin{tikzcd}
a \arrow[r, "\alpha"'] \arrow[r, dotted, no head, bend left, "m"] \arrow[rr, bend right, "-\gamma"'] & b \arrow[r, "-\beta"'] \arrow[r, dotted, no head, bend left, "n"] & c
\end{tikzcd}$$

We define $b \oplus c$ for $b, c \in G$. Firstly, we think all possible combinations for oxidation-reduction reactions about $b+c$. There are two cases about $b+c$ in all possible combinations for oxidation-reduction reactions as follows:

$$\begin{cases} b + c \xrightarrow[-\beta]{\gamma} c + a & (\gamma + (-\beta)) \\ b + c \xrightarrow[-\beta]{\beta} c + b & (\beta + (-\beta) = 0) \end{cases}$$

Hence $\{a, c\}$ or $\{b, c\}$ candidates for the hyperoperation $b \oplus c$. Since $\gamma + (-\beta) = \dfrac{-m\alpha + n\beta}{m+n} - \beta = \dfrac{-m\alpha + n\beta - m\beta - n\beta}{m+n} = \dfrac{-m(\alpha+\beta)}{m+n} < 0$, we define $b \oplus c = \{b, c\}$ as the major product.

When $G = \{a, b, c\}$, we want to define a hyperoperation $\oplus$ on G. In order to define the hyperoperation, it is sufficient to investigate all possible combinations for redox reactions which are $0, \pm(\alpha - \beta), \pm(\alpha - \gamma), \pm(\gamma - \beta)$ in Table 1. The following Lemma 4.6 gives the hyperoperation table in Theorem 4.7.

Lemma 4.6 *Let a, b and c be chemical species, real numbers α, β and γ be potential differences, and positive integers m and n be electronic differences with Figure 2. Then we have the following.*

(1) If $\alpha > \beta$, *then we have* $\begin{cases} \alpha-\gamma > 0 \\ \gamma-\beta > 0 \\ \alpha-\beta > \max\{\alpha-\gamma, \gamma-\beta\} \end{cases}$

(2) If $\beta > \alpha$, *then we have* $\begin{cases} \alpha-\gamma < 0 \\ \gamma-\beta < 0 \\ \alpha-\beta < \min\{\alpha-\gamma, \gamma-\beta\} \end{cases}$

Proof. (1) If $\alpha > \beta$, then

$$\begin{aligned} \alpha-\gamma &= \alpha - \frac{m\alpha+n\beta}{(m+n)} \\ &= \frac{m\alpha+n\alpha-m\alpha-n\beta}{(m+n)} \\ &= \frac{n(\alpha-\beta)}{m+n} > 0, \end{aligned}$$

$$\begin{aligned} \gamma-\beta &= \frac{m\alpha+n\beta}{(m+n)} - \beta \\ &= \frac{m\alpha+n\beta-m\beta-n\beta}{(m+n)} \\ &= \frac{m(\alpha-\beta)}{m+n} > 0, \end{aligned}$$

$(\alpha-\beta)-(\alpha-\gamma) = -\beta+\gamma = \gamma-\beta > 0$ and $(\alpha-\beta)-(\gamma-\beta) = \alpha-\gamma > 0$.

Then, the proof is completed from the above.

Similarly, we can prove (2).

Theorem 4.7 *With the same assumption in Lemma 4.6, let* $G = \{a,b,c\}$ *and* $\oplus$ *be a hyperoperation on* G. *Then we have the following.*

(1) When $\alpha > \beta$, *we have the following hyperoperation table, called a type* ${}_3SH_v$,

$\oplus$	a	b	c
a	a	a,b	b
b	a,b	b	b,c
c	b	b,c	c

and the hyperstructure $(G,\oplus)$ *with the type* ${}_3SH_v$ *is a commutative* H_v*-semigroup.*

(2) When $\beta > \alpha$, *we have the following hyperoperation table, called a type* ${}_3SH$,

$\oplus$	a	b	c
a	a	a,c	a,c
b	a,c	a,c	a,c
c	a,c	a,c	c

and the hyperstructure $(G, \oplus)$ *with the type* ${}_3SH$ *is a commutative semihypergroup.*

Proof. Using Table 1 and Lemma 4.6, we get the hyperoperation tables.

(1) For all $x, y, z \in G$, we show that $x \oplus (y \oplus z) \cap (x \oplus y) \oplus z \neq \emptyset$.

$$\begin{cases} x \oplus (y \oplus z) \cap (x \oplus y) \oplus z = \{a\}, & \text{if } x = a, y = a, z = a; \\ x \oplus (y \oplus z) \cap (x \oplus y) \oplus z = \{c\}, & \text{if } x = c, y = c, z = c; \\ x \oplus (y \oplus z) \cap (x \oplus y) \oplus z \ni b, & \text{otherwise.} \end{cases}$$

But it is not a semihypergroup; for example, $c \oplus (a \oplus a) = c \oplus a = \{b\}$ and $(c \oplus a) \oplus a = b \oplus a = \{a, b\}$. Hence $c \oplus (a \oplus a) \neq (c \oplus a) \oplus a$.

(2) For all $x, y, z \in G$, we show that $x \oplus (y \oplus z) = (x \oplus y) \oplus z$.

$$x \oplus (y \oplus z) = (x \oplus y) \oplus z = \begin{cases} \{a\}, & \text{if } x = a, y = a, z = a; \\ \{c\}, & \text{if } x = c, y = c, z = c; \\ \{a, c\}, & \text{otherwise.} \end{cases}$$

Next we give a more generalized hyperoperation on a set of two or more chemical species.

Definition 4.8 *Let G be a set of two or more chemical species and a hyperoperation $\oplus'$ on G is defined as follows;*

$$\oplus' : G \times G \to \wp^*(G)$$

where $\wp^*(G)$ is the set of all non-empty subset of G. For all $x, y, x', y' \in G$, consider the following all possible combinations for oxidation-reduction reactions

$$x + y \xrightarrow[\alpha]{\beta} x' + y'$$

where α and β are potential differences. We define $x \oplus' y$ as follows;

$$x \oplus' y = \begin{cases} \cup_{\alpha+\beta>0}\{x', y'\}, & \textit{whenever } x + y \to x' + y'; \\ \{x, y\}, & \textit{otherwise.} \end{cases}$$

Theorem 4.9 *With the same assumption in Lemma 4.6, let $G = \{a, b, c\}$ and $\oplus'$ be a hyperoperation on G. Then we have the following.*

(1) When $\alpha > \beta$, we have the following hyperoperation table, called a type ${}_3H'$,

$\oplus'$	a	b	c
a	a	a,b	a,b,c
b	a,b	b	b,c
c	a,b,c	b,c	c

and the hyperstructure $(G, \oplus')$ *with the type* ${}_3H'$ *is a commutative hypergroup.*

(2) *When* $\beta > \alpha$*, we have the following hyperoperation table, called a type* ${}_3SH'$*,*

$\oplus'$	a	b	c
a	a	a,c	a,c
b	a,c	a,c	a,c
c	a,c	a,c	c

and the hyperstructure $(G, \oplus')$ *with the type* ${}_3SH'$ *is a commutative semihypergroup. In this case, both* $(G, \oplus)$ *and* $(G, \oplus')$ *are the same semihypergroup.*

Proof. Using Table 1 and Lemma 4.6, we get the hyperoperation tables.

(1) For all $x, y, z \in G$, we show that $x \oplus' (y \oplus' z) = (x \oplus' y) \oplus' z$.

$$x \oplus' (y \oplus' z) = (x \oplus' y) \oplus' z = \begin{cases} \{a\}, & \text{if } x = a, y = a, z = a; \\ \{b\}, & \text{if } x = b, y = b, z = b; \\ \{c\}, & \text{if } x = c, y = c, z = c; \\ \{a, b\}, \{b, c\} \text{ or } \{a, b, c\}, & \text{otherwise.} \end{cases}$$

Clearly for all $x \in G$ we have $x \oplus' G = G \oplus' x = G$.

(2) The proof is the same as Theorem 4.7(2).

5 Examples and chemical analysis

From the standard reduction potentials data in the periodic table at [17], we get a hyperstructure by Theorem 4.7. Consider the following the standard reduction potentials data,

$$\begin{tikzcd} a \arrow[r, "x"] & b \arrow[r, "y"] & c \end{tikzcd}$$

where a, b and c are chemical species and x and y are potential differences. We denote $G_{element} = \{a, b, c\}$, and we investigate the hyperstructure $(G_{elemnt}, \oplus)$.

Example 5.1 *Consider the following standard reduction potentials data[17] of Hydrogen(H) in [group 1];*

$$\begin{tikzcd}\text{(acidic solution)}\ H^+ \arrow[r, "0.000"] \arrow[r, dotted, bend left=40, "1"] & H_2 \arrow[r, "-2.25"] \arrow[r, dotted, bend left=40, "1"] & H^-\end{tikzcd}$$

$$\begin{tikzcd}\text{(basic solution)}\ H_2O \arrow[r, "0.828"] \arrow[r, dotted, bend left=40, "1"] & H_2/OH^- \arrow[r, "-2.25"] \arrow[r, dotted, bend left=40, "1"] & H^-\end{tikzcd}$$

Let $G_{H(acidic)} = \{H^+, H_2, H^-\}$ *and* $G_{H(basic)} = \{H_2O, H_2/OH^-, H^-\}$*. Since* $(x, y) = (0.000, -2.25)$ *at acidic solution,* $(G_{H(acidic)}, \oplus)$ *is a 'type* ${}_3SH_v$*' by Theorem 4.7. Also* $(G_{H(basic)}, \oplus)$ *is the 'type* ${}_3SH_v$*'s by Theorem 4.7. We can be summarized as follows:*

Group	*Elements*	$(x,\ y)$	*Type*
1	$G_{H(acidic)} = \{H^+, H_2, H^-\}$	$(0.000, -2.25)$	${}_3SH_v$
1	$G_{H(basic)} = \{H_2O, H_2/OH^-, H^-\}$	$(0.828, -2.25)$	${}_3SH_v$

For the other cases[17], as follows:

Table 2: Chemical hyperstructures of three species

Group	*Elements*	$(x,\ y)$	*Type*
2	$G_{Mg(acidic)} = \{Mg^{2+}, Mg^+, Mg\}$	$(-2.054, -2.657)$	${}_3SH_v$
3	$G_{Ac(acidic)} = \{Ac^{3+}, Ac^{2+}, Ac\}$	$(-4.9, -0.7)$	${}_3SH$
4	$G_{Ce(acidic)} = \{Ce^{4+}, Ce^{3+}, Ce\}$	$(1.76, -2.34)$	${}_3SH_v$
4	$G_{Ce(basic)} = \{CeO_2, Ce(OH)_3, Ce\}$	$(-0.7, -2.78)$	${}_3SH_v$
5	$G_{Nb} = \{Nb_2O_5, Nb^{3+}, Nb\}$	$(-0.1, -1.1)$	${}_3SH_v$
5	$G_{Pr(acidic)} = \{Pr^{4+}, Pr^{3+}, Pr\}$	$(3.2, -2.35)$	${}_3SH_v$
5	$G_{Pr(basic)} = \{PrO_2, Pr(OH)_3, Pr\}$	$(0.8, -2.79)$	${}_3SH_v$
6	$G_{Cr(basic1)} = \{CrO_4^{2-}, Cr(OH)_3, Cr\}$	$(-0.11, -1.33)$	${}_3SH_v$
6	$G_{Cr(basic2)} = \{CrO_4^{2-}, Cr(OH)_4^-, Cr\}$	$(-0.72, -1.33)$	${}_3SH_v$
6	$G_{Mo(basic)} = \{MoO_4^{2-}, MoO_2, Mo\}$	$(-0.78, -0.98)$	${}_3SH_v$
6	$G_{W(basic)} = \{WO_4^{2-}, WO_2, W\}$	$(-1.259, -0.982)$	${}_3SH$
6	$G_{Nd(basic)} = \{NdO_2, Nd(OH)_3, Nd\}$	$(2.5, -2.78)$	${}_3SH_v$
7	$G_{Tc} = \{TcO_4^-, TcO_3, Tc\}$	$(0.70, 0.83)$	${}_3SH$
8	$G_{Fe(acidic1)} = \{Fe^{3+}, Fe^{2+}, Fe\}$	$(0.77, -0.44)$	${}_3SH_v$
8	$G_{Fe(acidic2)} = \{Fe(CN)_6^{3-},\ Fe(CN)_6^{4-}, Fe\}$	$(0.36, -1.16)$	${}_3SH_v$
8	$G_{Os(acidic)} = \{OsO_4, OsO_2, Os\}$	$(1.005, 0.687)$	${}_3SH_v$
8	$G_{Sm(acidic)} = \{Sm^{3+}, Sm^{2+}, Sm\}$	$(-1.55, -2.67)$	${}_3SH_v$
9	$G_{Ir(acidic1)} = \{IrO_2, Ir^{3+}, Ir\}$	$(0.223, 1.156)$	${}_3SH$
9	$G_{Ir(acidic2)} = \{IrCl_6^{2-}, IrCl_6^{3-}, Ir\}$	$(0.867, 0.86)$	${}_3SH_v$
9	$G_{Eu(acidic)} = \{Eu^{3+}, Eu^{2+}, Eu\}$	$(-0.35, -2.80)$	${}_3SH_v$
10	$G_{Cm(basic)} = \{CmO_2, Cm(OH)_3, Cm\}$	$(0.7, -2.5)$	${}_3SH_v$
11	$G_{Cu(acidic)} = \{Cu^{2+}, Cu^+, Cu\}$	$(0.159, 0.520)$	${}_3SH$
11	$G_{Au(acidic1)} = \{Au^{3+}, Au^+, Au\}$	$(1.36, 1.83)$	${}_3SH$
11	$G_{Au(acidic2)} = \{AuCl_4^-, AuCl_2^-, Au\}$	$(0.926, 1.154)$	${}_3SH$
11	$G_{Au(acidic3)} = \{AuBr_4^-, AuBr_2^-, Au\}$	$(0.802, 0.960)$	${}_3SH$
11	$G_{Au(acidic4)} = \{AuI_4^-, AuI_2^-, Au\}$	$(0.55, 0.578)$	${}_3SH$
11	$G_{Tb(acidic)} = \{Tb^{4+}, Tb^{3+}, Tb\}$	$(3.1, -2.31)$	${}_3SH_v$
11	$G_{Tb(basic)} = \{TbO_2, Tb(OH)_3, Tb\}$	$(0.9, -2.82)$	${}_3SH_v$

Group	*Elements*	$(x,\ y)$	*Type*
12	$G_{Hg(acidic)} = \{Hg^{2+}, Hg^{+}, Hg\}$	$(0.911, 0.796)$	${}_3SH_v$
12	$G_{Dy(basic)} = \{DyO_2, Dy(OH)_3, Dy\}$	$(3.5, -2.80)$	${}_3SH_v$
13	$G_{Ga(acidic)} = \{Ga^{3+}, Ga^{2+}, Ga\}$	$(-0.65, -0.45)$	${}_3SH$
13	$G_{In(acidic)} = \{In^{3+}, In^{+}, In\}$	$(-0.444, -0.126)$	${}_3SH$
13	$G_{Tl(acidic)} = \{Tl^{3+}, Tl^{+}, Tl\}$	$(1.25, -0.3363)$	${}_3SH_v$
14	$G_{Fm(acidic)} = \{Fm^{3+}, Fm^{2+}, Fm\}$	$(-1.15, -2.37)$	${}_3SH_v$
14	$G_{Sn(basic)} = \{Sn(OH)_6^{2-},\ SnO_2H^{-}, Sn\}$	$(-0.93, -0.91)$	${}_3SH$
15	$G_{Sb(acidic)} = \{Sb_2O_5, SbO^{+}, Sb\}$	$(0.605, 0.204)$	${}_3SH_v$
15	$G_{Tm(acidic)} = \{Tm^{3+}, Tm^{2+}, Tm\}$	$(-2.2, -2.4)$	${}_3SH_v$
15	$G_{Md(acidic)} = \{Md^{3+}, Md^{2+}, Md\}$	$(-0.15, -2.4)$	${}_3SH_v$
16	$G_{Yb(acidic)} = \{Yb^{3+}, Yb^{2+}, Yb\}$	$(-1.05, -2.8)$	${}_3SH_v$
16	$G_{No(acidic)} = \{No^{3+}, No^{2+}, No\}$	$(1.4, -2.5)$	${}_3SH_v$
18	$G_{Xe(acidic1)} = \{H_4XeO_6, XeO_3, Xe\}$	$(2.42, 2.12)$	${}_3SH_v$
18	$G_{Xe(acidic2)} = \{XeF_2, XeF, Xe\}$	$(0.9, 3.4)$	${}_3SH$
18	$G_{Xe(basic)} = \{[HXeO_6]_3^{-},\ [HXeO_4]^{-}, Xe\}$	$(0.99, 1.24)$	${}_3SH$

Based on the standard reduction potential for three consecutive oxidation states of elements in acidic or basic solutions, we showed that each element have the predominant type in Table 2. For the chromium(Cr) in [Group 6] element, It can be classified as the type ${}_3SH_v$ from mathematical operation through the Latimer diagram in Table 2, and also the result was considered from thermodynamic point of view.

In the case of indium(In), In^{3+} is the most thermodynamically stable and it is the type ${}_3SH$ in the hyperoperation table of Theorem 4.7(2). That is, chromium iron(type ${}_3SH_v$) and indium(type ${}_3SH$) have a different type.

The lanthanide elements have the type ${}_2H$ or ${}_3SH_v$, and here, the type ${}_2H$ is defined as the element that have 0 and only one form of a positive value in oxidation state. For example,

$$Gd^{3+} \xrightarrow{-2.28} Gd.$$

Cerium(Ce) in [group 4] and terbium(Tb) in [group 11] classify the type ${}_3SH_v$ and we can know the fact that Ce^{3+} and Tb^{3+} are present most abundant from the type ${}_3SH_v$, and they are thermodynamic point of view. Because it can be seen that the fact that Ce^{3+} to Ce^{4+} is to be reduced easily, and also 3+ is the most stable in the case of lanthanides[5]. The $4f$ orbital in the electron configuration of cerium 4+ are empty($4f^0$), thus additional stability may be present[5]. From Table 2, terbium(Tb) for the type ${}_3SH_v$ shows that 4+ can exist in a relatively stable. It can be seen due to the electronic configuration, $4f^7$, from the fact that the half-filled rule is applied.

As the type ${}_3SH_v$ for lanthanide elements of another tribe, we can find the results of samarium(Sm) in [group 8] and europium(Eu) in [group 9]. In terms of chemistry, Sm^{3+} and Eu^{3+} are the most stable. But Table 2 in the mathematical operation shows that Sm^{2+} and Eu^{2+} are regarded as the most representative chemical species respectively. Thus we can understand the fact that will possible predict the oxidation state of any element that was going to be the most predominant in the actual chemical reaction that is not visible through the results of the mathematical operation. This is a great significance. Because we only have experienced for the oxidation state of the elements which are in the most stable in the chemistry. This description is related to the Pourbaix diagram of the predominant area of the acidic solution in the f-block element[15].

Also on iron(Fe) in [group 8] that is other transition element. Both of Fe^{3+} and $Fe(CN)_6^{3-}$ are easy to be reduced to 2+, especially complex ion, $Fe(CN)_6^{3-}$ is more stable than free Fe^{3+} due to the standard reduction potential. They are the type ${}_3SH_v$ for mathematical operations, and they all belong to the reaction of 2+ state may be the most common.

In [group 2], magnesium(Mg) has the oxidation state 2+ and 0. But recently the chemists have created stable dimer of magnesium(I) compounds[16]. Our results classified the type ${}_3SH_v$ for magnesium and the actual energy of Mg^+ is thermodynamically unstable, but it can prove to be the most abundant from the mathematical operation.

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