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Erratum to "The Vertex PI and Szeged Indices of Chain Graphs" *

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Abstract

Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. The vertex PI index and Szeged index of G are defined as $PI_v(G) = \sum_{e=uv \in E(G)} \big(n_u(e|G) + n_v(e|G)\big)$ and $Sz(G) = \sum_{e=uv \in E(G)} n_u(e|G) n_v(e|G)$, where $n_u(e|G)$ denotes the number of vertices of G lying closer to the vertex u than to the vertex v. Analogously, the number of vertices of G lying closer to the vertex v than to the vertex u denoted by $n_v(e|G)$. In this paper, using the same discussion methods in [MATCH 68 (2012) 349-356], we completely correct "the vertex PI and Szeged indices of chain graphs".

Throughout this paper, the notation and terminology still refer to [3].

A special spiro hexagonal chain with length d is shown in Fig. 1. It is also called the spiro para-chain and denoted by $\mathcal{P}_d = H_1 H_2 \cdots H_d$, where $H_i (i = 1, 2, \cdots, d)$ are hexagons. See [1].

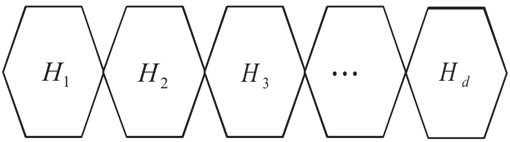

Figure 1: The spiro para-chain $\mathcal{P}_d$ of hexagons with length d.

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In Example 1 of [3], the authors computed the vertex PI and Szeged indices of the spiro para-chain $\mathcal{P}_d = H_1H_2\cdots H_d$ using their main results, respectively. i.e., $PI_v(\mathcal{P}_d) = 6d^2 + 30d$ and $Sz(\mathcal{P}_d) = d^3 + 15d^2 + 38d$. But we think that our expressions about the two indices of $\mathcal{P}_d$ as presented below are right versions.

Lemma 1. [2] *Let G be a connected graph. Then $PI_v(G) \leq |E(G)||V(G)|$ with equality if and only if G is bipartite.*

Obviously, a spiro hexagonal chain with length d is bipartite and has $5d+1$ vertices, $6d$ edges. Thus by Lemma 1, we have

$$PI_v(\mathcal{P}_d) = 6d(5d+1) = 30d^2 + 6d. \tag{1}$$

On the other hand, it is easy to see that $n_u(e|\mathcal{P}_d) = 5i-2$ and $n_v(e|\mathcal{P}_d) = 5d+1-(5i-2) = 5d+3-5i$ for each $e = uv \in E(H_i)$, $i = 1, 2, \cdots, d$. Then by the definition of the Szeged index, we have

$$\begin{aligned} Sz(\mathcal{P}_d) &= \sum_{i=1}^{d} \sum_{e\in E(H_i)} n_u(e|\mathcal{P}_d)n_v(e|\mathcal{P}_d) \\ &= 6\sum_{i=1}^{d}(5i-2)(5d+3-5i) \\ &= 25d^3 + 15d^2 + 14d. \end{aligned} \tag{2}$$

Now we give the correct versions of corresponding results in [3].

For the chain graph $G = C(G_1, G_2, \cdots, G_d; v_1, w_1, v_2, w_2, \cdots, v_d, w_d)$, we have $|V(G)| = \sum_{i=1}^{d} |V(G_i)| - d + 1$ and $|E(G)| = \sum_{i=1}^{d} |E(G_i)|$. Then $|\cup_{j=1}^{i-1} V(G_j)| = \sum_{j=1}^{i-1} |V(G_j)| - i + 2$ and $|\cup_{j=i+1}^{d} V(G_j)| = \sum_{j=i+1}^{d} |V(G_j)| + i - d + 1$.

Theorem 2. (Corresponding to Theorem 1 in [3])

Let $G = C(G_1, G_2, \cdots, G_d; v_1, w_1, v_2, w_2, \cdots, v_d, w_d)$ be a chain graph. Then

$$\begin{aligned} PI_v(G) &= \sum_{i=1}^{d} PI_v(G_i) + \sum_{i=2}^{d} \big(|E(G_i)| - |M_{v_i}(G_i)|\big)\big(\alpha_i - 1\big) \\ &\quad + \sum_{i=1}^{d-1} \big(|E(G_i)| - |M_{w_i}(G_i)|\big)\big(\beta_i - 1\big), \end{aligned}$$

where $\alpha_i = \sum_{j=1}^{i-1} |V(G_j)| - i + 2$ and $\beta_i = \sum_{j=i+1}^{d} |V(G_j)| + i - d + 1$.

Proof. If $e \in E(G_1) \setminus M_{w_1}(G_1)$, then $n_e(G) = n_e(G_1) + \beta_1 - 1$. If $e \in M_{w_1}(G_1)$, then $n_e(G) = n_e(G_1)$. Thus we have

$$\sum_{e \in E(G_1)} n_e(G) = PI_v(G_1) + \big(|E(G_1)| - |M_{w_1}(G_1)|\big)\big(\beta_1 - 1\big).$$

Similarly, we get

$$\sum_{e \in E(G_d)} n_e(G) = PI_v(G_d) + \big(|E(G_d)| - |M_{w_d}(G_d)|\big)\big(\alpha_d - 1\big).$$

For $e \in E(G_i)(i = 2, 3, \cdots, d-1)$, if $e \in M_{v_i}(G_i) \cap M_{w_i}(G_i)$, then $n_e(G) = n_e(G_i)$; if $e \in M_{v_i}(G_i) \setminus M_{w_i}(G_i)$, then $n_e(G) = n_e(G_i) + \beta_i - 1$; if $e \in M_{w_i}(G_i) \setminus M_{v_i}(G_i)$, then $n_e(G) = n_e(G_i) + \alpha_i - 1$; if $e \in E(G_i) \setminus (M_{w_i}(G_i) \cup M_{v_i}(G_i))$, then $n_e(G) = n_e(G_i) + \alpha_i + \beta_i - 2$. Thus we have

$$\begin{aligned} \sum_{i=2}^{d-1} \sum_{e \in E(G_i)} n_e(G) &= \sum_{i=2}^{d-1} PI_v(G_i) + \sum_{i=2}^{d-1} \big(|E(G_i)| - |M_{v_i}(G_i)|\big)\big(\alpha_i - 1\big) \\ &\quad + \sum_{i=2}^{d-1} \big(|E(G_i)| - |M_{w_i}(G_i)|\big)\big(\beta_i - 1\big). \end{aligned}$$

Combining the above argument, we obtain the assertion. □

Corollary 3. (Corresponding to Corollary 2 in [3]) *The vertex PI index of the chain graph* $G = C(\underbrace{H, \cdots, H}_{d \ times}; \underbrace{v, w, \cdots, v, w}_{d \ times})$ *is given by*

$$PI_v(G) = dPI_v(H) + \big(|V(H)| - 1\big)\binom{d}{2}\Big[2|E(H)| - |M_v(H)| - |M_w(H)|\Big].$$

By the similar method, the result on the Szeged index of chain graph in [3] can also be corrected as below.

Theorem 4. (Corresponding to Theorem 3 in [3])

Let $G = C(G_1, G_2, \cdots, G_d; v_1, w_1, v_2, w_2, \cdots, v_d, w_d)$ *be a chain graph. Then*

$$\begin{aligned} Sz(G) &= \sum_{i=1}^{d} Sz(G_i) + \sum_{i=2}^{d} \big(\alpha_i - 1\big)\Big[\sum_{e \in L_{v_i}} n_v(e|G_i) + \sum_{e \in R_{v_i}} n_u(e|G_i)\Big] \\ &\quad + \sum_{i=1}^{d-1} \big(\beta_i - 1\big)\Big[\sum_{e \in L_{w_i}} n_v(e|G_i) + \sum_{e \in R_{w_i}} n_u(e|G_i)\Big] \\ &\quad + \sum_{i=2}^{d-1} \Big[|L_{v_i} \cap R_{w_i}| + |L_{w_i} \cap R_{v_i}|\Big]\big(\alpha_i - 1\big)\big(\beta_i - 1\big), \end{aligned}$$

where $\alpha_i = \sum_{j=1}^{i-1} |V(G_j)| - i + 2$ *and* $\beta_i = \sum_{j=i+1}^{d} |V(G_j)| + i - d + 1$.

Corollary 5. (Corresponding to Corollary 4 in [3]) *The Szeged index of the chain graph* $G = C(\underbrace{H, \cdots, H}_{d\ times}; \underbrace{v, w, \cdots, v, w}_{d\ times})$ *is given by*

$$\begin{aligned} Sz(G) &= dSz(H) + \big(|V(H)| - 1\big)\binom{d}{2}\Big[\sum_{e \in R_v \cup R_w} n_u(e|H) + \sum_{e \in L_v \cup L_w} n_v(e|H)\Big] \\ &\quad + \big(|V(H)| - 1\big)^2 \binom{d}{3}\big[|L_v \cap R_w| + |L_w \cap R_v|\big]. \end{aligned}$$

Remark 1. *Now we use Corollaries* 3 *and* 5 *to recompute the vertex PI and Szeged indices of* $\mathcal{P}_d$, *respectively. The spiro para-chain is a special chain graph, i.e.,* $\mathcal{P}_d = C(\underbrace{C_6, \cdots, C_6}_{d\ times}; \underbrace{v, w, \cdots, v, w}_{d\ times})$. *Since* $PI_v(C_6) = 36$, $|M_v(C_6)| = |M_w(C_6)| = 0$, *by Corollary* 3, *we have* $PI_v(\mathcal{P}_d) = 36d + 5\binom{d}{2} \cdot 12 = 30d^2 + 6d$. *On the other hand, since* $Sz(C_6) = 54$, $R_v \cup R_w = L_v \cup L_w = E(C_6)$ *and* $|L_v \cap R_w| = |L_w \cap R_v| = 3$ *by Corollary* 5, *we have* $Sz(\mathcal{P}_d) = 54d + 5\binom{d}{2}PI_v(C_6) + 25\binom{d}{3} \cdot 6 = 25d^3 + 15d^2 + 14d$.

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