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Describing the Algebraic Hyperstructure of All Elements in Radiolytic Processes in Cement Medium

Bijan Davvaz[a], **Akbar Dehghan Nezhad**[a], **Mohammad Mazloum-Ardakani**[b] and **M. A. Sheikh-Mohsenei**[b]

[a]Department of Mathematics, Yazd University, Yazd, Iran
davvaz@yazd.ac.ir , anezhad@yazd.ac.ir
[b]Department of Chemistry, Yazd University, Yazd, Iran
mazloum@yazd.ac.ir

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Abstract

Hyperstructures are algebraic structures equipped with at least one multi-valued operation, called a hyperoperation. The largest classes of hyperstructures are the ones called H_v-structures. In the present paper, the algebraic hyperstructures (H_v-structures) are described for the elements in radiolytic processes in cement medium. These elements include e^-_{aq}, H_2, OH^-, H, O^-, OH, H_2O, HO_2^-, H_2O_2, O_2^-, O_2^{2-}, HO_2, O_2, O_3^-, O_3 and O^{2-} which were reported by Bouniol and Bjergbakke, within a comprehensive model for describing radiolytic processes in cement medium.

1 Introduction

Radicals (or free radicals) are atoms, molecules, or ions with unpaired electrons on an open shell configuration. These unpaired electrons cause radicals to be highly chemically reactive. If the free electron strikes a water molecule, the ionized molecule will produce a free radical. Free radicals play an important role in combustion, atmospheric chemistry, polymerization, plasma chemistry, biochemistry, and many other chemical processes. They also play a key role in the intermediary metabolism of various biological compounds [16,17]. A free radical reaction is any chemical reaction involving free radicals. For example with reaction of two radicals of H, the H_2 molecule prepares. In this reaction, combination of unpaired electrons of each H, leads to formation of a covalence bond. In another example, with reaction between radicals of H and OH, the water molecule (H_2O) forms. Also, reaction between two radicals of OH causes to formation of H_2O_2 molecule, due to combination of unpaired electrons of each O in OH.

Portland cement (OPC) is the most common type of cement in general use around the world because it is a basic ingredient of concrete, mortar, stucco and most non-specialty grout. Understanding of the basic mechanisms of radiolysis within the cementitious matrices is important. In this field, Bouniol and Bjergbakke in 2008 reported a comprehensive model for describing radiolytic processes in cement medium [1]. They started from a standard configuration: matrix of ordinary portland cement subjected to a gamma irradiation in closed system at a temperature of $25°C$. They studied and described free radical reactions which may take place in cement medium.

In the present paper, we used the elements in cement medium reactions, reported by Bouniol and Bjergbakke [1], for investigation the algebraic hyperstructure. These elements, including radicals or compounds, are e^-_{aq}, H_2, OH^-, H, O^-, OH, H_2O, HO_2^-, H_2O_2, O_2^-, O_2^{2-}, HO_2, O_2, O_3^-, O_3 and O^{2-}. We suggest that any product which can be produced by reactions between these components, is based on the reactions reported in the stated article. However, between some components no reaction takes place or no new species are produced. Therefore, in these cases, the reagents are considered as products.

2 Weak hyperstructures or H_v-structures

The first definition of hyperstructures was announcement by Marty on the 8^{th} Congress of Scandinavian Mathematicians in 1934. The theory of H_v-structures or weak hyperstructures has been introduced by Vougiouklis in 1990 during the fourth AHA congress [21], also see [22]. Vougiouklis defined the notions of H_v-semigroup and H_v-group. Since then, many researchers have worked on this new topic of algebra and developed it [2–5, 7, 13–15, 19, 20, 22–24].

Let $\mathcal{H}$ be a non-empty set and $\mathcal{P}^*(\mathcal{H})$ be the set of all non-empty subsets of $\mathcal{H}$. Let $\circ$ be a *hyperoperation* on $\mathcal{H}$, that is, $\circ$ is a function from $\mathcal{H} \times \mathcal{H}$ into $\mathcal{P}^*(\mathcal{H})$. If

$(a, b) \in \mathcal{H} \times \mathcal{H}$, its image under $\circ$ in $\mathcal{P}^*(\mathcal{H})$ is denoted by $a \circ b$. The hyperoperation is extended to subsets of $\mathcal{H}$ in a natural way, that is, for non-empty subsets $\mathcal{A}, \mathcal{B}$ of $\mathcal{H}$, $\mathcal{A} \circ \mathcal{B} = \cup\{a \circ b \mid a \in \mathcal{A}, b \in \mathcal{B}\}$. The notation $a \circ \mathcal{A}$ is used for $\{a\} \circ \mathcal{A}$ and $\mathcal{A} \circ a$ for $\mathcal{A} \circ \{a\}$. Generally, the singleton $\{a\}$ is identified with its member a. The structure $(\mathcal{H}, \circ)$ is called a *semihypergroup* if $a \circ (b \circ c) = (a \circ b) \circ c$ for all $a, b, c \in \mathcal{H}$, and is called a *hypergroup* if it is a semihypergroup and $a \circ \mathcal{H} = \mathcal{H} \circ a = \mathcal{H}$ for all $a \in \mathcal{H}$.

Definition 2.1. The hyperoperation $\circ : \mathcal{H} \times \mathcal{H} \to \mathcal{P}^*(\mathcal{H})$ is called *weakly associative hyperoperation* if

$$a \circ (b \circ c) \cap (a \circ b) \circ c \neq \emptyset,$$

for all $a, b, c \in \mathcal{H}$.

The hyperoperation $\circ$ is *weakly commutative* if

$$a \circ b \cap b \circ a \neq \emptyset,$$

for all $a, b \in \mathcal{H}$.

Definition 2.2. A *weak semihypergroup* (or H_v-semigroup) is a non-empty set $\mathcal{H}$ equipped with a weakly associative hyperoperation. An H_v-semigroup is called a *weak hypergroup* (or H_v-group) if moreover the reproduction axiom, i.e., $a \circ \mathcal{H} = \mathcal{H} = \mathcal{H} \circ a$ is satisfied for any $a \in \mathcal{H}$.

Definition 2.3. Let $(\mathcal{H}_1, \circ)$, $(\mathcal{H}_2, \star)$ be two H_v-semigroups. A map $\phi : \mathcal{H}_1 \to \mathcal{H}_2$ is called an *H_v-homomorphism* or *weak homomorphism* if

$$\phi(x \circ y) \cap \phi(x) \star \phi(y) \neq \emptyset \quad \text{for all } x, y \in \mathcal{H}_1.$$

ϕ is called an *inclusion homomorphism* if

$$\phi(x \circ y) \subseteq \phi(x) \star \phi(y) \quad \text{for all } x, y \in \mathcal{H}_1.$$

Finally, ϕ is called a *strong homomorphism* if

$$\phi(x \circ y) = \phi(x) \star \phi(y) \quad \text{for all } x, y \in \mathcal{H}_1.$$

If ϕ is onto, one to one and strong homomorphism, then it is called *isomorphism*, if moreover ϕ is defined on the same H_v-semigroup then it is called automorphism.

Algebraic hyperstructure theory has many applications in other disciplines. In [6], inheritance issue based on genetic information is looked at carefully via a new hyperalgebraic approach. Several examples are provided from different biology points of view, and it is shown that the theory of hyperstructures exactly fits the inheritance issue. In [12], the authors used the concept of algebraic hyperstructures in the F_2-genotypes with cross operation. A physical example of hyperstructures associated with the elementary particle physics is presented in [11]. The theory of algebraic hyperstructures allows us to expand the group theory to much more sets of objects. In [11], the authors have shown the leptons set, as an important group of elementary particles, along with a hyperoperation form a hyperstructure. The hyperoperation is the interaction between the leptons considering the conservation rules. This theory provides a new view to the elementary particle physics and helps us to make a new arrangement to the elementary particles. Another applications of algebraic hyperstructures is in chemistry. In [8–10], Davvaz et al. presented examples of algebraic hyperstructures associated with chain reactions and dismutation reactions.

3 Tables of hyperoperations

Consider $A1 = \{{e_{aq}}^-, H_2, OH^-, H, O^-, OH, H_2O, {HO_2}^-\}$ by convention, ${e_{aq}}^- = H_2O^-$ and $A2 = \{H_2O_2, {O_2}^-, {O_2}^{2-}, HO_2, O_2, {O_3}^-, O_3, O^{2-}\}$.

$\oplus$	$A1$	$A2$
$A1$	$A11$	$A12$
$A2$	$A21$	$A22$

The above hyperoperation $\oplus$ obtained from $A11, A12, A21, A22$, i.e.,

A11	e_{aq}^-	H_2	OH^-	H	O^-	OH	H_2O	HO_2^-
e_{aq}^-	H_2 OH^-	e_{aq}^- H_2	e_{aq}^- OH^-	H_2 OH^-	OH^-	H_2O OH^-	e_{aq}^- H_2O	H_2O O^- OH^-
H_2	e_{aq}^- H_2	H_2	H_2 OH^-	H_2 H	OH^- H	H_2O H	H_2 H_2O	H_2 HO_2^-
OH^-	e_{aq}^- OH^-	H_2 OH^-	OH^-	OH^- H	O^- OH^-	OH OH^-	H_2O OH^-	HO_2^- OH^-
H	H_2 OH^-	H_2 H	OH^- H	H_2	OH^-	H_2O	H_2 OH	OH OH^-
O^-	OH^-	OH^- H	OH^- O^-	OH^-	O_2^{2-}	HO_2^-	O^- H_2O	OH^- O_2^-
OH	H_2O OH^-	H_2O H	OH^- OH	H_2O	HO_2^-	H_2O_2	OH H_2O	H_2O O_2
H_2O	e_{aq}^- H_2O	H_2 H_2O	OH^- H_2O	H_2 OH	O^- H_2O	OH H_2O	H_2O	H_2O HO_2^-
HO_2^-	H_2O O^- OH^-	H_2 HO_2^-	OH^- HO_2^-	OH OH^-	OH^- O_2^-	H_2O O_2^-	H_2O HO_2^-	HO_2^-

A12	H_2O_2	$O_2{}^-$	$O_2{}^{2-}$	HO_2	O_2	$O_3{}^-$	O_3	O^{2-}
$e_{aq}{}^-$	H_2 OH^- OH	H_2O $O_2{}^{2-}$	$e_{aq}{}^-$ $O_2{}^{2-}$	H_2O $HO_2{}^-$	H_2O $O_2{}^-$	OH^- O_2	H_2O $O_3{}^-$	$e_{aq}{}^-$ O^{2-}
H_2	H_2O_2 H_2	H_2 $O_2{}^-$	H_2 $O_2{}^{2-}$	H_2 HO_2	H_2 O_2	H_2 $O_3{}^-$	H_2 O_3	H_2 O^{2-}
OH^-	H_2O_2 OH^-	$O_2{}^-$ OH^-	OH^- $O_2{}^{2-}$	OH^- HO_2	O_2 OH^-	$O_3{}^-$ OH^-	O_2 $HO_2{}^-$	O^{2-} OH^-
H	H_2O OH	$HO_2{}^-$	$O_2{}^{2-}$ H	H_2O_2	HO_2	H $O_3{}^-$	O_2 OH	H O^{2-}
O^-	$O_2{}^-$ H_2O	O_2 O^{2-}	O^- $O_2{}^{2-}$	O^- HO_2	$O_3{}^-$	$O_2{}^-$	$O_2{}^-$ O_2	O^- O^{2-}
OH	H_2O HO_2	OH^- O_2	OH $O_2{}^{2-}$	H_2O O_2	OH O_2	OH O_3	HO_2 O_2	OH O^{2-}
H_2O	H_2O H_2O_2	O_2 H_2O	$O_2{}^{2-}$ H_2O	H_2O HO_2	H_2O O_2	H_2O $O_3{}^-$	H_2O O_3	H_2O O^{2-}
$HO_2{}^-$	H_2O O_2 OH^-	O_2 OH^- O^-	$O_2{}^{2-}$ $HO_2{}^-$	$HO_2{}^-$ HO_2	$HO_2{}^-$ O_2	O_2 OH^- $O_2{}^-$	$O_2{}^-$ O_2 OH	$HO_2{}^-$ O^{2-}

A21	e_{aq}^-	H_2	OH^-	H	O^-	OH	H_2O	HO_2^-
H_2O_2	H_2O OH OH^-	H_2O_2 H_2	H_2O_2 OH^-	H_2O OH	O_2^- H_2O	H_2O HO_2	H_2O_2 H_2O	H_2O O_2 OH^-
O_2^-	O_2^{2-} H_2O	H_2 O_2^-	O_2^- OH^-	HO_2^-	O^{2-} O_2	OH^- O_2	H_2O O_2^-	O_2 OH^- O^-
O_2^{2-}	e_{aq}^- O_2^{2-}	H_2 O_2^{2-}	OH^- O_2^{2-}	O_2^{2-} H	O^- O_2^{2-}	OH O_2^{2-}	H_2O O_2^{2-}	HO_2^- O_2^{2-}
HO_2	H_2O HO_2^-	H_2 HO_2	OH^- HO_2	H_2O_2	O^- HO_2	H_2O O_2	H_2O HO_2	HO_2 HO_2^-
O_2	H_2O O_2	H_2 O_2	OH^- O_2	HO_2	O_3^-	OH O_2	O_2 H_2O	HO_2^- O_2
O_3^-	OH^- O_2	H_2 O_3^-	OH^- O_3^-	H O_3^-	O_2^-	OH^- O_3	O_3^- H_2O	O_2^- OH^- O_2
O_3	O_3 H_2O	H_2 O_3	O_2 HO_2^-	O_2 OH	O_2^- O_2	HO_2 O_2	H_2O O_2	O_2 OH O_2^-
O^{2-}	e_{aq}^- O^{2-}	H_2 O^{2-}	OH^- O^{2-}	H O^{2-}	O^- O^{2-}	OH O^{2-}	H_2O O^{2-}	HO_2^- O^{2-}

A22	H_2O_2	$O_2{}^-$	$O_2{}^{2-}$	HO_2	O_2	$O_3{}^-$	O_3	O^{2-}
H_2O_2	H_2O_2	O_2 OH^- OH	H_2O_2 $O_2{}^{2-}$	O_2 OH H_2O	H_2O_2 O_2	$O_2{}^-$ H_2O O_2	OH HO_2 O_2	H_2O_2 O^{2-}
$O_2{}^-$	O_2 OH^- OH	O_2 $O_2{}^{2-}$	$O_2{}^-$ $O_2{}^{2-}$	O_2 HO_2	O^{2-} O_2	O^{2-} O_2	O_2 $O_3{}^-$	$O_2{}^-$ O^{2-}
$O_2{}^{2-}$	H_2O_2 $O_2{}^{2-}$	$O_2{}^-$ $O_2{}^{2-}$	$O_2{}^{2-}$	$O_2{}^{2-}$ HO_2	$O_2{}^{2-}$ O_2	$O_3{}^-$ $O_2{}^{2-}$	O_2^{2-} O_3	O_2^{2-} O^{2-}
HO_2	O_2 OH H_2O	O_2 HO_2	HO_2 O_2^{2-}	HO_2	HO_2 O_2	HO_2 $O_3{}^-$	O_2 OH	HO_2 O^{2-}
O_2	H_2O_2 O_2	O^{2-} O_2	$O_2{}^{2-}$ O_2	HO_2 O_2	O_2	O_2 $O_3{}^-$	O_2 O_3	O^{2-} O_2
$O_3{}^-$	$O_2{}^-$ H_2O O_2	O_2 O^{2-}	$O_3{}^-$ $O_2{}^{2-}$	HO_2 $O_3{}^-$	$O_3{}^-$ O_2	$O_3{}^-$	$O_3{}^-$ O_3	$O_3{}^-$ O^{2-}
O_3	O_2 OH HO_2	O_2 $O_3{}^-$	O_3 $O_2{}^{2-}$	O_2 OH	O_2 O_3	O_3 $O_3{}^-$	O_3	O_3 O^{2-}
O^{2-}	H_2O_2 O^{2-}	$O_2{}^-$ O^{2-}	$O_2{}^{2-}$ O^{2-}	HO_2 O^{2-}	O_2 O^{2-}	$O_3{}^-$ O^{2-}	O_3 O^{2-}	O^{2-}

We put

$$a_1 = e_{aq}{}^-, \quad a_2 = H_2, \quad a_3 = OH^-, \quad a_4 = H,$$
$$a_5 = O^-, \quad a_6 = OH, \quad a_7 = H_2O, \quad a_8 = HO_2{}^-,$$
$$a_9 = H_2O_2, \quad a_{10} = O_2{}^-, \quad a_{11} = O_2{}^{2-}, \quad a_{12} = HO_2,$$
$$a_{13} = O_2, \quad a_{14} = O_3{}^-, \quad a_{15} = O_3, \quad a_{16} = O^{2-},$$

and we suppose that $\mathcal{H} = \{a_1, \ldots, a_{16}\}$. Then, we have the following table:

$\circ$	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
a_1	a_2 a_3	a_1 a_2	a_1 a_3	a_2 a_3	a_3	a_7 a_3	a_1 a_7	a_7 a_5 a_3	a_2 a_3 a_6	a_7 a_{11}	a_1 a_{11}	a_7 a_8	a_7 a_{10}	a_3 a_{13}	a_7 a_{14}	a_1 a_{16}
a_2	a_1 a_2	a_2	a_2 a_3	a_2 a_4	a_3 a_4	a_7 a_4	a_2 a_7	a_2 a_8	a_9 a_2	a_2 a_{10}	a_2 a_{11}	a_2 a_{12}	a_2 a_{13}	a_2 a_{14}	a_2 a_{15}	a_2 a_{16}
a_3	a_1 a_3	a_2 a_3	a_3	a_3 a_4	a_5 a_3	a_6 a_3	a_7 a_3	a_8 a_3	a_9 a_3	a_{10} a_3	a_3 a_{11}	a_3 a_{12}	a_{13} a_3	a_{14} a_3	a_{13} a_8	a_{16} a_3
a_4	a_2 a_3	a_2 a_4	a_3 a_4	a_2	a_3	a_7	a_2 a_6	a_6 a_3	a_7 a_6	a_8	a_{11} a_4	a_9	a_{12}	a_4 a_{14}	a_{13} a_6	a_4 a_{16}
a_5	a_3	a_3 a_4	a_3 a_5	a_3	a_{11}	a_8	a_5 a_7	a_3 a_{10}	a_{10} a_7	a_{13} a_{16}	a_5 a_{11}	a_5 a_{12}	a_{14}	a_{10}	a_{10} a_{13}	a_5 a_{16}
a_6	a_7 a_3	a_7 a_4	a_3 a_6	a_7	a_8	a_9	a_6 a_7	a_7 a_{13}	a_7 a_{12}	a_3 a_{13}	a_6 a_{11}	a_7 a_{13}	a_6 a_{13}	a_6 a_{15}	a_{12} a_{13}	a_6 a_{16}
a_7	a_1 a_7	a_2 a_7	a_3 a_7	a_2 a_6	a_5 a_7	a_6 a_7	a_7	a_7 a_8	a_7 a_9	a_{13} a_7	a_{11} a_7	a_7 a_{12}	a_7 a_{13}	a_7 a_{14}	a_7 a_{15}	a_7 a_{16}
a_8	a_7 a_5 a_3	a_2 a_8	a_3 a_8	a_6 a_3	a_3 a_{10}	a_7 a_{10}	a_7 a_8	a_8	a_7 a_{13} a_3	a_{13} a_3 a_5	a_{11} a_8	a_8 a_{12}	a_8 a_{13}	a_{13} a_3 a_{10}	a_{10} a_{13} a_6	a_8 a_{16}
a_9	a_7 a_6 a_3	a_9 a_2	a_9 a_3	a_7 a_6	a_{10} a_7	a_7 a_{12}	a_9 a_7	a_7 a_{13} a_3	a_9	a_{13} a_3 a_6	a_9 a_{11}	a_{13} a_6 a_7	a_9 a_{13}	a_{10} a_7 a_{13}	a_6 a_{12} a_{13}	a_9 a_{16}
a_{10}	a_{11} a_7	a_2 a_{10}	a_{10} a_3	a_8	a_{16} a_{13}	a_3 a_{13}	a_7 a_{10}	a_{13} a_3 a_5	a_{13} a_3 a_6	a_{13} a_{11}	a_{10} a_{11}	a_{13} a_{12}	a_{16} a_{13}	a_{16} a_{13}	a_{13} a_{14}	a_{10} a_{16}
a_{11}	a_1 a_{11}	a_2 a_{11}	a_3 a_{11}	a_{11} a_4	a_5 a_{11}	a_6 a_{11}	a_7 a_{11}	a_8 a_{11}	a_9 a_{11}	a_{10} a_{11}	a_{11}	a_{11} a_{12}	a_{11} a_{13}	a_{14} a_{11}	a_{10} a_{15}	a_{11} a_{16}
a_{12}	a_7 a_8	a_2 a_{12}	a_3 a_{12}	a_9	a_5 a_{12}	a_7 a_{13}	a_7 a_{12}	a_{12} a_8	a_{13} a_6 a_7	a_{13} a_{12}	a_{12} a_{10}	a_{12}	a_{12} a_{13}	a_{12} a_{14}	a_{13} a_6	a_{12} a_{16}
a_{13}	a_7 a_{13}	a_2 a_{13}	a_3 a_{13}	a_{12}	a_{14}	a_6 a_{13}	a_{13} a_7	a_8 a_{13}	a_9 a_{13}	a_{16} a_{13}	a_{11} a_{13}	a_{12} a_{13}	a_{13}	a_{13} a_{14}	a_{13} a_{15}	a_{16} a_{13}
a_{14}	a_3 a_{13}	a_2 a_{14}	a_3 a_{14}	a_4 a_{14}	a_{10}	a_3 a_{15}	a_{14} a_7	a_{10} a_3 a_{13}	a_{10} a_7 a_{13}	a_{13} a_{16}	a_{14} a_{11}	a_{12} a_{14}	a_{14} a_{13}	a_{14}	a_{14} a_{15}	a_{14} a_{16}
a_{15}	a_{15} a_7	a_2 a_{15}	a_{13} a_8	a_{13} a_6	a_{10} a_{13}	a_{12} a_{13}	a_7 a_{13}	a_{13} a_6 a_{10}	a_{13} a_6 a_{12}	a_{13} a_{14}	a_{15} a_{11}	a_{13} a_6	a_{13} a_{15}	a_{15} a_{14}	a_{15}	a_{15} a_{16}
a_{16}	a_1 a_{16}	a_2 a_{16}	a_3 a_{16}	a_4 a_{16}	a_5 a_{16}	a_6 a_{16}	a_7 a_{16}	a_8 a_{16}	a_9 a_{16}	a_{10} a_{16}	a_{11} a_{16}	a_{12} a_{16}	a_{13} a_{16}	a_{14} a_{16}	a_{15} a_{16}	a_{16}

Table 1. All reactions.

Theorem 3.1. $(\mathcal{H}, \circ)$ *is an* H_v*-semigroup.*

Proof. Weak associativity is valid. As a sample of how to calculate the weak associativity, we illustrate three cases:

$$\begin{aligned} a_5 \circ (a_8 \circ a_{12}) &= a_5 \circ \{a_8,\ a_{12}\} \\ &= \{a_3,\ a_{10}\} \cup \{a_5,\ a_{12}\} \\ &= \{a_3,\ a_5,\ a_{10},\ a_{12}\}, \end{aligned}$$

$$\begin{aligned} (a_5 \circ a_8) \circ a_{12} &= \{a_3,\ a_{10}\} \circ a_{12} \\ &= a_3 \circ a_{12} \cup a_{10} \circ a_{12} \\ &= \{a_3,\ a_{12}\} \cup \{a_{12},\ a_{13}\} \\ &= \{a_3,\ a_{12},\ a_{13}\}, \end{aligned}$$

and so $a_5 \circ (a_8 \circ a_{12}) \cap (a_5 \circ a_8) \circ a_{12} \neq \emptyset$.

Similarly, we have

$$\begin{aligned} a_4 \circ (a_7 \circ a_{10}) &= a_4 \circ \{a_7,\ a_{13}\} \\ &= \{a_2,\ a_6\} \cup \{a_{12}\} \\ &= \{a_2,\ a_6,\ a_{12}\}, \end{aligned}$$

$$\begin{aligned} (a_4 \circ a_7) \circ a_{10} &= \{a_2,\ a_6\} \circ a_{10} \\ &= a_2 \circ a_{10} \cup a_6 \circ a_{10} \\ &= \{a_2,\ a_{10}\} \cup \{a_3,\ a_{13}\} \\ &= \{a_2,\ a_3,\ a_{10},\ a_{13}\}, \end{aligned}$$

and so $a_4 \circ (a_7 \circ a_{10}) \cap (a_4 \circ a_7) \circ a_{10} \neq \emptyset$.

Also, we have

$$\begin{aligned} a_7 \circ (a_8 \circ a_9) &= a_7 \circ \{a_3,\ a_7,\ a_{13}\} \\ &= a_7 \circ a_3 \cup a_7 \circ a_7 \cup a_7 \circ a_{13} \\ &= \{a_3,\ a_7\} \cup \{a_7\} \cup \{a_7,\ a_{13}\} \\ &= \{a_3,\ a_7,\ a_{13}\}, \end{aligned}$$

$$\begin{aligned} (a_7 \circ a_8) \circ a_9 &= \{a_7,\ a_8\} \circ a_9 \\ &= a_7 \circ a_9 \cup a_8 \circ a_9 \\ &= \{a_7,\ a_9\} \cup \{a_3,\ a_7,\ a_{13}\} \\ &= \{a_3,\ a_7,\ a_9,\ a_{13}\}, \end{aligned}$$

and so $a_7 \circ (a_8 \circ a_9) \cap (a_7 \circ a_8) \circ a_9 \neq \emptyset$. □

Conclusion of Theorem 3.1. H_v-semigroup is an algebraic structure as a generalization of semigroups and semihypergroups. Theorem 3.1 proved that in Radiolytic Processes in Cement Medium, in general, the associative law does not hold. For example, if we combine O^- with the result of combination of HO_2^- and HO_2, we obtain OH^-, O^-, O_2^-, HO_2. Meanwhile, if at the first we combine O^- with HO_2^- and latter combine the obtained results with HO_2, we obtain OH^-, HO_2, O_2. We observe that the final results of the above two cases are different but it is important to see the results have common OH^-, HO_2.

As another example, if we combine H with the results of combination of H_2O and O_2 we obtain $H_2,\ OH,\ HO_2$. Meanwhile, if we combine H with H_2O and then combine the obtained results with H_2O, we obtain $H_2,\ OH^-,\ O_2^-,\ O_2$. The final results of these cases are also different but the results have the common H_2. In conclusion, the order of elements in this chemical composition is very important.

Remark 1. It can be seen that $a_2 \notin a_6 \circ H$. Therefore, the reproduction axiom does not hold. As a result, $(\mathcal{H}, \circ)$ is not an H_v-group.

Definition 3.2. A non-empty set $\mathcal{A}$ with two hyperoperations $\circ$ and $\star$ is called a *bi-H_v-semigroup* if there exist two proper subsets $\mathcal{B}$ and $\mathcal{C}$ such that

(1) $\mathcal{A} = \mathcal{B} \cup \mathcal{C}$;

(2) $(\mathcal{B}, \circ)$ is an H_v-semigroup;

(3) $(\mathcal{C}, \star)$ is an H_v-semigroup.

Theorem 3.3. *Let*

$$\begin{aligned}\mathcal{A} &= \{a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_{11}, a_{13},\ a_{14},\ a_{15},\ a_{16}\},\\ \mathcal{B} &= \{a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_{11}, a_{16}\},\\ \mathcal{C} &= \{a_{13},\ a_{14},\ a_{15},\ a_{16}\}.\end{aligned}$$

Then, $\mathcal{A}$ is a bi-H_v-semigroup.

Proof. The following tables of combinations of elements of $\mathcal{B}$ and $\mathcal{C}$ show that $\mathcal{A}$ is a bi-H_v-semigroup.

$\circ$	a_1	a_2	a_3	a_4	a_5	a_{11}	a_{16}
a_1	a_2 a_3	a_1 a_2	a_1 a_3	a_2 a_3	a_3	a_1 a_{11}	a_1 a_{16}
a_2	a_1 a_2	a_2	a_2 a_3	a_2 a_4	a_3 a_4	a_2 a_{11}	a_2 a_{16}
a_3	a_1 a_3	a_2 a_3	a_3	a_3 a_4	a_5 a_3	a_3 a_{11}	a_3 a_{16}
a_4	a_2 a_3	a_2 a_4	a_3 a_4	a_2	a_3	a_4 a_{11}	a_4 a_{16}
a_5	a_3	a_3 a_4	a_3 a_5	a_3	a_{11}	a_5 a_{11}	a_5 a_{16}
a_{11}	a_1 a_{11}	a_2 a_{11}	a_3 a_{11}	a_{11} a_4	a_5 a_{11}	a_{11}	a_{11} a_{16}
a_{16}	a_1 a_{16}	a_2 a_{16}	a_3 a_{16}	a_4 a_{16}	a_5 a_{16}	a_{11} a_{16}	a_{16}

$\circ$	a_{13}	a_{14}	a_{15}	a_{16}
a_{13}	a_{13}	a_{13} a_{14}	a_{13} a_{15}	a_{13} a_{16}
a_{14}	a_{13} a_{14}	a_{14}	a_{14} a_{15}	a_{14} a_{16}
a_{15}	a_{13} a_{15}	a_{14} a_{15}	a_{15}	a_{15} a_{16}
a_{16}	a_{13} a_{16}	a_{14} a_{16}	a_{15} a_{16}	a_{16}

□

Conclusion of Theorem 3.3. Theorem 3.3 shows that the results of combinations between the family of e^-_{aq}, H_2, OH^-, H, O^-, O_2^{2-}, O^{2-} are belong to itself, i.e., this family is independent. Moreover, the results of combinations between the family of O_2, O_3^-, O_3, O^{2-} are belong to itself, i.e., his family is independent, too.

The physical and chemical properties of compounds are closely related to their structure. Within aqueous solutions the most direct means to the electron-transfer reduction of dioxygen is by pulse radiolysis. Irradiation of an aqueous solution by an electron beam yields (almost instantly; 10-12s) solvated electrons (e^-_{aq}), hydrogen atoms ($H.$), and hydroxyl radicals (OH) [18].

Widely employed in nuclear industry, concretes, mortars and grouts can be subjected to radiations in a number of situations (structural concretes and shield materials, matrices for conditioning or embedding of radioactive wastes). Residual pore water in these materials is affected by a decomposition (radiolysis) whose intensity is a function of many factors: dose rate, radiation type, and initial chemical composition of the pore solution. With the in situ emission of hydrogen gas or escape into the operational area, consequences of the radiolysis create problem of safety whose analysis is essentially based on the accurate evaluation of the $H_2 2$ source-term, in other words on the complete description of the radiolysis and its associated phenomena. This evaluation is made particularly difficult because of the composite and porous character of the cementitious materials where the presence of gas and solid phases strongly interfere with the pore water radiolysis [1].

In this work we suggested that any product which can be produced by reactions between these components is based on all reactions reported. However, between some components no reaction takes placed or no new species produced. Therefore, in these cases, the reagents are considered as products. A brief of all reactions illustrated in Table 1, i.e., the theory of algebraic hyperstructure uses a mathematical model to describe radiolytic processes in cement medium in mathematical terms.

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