

# Chemical Hyperalgebra: Redox Reactions

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## Abstract

Algebraic hyperstructure theory has a multiplicity of applications to other disciplines. This paper deals with an outline of applications in chemistry. Indeed, we provide examples of hyperstructures and  $H_v$ -structures associated with redox reactions (*Ag*, *Cu*, *Am* and *Au*).

## 1 Introduction

The hyperstructures were introduced by Marty when he first defined a hypergroup as a set equipped with an associative and reproductive hyperoperation. The motivating example was the quotient of a group by any, not necessary normal, subgroup. Algebraic hyperstructures represent a natural extension of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set, see [1]. In [2], Corsini and Leoreanu presented some of the numerous applications of hyperstructures, especially those that were found and studied in the last fifteen years. There are applications to the following subjects: geometry; hypergraphs; binary relations; lattices; fuzzy sets and rough sets; automata; cryptography; median algebras, relation algebras; combinatorics; codes; artificial intelligence and probabilities. Moreover, algebraic hyperstructures theory

has a multiplicity of applications to other disciplines, for example see [3-7,9,10]. Another book [8] is devoted especially to the study of hyperring theory. Several kinds of hyper-rings are introduced and analyzed. The volume ends with an outline of applications in chemistry and physics, analyzing several special kinds of hyperstructures.

## 2 Algebraic hyperstructures

Let  $H$  be a non-empty set,  $\mathcal{P}^*(H)$  be the set of all non-empty subsets of  $H$ . A hyperoperation on  $H$  is a map  $\odot : H \times H \longrightarrow \mathcal{P}^*(H)$  and the couple  $(H, \odot)$  is called a hypergroupoid (or hyperstructure). If  $A$  and  $B$  are non-empty subsets of  $H$ , then we denote

$$A \odot B = \bigcup_{a \in A, b \in B} a \odot b,$$

and if  $x \in H$ , then we denote  $A \odot x = A \odot \{x\}$  and  $x \odot B = \{x\} \odot B$ . A hypergroupoid  $(H, \odot)$  is called a semihypergroup if for all  $x, y, z$  of  $H$  we have  $(x \odot y) \odot z = x \odot (y \odot z)$ . That is,

$$\bigcup_{u \in x \odot y} u \odot z = \bigcup_{v \in y \odot z} x \odot v.$$

A semihypergroup  $(H, \odot)$  is called a hypergroup if for all  $x \in H$ ,  $x \odot H = H \odot x = H$ . A non-empty subset  $S$  of the hypergroup  $H$  is called a subhypergroup if  $a \odot S = S \odot a = S$  for all  $a \in S$ .  $H_v$ -structures first were introduced by Vougiouklis in Fourth AHA congress [15]. The concept of  $H_v$ -structures constitute a generalization of the well-known algebraic hyperstructures (hypergroup, hyperring, hypermodule and so on). Actually some axioms concerning the above hyperstructures such as the associative law, the distributive law and so on are replaced by their corresponding weak axioms. The reader will find in [14] some basic definitions and theorems about the  $H_v$ -structures.

Let  $(H_1, \odot)$  and  $(H_2, \star)$  be two hypergroupoids. A map  $f : H_1 \rightarrow H_2$ , is called

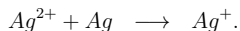
- an inclusion homomorphism if for all  $x, y$  of  $H$ , we have  $f(x \odot y) \subseteq f(x) \star f(y)$ ;
- a homomorphism if for all  $x, y$  of  $H$ , we have  $f(x \odot y) = f(x) \star f(y)$ ;
- an isomorphism if it is a one to one and onto homomorphism. In this case, we say  $H_1$  and  $H_2$  are isomorphic and we write  $H_1 \cong H_2$ .

### 3 Redox reactions

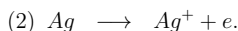
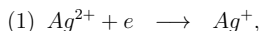
Redox (reduction-oxidation) reactions include all chemical reactions in which atoms have their oxidation state changed. This can be either a simple redox process, such as the oxidation of carbon to yield carbon dioxide ( $CO_2$ ) or the reduction of carbon by hydrogen to yield methane ( $CH_4$ ), or a complex process such as the oxidation of glucose ( $C_6H_{12}O_6$ ) in the human body through a series of complex electron transfer processes. Oxidation is the loss of electrons or an increase in oxidation state, and reduction is the gain of electrons or a decrease in oxidation state by an analyte (molecule, atom or ion). There can not be an oxidation reaction without a reduction reaction happening simultaneously. Therefore the oxidation alone and the reduction alone are each called a half-reaction, because two half-reactions always occur together to form a whole reaction [13].

Each half-reaction has a standard reduction potential ( $E^0$ ), which is equal to the potential difference at equilibrium under standard conditions of an electrochemical cell in which the cathode reaction is the half-reaction considered, and the anode is a standard hydrogen electrode (SHE). For a redox reaction, the potential of the cell is defined by:  $E_{cell}^0 = E_{cathode}^0 - E_{anode}^0$ . If the potential of a redox reaction ( $E_{cell}^0$ ) is positive, this reaction will spontaneous [13].

For example, consider the redox reaction of  $Ag^{2+}$  with  $Ag$ :



We can write two half-reactions for this reaction:



The  $E^0$  of the first reaction ( $E_{cathode}^0$ ) is 1.98 V (vs. SHE) and the  $E^0$  of the second reaction ( $E_{anode}^0$ ) is 0.799 V (vs. SHE) [11]. Therefore, in this case, the  $E_{cell}^0$  ( $E_{cathode}^0 - E_{anode}^0 = 1.181$ ) is positive and the above redox reaction between  $Ag^{2+}$  and  $Ag$  is spontaneous.

Silver ( $Ag$ ) is a transition metal and has a large number of applications in jewelry, electrical contacts and conductors, catalysis of chemical reactions, disinfectants and microbiocides. Silver plays no known natural biological role in humans and itself is not toxic, but most silver salts are toxic, and some may be carcinogenic.  $Ag$  can be in three

oxidation state:  $Ag(0)$ ,  $Ag(I)$  and  $Ag(II)$ . Among  $Ag(I)$  and  $Ag(II)$ ,  $Ag(I)$  is very well characterized and many simple ionic compounds are known containing  $Ag^+$ . However,  $AgF_2$  is known which  $Ag$  has oxidation state of  $II$  in it.  $AgF_2$  is strongly oxidizing and a good fluorinating agent. But  $Ag(II)$  is more stable in complex forms. A number of  $Ag(II)$  complexes have been obtained by oxidation of  $Ag(I)$  salts in aqueous solution in the presence of the ligand. For example,  $[Ag(pyridine)_4]^{2+}$  and  $[Ag(bipyridine)_2]^{2+}$  are quite stable. The +1 oxidation state is the best known oxidation state of silver.  $Ag^+$  salts are generally insoluble in water with the exception of nitrate, fluoride and perchlorate. Most stable  $Ag(I)$  complexes have a linear structure [12].

As described above,  $Ag$  species with different oxidation state can react with themselves. All possible products for spontaneous reactions are presented in the following table:

| $\oplus$  | $Ag^{2+}$       | $Ag^+$          | $Ag$       |
|-----------|-----------------|-----------------|------------|
| $Ag^{2+}$ | $Ag^{2+}$       | $Ag^+, Ag^{2+}$ | $Ag^+$     |
| $Ag^+$    | $Ag^+, Ag^{2+}$ | $Ag^+$          | $Ag, Ag^+$ |
| $Ag$      | $Ag^+$          | $Ag^+, Ag$      | $Ag$       |

Table 1. Redox reactions  $Ag$

Copper ( $Cu$ ) is a ductile metal with very high thermal and electrical conductivity. It is used as a conductor of heat and electricity, a building material, and a constituent of various metal alloys.  $Cu$  can be in four oxidation state:  $Cu(0)$ ,  $Cu(I)$ ,  $Cu(II)$  and  $Cu(III)$ . In nature, copper mainly is as  $CuFeS_2$ , with oxidation state of  $II$  for  $Cu$ . Also,  $Cu$  can be as  $Cu_2S$  or  $Cu_2O$  with the oxidation state of  $I$ . Pure copper is obtained by electrolytic refining using sheets of pure copper as cathode and impure copper as anode. In this process different ions of  $Cu$ ,  $Cu(II)$  or  $Cu(I)$ , reduced to  $Cu(0)$  at cathode.  $Cu(III)$  is generally uncommon, however some its complexes are known [12].

The standard reduction potential ( $E^0$ ) for conversion of each oxidation state to another are:  $E^0(Cu^{3+}/Cu^{2+}) = 2.4 V$ ,  $E^0(Cu^{2+}/Cu^+) = 0.153 V$ ,  $E^0(Cu^{2+}/Cu) = 0.342 V$  and  $E^0(Cu^+/Cu) = 0.521 V$ , where potentials are versus SHE [11]. According to these standard potentials, and similar to example of  $Ag$ , the following reactions are spontaneous:

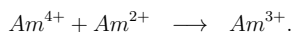
- (1)  $Cu^{3+} + Cu^+ \longrightarrow Cu^{2+}$ ,
- (2)  $Cu^{3+} + Cu \longrightarrow Cu^{2+} + Cu^+$ .

Therefore, all possible products in reactions between oxidation states of *Cu* which can be produced spontaneously are listed in the below table:

| $\odot$                 | <i>Cu</i>                                        | <i>Cu</i> <sup>+</sup>                           | <i>Cu</i> <sup>2+</sup>                           | <i>Cu</i> <sup>3+</sup>                           |
|-------------------------|--------------------------------------------------|--------------------------------------------------|---------------------------------------------------|---------------------------------------------------|
| <i>Cu</i>               | <i>Cu</i>                                        | <i>Cu</i> , <i>Cu</i> <sup>+</sup>               | <i>Cu</i> <sup>2+</sup> , <i>Cu</i>               | <i>Cu</i> <sup>2+</sup> , <i>Cu</i> <sup>+</sup>  |
| <i>Cu</i> <sup>+</sup>  | <i>Cu</i> , <i>Cu</i> <sup>+</sup>               | <i>Cu</i> <sup>+</sup>                           | <i>Cu</i> <sup>2+</sup> , <i>Cu</i> <sup>+</sup>  | <i>Cu</i> <sup>2+</sup>                           |
| <i>Cu</i> <sup>2+</sup> | <i>Cu</i> , <i>Cu</i> <sup>2+</sup>              | <i>Cu</i> <sup>2+</sup> , <i>Cu</i> <sup>+</sup> | <i>Cu</i> <sup>2+</sup>                           | <i>Cu</i> <sup>2+</sup> , <i>Cu</i> <sup>3+</sup> |
| <i>Cu</i> <sup>3+</sup> | <i>Cu</i> <sup>+</sup> , <i>Cu</i> <sup>2+</sup> | <i>Cu</i> <sup>2+</sup>                          | <i>Cu</i> <sup>2+</sup> , <i>Cu</i> <sup>3+</sup> | <i>Cu</i> <sup>3+</sup>                           |

Table 2. Redox reactions *Cu*

Americium (*Am*) is a transuranic radioactive chemical element in actinide series. It have four oxidation states of 0, 2, 3 and 4. The standard reduction potential ( $E^0$ ) for conversion of each oxidation state to another are:  $E^0$  ( $Am^{4+}/Am^{3+}$ )= 2.6 V,  $E^0$  ( $Am^{3+}/Am^{2+}$ )= -2.3 V,  $E^0$  ( $Am^{3+}/Am$ )= -2.048 V and  $E^0$  ( $Am^{2+}/Am$ )= -1.9 V, where potentials are versus SHE [11]. Therefore, the following reaction is spontaneous:



Therefore, all possible combinations for different oxidation states of *Am* which can be produced without energy are presented at the following table:

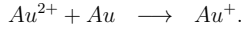
| $\otimes$               | <i>Am</i>                           | <i>Am</i> <sup>2+</sup>                           | <i>Am</i> <sup>3+</sup>                           | <i>Am</i> <sup>4+</sup>                           |
|-------------------------|-------------------------------------|---------------------------------------------------|---------------------------------------------------|---------------------------------------------------|
| <i>Am</i>               | <i>Am</i>                           | <i>Am</i> , <i>Am</i> <sup>2+</sup>               | <i>Am</i> , <i>Am</i> <sup>3+</sup>               | <i>Am</i> , <i>Am</i> <sup>4+</sup>               |
| <i>Am</i> <sup>2+</sup> | <i>Am</i> , <i>Am</i> <sup>2+</sup> | <i>Am</i> <sup>2+</sup>                           | <i>Am</i> <sup>2+</sup> , <i>Am</i> <sup>3+</sup> | <i>Am</i> <sup>3+</sup>                           |
| <i>Am</i> <sup>3+</sup> | <i>Am</i> , <i>Am</i> <sup>3+</sup> | <i>Am</i> <sup>2+</sup> , <i>Am</i> <sup>3+</sup> | <i>Am</i> <sup>3+</sup>                           | <i>Am</i> <sup>3+</sup> , <i>Am</i> <sup>4+</sup> |
| <i>Am</i> <sup>4+</sup> | <i>Am</i> , <i>Am</i> <sup>4+</sup> | <i>Am</i> <sup>3+</sup>                           | <i>Am</i> <sup>3+</sup> , <i>Am</i> <sup>4+</sup> | <i>Am</i> <sup>4+</sup>                           |

Table 3. Redox reactions *Am*

Gold (*Au*) is a dense, soft, shiny, malleable and ductile metal and can be in four oxidation states of *Au* (0), *Au* (I), *Au* (II) and *Au* (III). *Au* (III) is common for gold compounds and exist as: *Au*<sub>2</sub>O<sub>3</sub>, *AuF*<sub>3</sub>, *AuCl*<sub>3</sub>, *AuBr*<sub>3</sub> and *Au* (OH)<sub>3</sub>. *Au* (I) is much less stable in solution and is stabilized in complexes [12].

The standard reduction potential ( $E^0$ ) for conversion of each oxidation state to another are:  $E^0$  ( $Au^{3+}/Au^{+}$ )=1.401 V,  $E^0$  ( $Au^{3+}/Au$ )= 1.498 V,  $E^0$  ( $Au^{2+}/Au^{+}$ )= 1.8 V and  $E^0$  ( $Au^{+}/Au$ )= 1.692 V, where potentials are versus SHE [11]. According to these

standard potentials, the following reaction is spontaneous:



Therefore, the major products in reactions between oxidation states of Au which can be produced spontaneously are listed in below table:

| $\oplus$  | $Au$          | $Au^+$          | $Au^{2+}$          | $Au^{3+}$          |
|-----------|---------------|-----------------|--------------------|--------------------|
| $Au$      | $Au$          | $Au, Au^+$      | $Au^+$             | $Au, Au^{3+}$      |
| $Au^+$    | $Au, Au^+$    | $Au^+$          | $Au^+, Au^{2+}$    | $Au^+, Au^{3+}$    |
| $Au^{2+}$ | $Au^+$        | $Au^+, Au^{2+}$ | $Au^{2+}$          | $Au^{2+}, Au^{3+}$ |
| $Au^{3+}$ | $Au, Au^{3+}$ | $Au^+, Au^{3+}$ | $Au^{2+}, Au^{3+}$ | $Au^{3+}$          |

Table 4. Redox reactions  $Au$

## 4 Some results

### 4.1 About Table 1

In Table 1, if we rename  $Ag^{2+} := a$ ,  $Ag^+ := b$  and  $Ag := c$ , then we obtain the following table:

| $\oplus$ | $a$    | $b$    | $c$    |
|----------|--------|--------|--------|
| $a$      | $a$    | $a, b$ | $b$    |
| $b$      | $a, b$ | $b$    | $b, c$ |
| $c$      | $b$    | $b, c$ | $c$    |

This table is isomorphic to the first table of dismutation reactions (page 58, [6]). Therefore,  $\oplus$  is weak associative. Also, we conclude that  $(\{a, b\}, \oplus)$  and  $(\{b, c\}, \oplus)$  are hypergroups.

### 4.2 About Table 2

In Table 2, we rename  $Cu$ ,  $Cu^+$ ,  $Cu^{2+}$  and  $Cu^{3+}$  as follows:

$$Cu := a, Cu^+ := b, Cu^{2+} := c \text{ and } Cu^{3+} := d.$$

Then, we obtain the following table:

|         |        |        |        |        |
|---------|--------|--------|--------|--------|
| $\odot$ | $a$    | $b$    | $c$    | $d$    |
| $a$     | $a$    | $a, b$ | $a, c$ | $b, c$ |
| $b$     | $a, b$ | $b$    | $b, c$ | $c$    |
| $c$     | $a, c$ | $b, c$ | $c$    | $c, d$ |
| $d$     | $b, c$ | $c$    | $c, d$ | $d$    |

The hyperoperation  $\odot$  is weak associative, for instance

$$(a \odot b) \odot d = \{a, b\} \odot d = \{b, c\},$$

$$a \odot (b \odot d) = a \odot c = \{a, c\}.$$

Hence, we have an  $H_v$ -semigroup.

The hyperstructures  $(\{a, b\}, \odot)$ ,  $(\{a, c\}, \odot)$ ,  $(\{b, c\}, \odot)$  and  $(\{c, d\}, \odot)$  are hypergroups.

Let  $H$  be a set with three elements. On  $H$ , we define the following hyperoperation:

$$x \star y = \{x, y\}, \text{ for all } x, y \in H.$$

It is easy to see that  $\star$  is associative and so  $(H, \star)$  is a hypergroup. Now, we have

$$(\{a, b, c\}, \odot) \cong (H, \star).$$

Note that  $(\{b, c, d\}, \odot)$  is not semihypergroup, because

$$(b \odot c) \odot d = \{b, c\} \odot d = \{c, d\},$$

$$b \odot (c \odot d) = b \odot \{c, d\} = \{b, c\}$$

### 4.3 About Table 3

In Table 3, we rename  $Am$ ,  $Am^{2+}$ ,  $Am^{3+}$  and  $Am^{4+}$  as follows:

$$Am := x, Am^{2+} := y, Am^{3+} := z \text{ and } Am^{4+} := t.$$

Then, we obtain the following table:

|           |        |        |        |        |
|-----------|--------|--------|--------|--------|
| $\otimes$ | $x$    | $y$    | $z$    | $t$    |
| $x$       | $x$    | $x, y$ | $x, z$ | $x, t$ |
| $y$       | $x, y$ | $y$    | $y, z$ | $z$    |
| $z$       | $x, z$ | $y, z$ | $z$    | $z, t$ |
| $t$       | $x, t$ | $z$    | $z, t$ | $t$    |

Similar to Table 2, we have

$$(\{x, y, z\}, \otimes) \cong (H, \star).$$

Note that  $(\{y, z, t\}, \otimes)$  is not semihypergroup, because

$$\begin{aligned} (y \otimes z) \otimes t &= \{y, z\} \otimes t = \{z, t\} \\ y \otimes (z \otimes t) &= y \otimes \{z, t\} = \{y, z\}. \end{aligned}$$

#### 4.4 About Table 4

In Table 4, we rename  $Au$ ,  $Cu^+$ ,  $Au^{2+}$  and  $Au^{3+}$  as follows:

$$Au := a, Au^+ := b, Au^{2+} := c \text{ and } Au^{3+} := d.$$

Then, we obtain the following table:

| $\oplus$ | $a$    | $b$    | $c$    | $d$    |
|----------|--------|--------|--------|--------|
| $a$      | $a$    | $a, b$ | $b$    | $a, d$ |
| $b$      | $a, b$ | $b$    | $b, c$ | $b, d$ |
| $c$      | $b$    | $b, c$ | $c$    | $c, d$ |
| $d$      | $a, d$ | $b, d$ | $c, d$ | $d$    |

Let us define the following map

$$\begin{aligned} a &\mapsto y \\ c &\mapsto t \\ b &\mapsto z \\ d &\mapsto x. \end{aligned}$$

Then, the  $H_v$ -semigroups defined in Table 3 and Table 4 are isomorphic.

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