

Dismutation Reactions as Experimental Verifications of Ternary Algebraic Hyperstructures

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Abstract

Algebraic hyperstructures theory has a multiplicity of applications to other disciplines. In this paper, we present examples of ternary hyperstructures associated with dismutation reactions.

1 A brief history of applications of algebraic hyperstructures

The hyperstructures were introduced by Marty [17] when he first defined a hypergroup as a set equipped with an associative and reproductive hyperoperation. The motivating example was the quotient of a group by any, not necessary normal, subgroup. Algebraic hyperstructures represent a natural extension of classical algebraic structures. In a classical algebraic structure, the composition of two elements is an element, while in an algebraic hyperstructure, the composition of two elements is a set. In [1], Corsini and Leoreanu presented some of the numerous applications of hyperstructures, especially those that were found and studied in the last fifteen years. There are applications to the following subjects: geometry; hypergraphs; binary relations; lattices; fuzzy sets and rough sets; automata; cryptography; median algebras, relation algebras; combinatorics; codes; artificial intelligence and probabilities. Moreover, algebraic hyperstructures theory has a multiplicity of applications to other disciplines. In [11], Delavar Khalafi and Davvaz generalized some concepts of convex analysis such as convex functions and linear

functions on hyperstructures. In [20], Santilli and Vougiouklis outlined a hierarchy of methods of increasing complexity, known under the names of isotopies, genotopies and hyperstructures, for the characterization of physical or biological systems of increasing methodological needs. In particular, they studied the class of hyperstructures with right and left units which are such to admit as particular cases the isotopies, genotopies called e -hyperstructures. These elements can be found in hyperstructures theory, especially in H_v -structure theory introduced in 1990 by Vougiouklis [23, 24]. In [21], Vougiouklis reviewed applicable hyperstructures in Lie Santilli theory especially when multivalued problems appeared, either in finite or in infinite case. Also, we refer the readers to [2, 7, 10, 13, 18, 19]. In [8], Davvaz, Santilli and Vougiouklis studied multi-valued hyperstructures following the apparent existence in nature of a realization of two-valued hyperstructures with hyperunits characterized by matter-antimatter systems and their extensions where matter is represented with conventional mathematics and antimatter is represented with isodual mathematics. Mendel, the father of genetics took the first steps in defining “contrasting characters, genotypes in F_1 and $F_2 \dots$ and setting different laws”. The genotypes of F_2 is dependent on the type of its parents genotype and it follows certain roles. In [14], Ghadiri, Davvaz and Nekouian analyzed the second generation genotypes of monohybrid and a dihybrid with a mathematical structure. They used the concept of H_v -semigroup structure in the F_2 -genotypes with cross operation and proved that this is an H_v -semigroup. They determined the kinds of number of the H_v -subsemigroups of F_2 -genotypes. Another motivation for the study of hyperstructures comes from physical phenomenon as the nuclear fission. This motivation and the results were presented by S. Hošková, J. Chvalina and P. Račková (see [15], [16]). In [4], Davvaz, Dehghan Nezhad and Benvidi presented examples of ternary hyperstructures associated with chain reactions. Also, in [5], they provided examples of hyperstructures and weak hyperstructures associated with dismutation reactions.

2 Hyperstructures and weak hyperstructures

Let H be a non-empty set and let $\wp^*(H)$ be the set of all non-empty subsets of H . A *hyperoperation* on H is a map $\circ : H \times H \longrightarrow \wp^*(H)$ and the couple (H, \circ) is called a *hypergroupoid*. If A and B are non-empty subsets in H , then we denote

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b, \quad x \circ A = \{x\} \circ A \quad \text{and} \quad A \circ x = A \circ \{x\}.$$

A hypergroupoid (H, \circ) is called a *semihypergroup* if for all x, y, z of H , we have $(x \circ y) \circ z = x \circ (y \circ z)$, which means that

$$\bigcup_{u \in x \circ y} u \circ z = \bigcup_{v \in y \circ z} x \circ v.$$

We say that a semihypergroup (H, \circ) is a *hypergroup* if for all $x \in H$, we have $x \circ H = H \circ x = H$.

The concept of H_v -structures first introduced by Vougiouklis at Fourth Algebraic Hyperstructures and Applications Congress [23]. The concept of H_v -structures constitutes a generalization of the well-known algebraic hyperstructures (hypergroup, hyperring, hypermodule and so on). Actually some axioms concerning the above hyperstructures such as the associative law, the distributive law and so on are replaced by their corresponding weak axioms.

The hyperstructure (H, \circ) is called an H_v -group if

- (1) $x \circ (y \circ z) \cap (x \circ y) \circ z \neq \emptyset$ for all $x, y, z \in H$,
- (2) $a \circ H = H \circ a = H$ for all $a \in H$.

A motivation to obtain the above structures is the following. Let (G, \cdot) be a group and ρ an equivalence relation on G . In G/ρ consider the hyperoperation \odot such that $\bar{x} \odot \bar{y} = \{\bar{z} \mid z \in \bar{x} \cdot \bar{y}\}$, where \bar{x} denotes the class of the element x . Then (G, \odot) is an H_v -group which is not always a hypergroup.

Let H be a non-empty set. Then, a map $f : H \times H \times H \rightarrow \wp^*(H)$ is called a *ternary hyperoperation* on H and the pair (H, f) is called a *ternary hypergroupoid*. If A, B, C are non-empty subsets of H , then we define

$$f(A, B, C) = \bigcup_{a \in A, b \in B, c \in C} f(a, b, c).$$

The ternary hypergroupoid (H, f) is called a *ternary semihypergroup* if for every $a_1, \dots, a_5 \in H$, we have

$$f(f(a_1, a_2, a_3), a_4, a_5) = f(a_1, f(a_2, a_3, a_4), a_5) = f(a_1, a_2, f(a_3, a_4, a_5)).$$

Since we can identify the set $\{x\}$ with the element x , any ternary semigroup is a ternary semihypergroup. The hyperoperation f is called *weak associative* and we write WASS, if

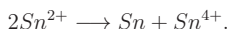
$f(f(a_1, a_2, a_3), a_4, a_5) \cap f(a_1, f(a_2, a_3, a_4), a_5) \cap f(a_1, a_2, f(a_3, a_4, a_5)) \neq \emptyset$. The hyperoperation f is called *weak commutative* and we write COW, if

$$\bigcap_{\sigma \in S_3} f(a_{\sigma(1)}, a_{\sigma(2)}, a_{\sigma(3)}) \neq \emptyset, \text{ for all } a_1, a_2, a_3 \in H.$$

Also, if for any $a_1, a_2, a_3 \in H$ and $\sigma \in S_3$, $f(a_{\sigma(1)}, a_{\sigma(2)}, a_{\sigma(3)}) = f(a_1, a_2, a_3)$, then f is called *commutative*. A ternary hypergroupoid (H, f) is called *ternary H_v -semigroup* if f is WASS. (H, f) is called a *ternary H_v -quasigroup* if the reproduction axiom is valid, i.e., $f(H, x, y) = f(x, H, y) = f(x, y, H) = H$, for all $x, y \in H$. The pair (H, f) is called a *ternary H_v -group* if it is a ternary H_v -semigroup and a ternary quasigroup. If (H, f) is a ternary quasigroup and f is associative, then (H, f) is called a *ternary hypergroup*.

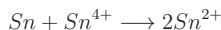
3 Ternary chemical hyperstructures

A chemical reaction in which two or more atoms of the same element originally having the same oxidation state react with other chemical(s) or themselves to give different oxidation numbers. In other words, disproportionation is a reaction in which a species is simultaneously reduced and oxidized to form two different oxidation numbers. The first disproportionation reaction to be studied in detail was:



This was examined using tartarates by Johan Gadolin in 1788 [12].

The reverse of disproportionation is called comproportionation. *Comproportionation* is a chemical reaction where two reactants, each containing the same element but with a different oxidation number, will form a product with an oxidation number intermediate of the two reactants. For example, an element tin in the oxidation states 0 and +4 can comproportionate to the state +2. The standard reduction potentials of all half reactions are: $E^\circ_{Sn^{4+}/Sn^{2+}} = 0.154 \text{ V}$, $E^\circ_{Sn^{2+}/Sn} = -0.136 \text{ V}$, $E^\circ_{Sn^{4+}/Sn} = 0.009 \text{ V}$. Therefore, the comproportionation reaction is spontaneous.



All possible combinations for the set $S = \{Sn, Sn^{2+}, Sn^{4+}\}$ to do without energy can be displayed as follows. The major products are written in the following table:

\oplus	Sn	Sn^{2+}	Sn^{4+}
Sn	Sn	Sn, Sn^{2+}	Sn^{2+}
Sn^{2+}	Sn, Sn^{2+}	Sn^{2+}	Sn^{2+}, Sn^{4+}
Sn^{4+}	Sn^{2+}	Sn^{2+}, Sn^{4+}	Sn^{4+}

Consider $S = \{Sn, S_n^{2+}, S_n^{4+}\}$ and the following ternary hyperoperation f obtained

from \oplus , i.e.,

$f(S_n, -, -)$	S_n	S_n^{2+}	S_n^{4+}
S_n	S_n	S_n, S_n^{2+}	S_n, S_n^{2+}
S_n^{2+}	S_n, S_n^{2+}	S_n, S_n^{2+}	S_n, S_n^{2+}
S_n^{4+}	S_n, S_n^{2+}	S_n, S_n^{2+}	S_n^{2+}
$f(S_n^{2+}, -, -)$	S_n	S_n^{2+}	S_n^{4+}
S_n	S_n, S_n^{2+}	S_n, S_n^{2+}	S_n^{2+}
S_n^{2+}	S_n, S_n^{2+}	S_n^{2+}	S_n^{2+}, S_n^{4+}
S_n^{4+}	S_n^{2+}	S_n^{2+}, S_n^{4+}	S_n^{2+}, S_n^{4+}
$f(S_n^{4+}, -, -)$	S_n	S_n^{2+}	S_n^{4+}
S_n	S_n^{2+}	S_n^{2+}, S_n^{4+}	S_n^{2+}, S_n^{4+}
S_n^{2+}	S_n^{2+}, S_n^{4+}	S_n^{2+}, S_n^{4+}	S_n^{2+}, S_n^{4+}
S_n^{4+}	S_n^{2+}, S_n^{4+}	S_n^{2+}, S_n^{4+}	S_n^{4+}

Then, (S, f) is a ternary H_v -semigroup and $(\{S_n, S_n^{2+}\}, f)$ is a ternary hypergroup.

As a simple example of how to calculate the weak associativity, we illustrate two of the cases:

$$\begin{aligned}
 f(f(S_n, S_n^{2+}, S_n^{4+}), S_n, S_n^{4+}) &= f(\{S_n, S_n^{2+}\}, S_n, S_n^{4+}) \\
 &= f(S_n, S_n, S_n^{4+}) \cup f(S_n^{2+}, S_n, S_n^{4+}) \\
 &= \{S_n, S_n^{2+}\} \cup \{S_n^{2+}\} = \{S_n, S_n^{2+}\}; \\
 f(S_n, (S_n^{2+}, S_n^{4+}, S_n), S_n^{4+}) &= f(S_n, \{S_n^{2+}\}, S_n^{4+}) = \{S_n, S_n^{2+}\}; \\
 f(S_n, S_n^{2+}, f(S_n^{4+}, S_n, S_n^{4+})) &= f(S_n, S_n^{2+}, \{S_n^{2+}, S_n^{4+}\}) \\
 &= f(S_n, S_n^{2+}, S_n^{2+}) \cup f(S_n, S_n^{2+}, S_n^{4+}) \\
 &= \{S_n, S_n^{2+}\} \cup \{S_n, S_n^{2+}\} = \{S_n, S_n^{2+}\}.
 \end{aligned}$$

Note that in the above case we have strong associativity, i.e.,

$$\begin{aligned}
 f(f(S_n, S_n^{2+}, S_n^{4+}), S_n, S_n^{4+}) &= f(S_n, (S_n^{2+}, S_n^{4+}, S_n), S_n^{4+}) \\
 &= f(S_n, S_n^{2+}, f(S_n^{4+}, S_n, S_n^{4+})).
 \end{aligned}$$

But, in general we have weak associativity. Consider the following case:

$$\begin{aligned}
 f(f(S_n^{2+}, S_n, S_n^{4+}), S_n^{2+}, S_n^{2+}) &= f(\{S_n^{2+}\}, S_n^{2+}, S_n^{2+}) \\
 &= \{S_n^{2+}\}; \\
 f(S_n^{2+}, f(S_n, S_n^{4+}, S_n^{2+}), S_n^{2+}) &= f(S_n^{2+}, \{S_n, S_n^{2+}\}, S_n^{2+}) \\
 &= f(S_n^{2+}, S_n, S_n^{2+}) \cup f(S_n^{2+}, S_n^{2+}, S_n^{2+}) \\
 &= \{S_n, S_n^{2+}\} \cup \{S_n^{2+}\} \\
 &= \{S_n, S_n^{2+}\}; \\
 f(S_n^{2+}, S_n, f(S_n^{4+}, S_n^{2+}, S_n^{2+})) &= f(S_n^{2+}, S_n, \{S_n^{2+}, S_n^{4+}\})
 \end{aligned}$$

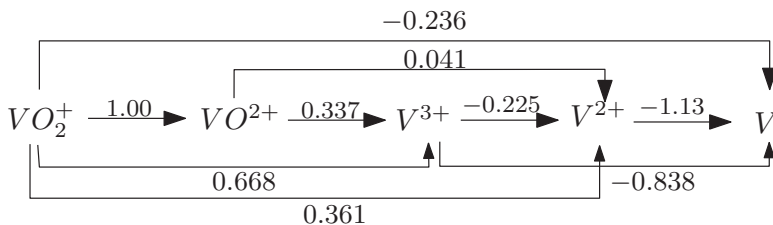
$$\begin{aligned}
 &= f(S_n^{2+}, S_n, S_n^{2+}) \cup f(S_n^{2+}, S_n, S_n^{4+}) \\
 &= \{S_n, S_n^{2+}\} \cup \{S_n^{2+}\} \\
 &= \{S_n, S_n^{2+}\}.
 \end{aligned}$$

Therefore,

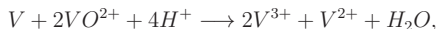
$$\begin{aligned}
 &f(f(S_n^{2+}, S_n, S_n^{4+}), S_n^{2+}, S_n^{2+}) \cap f(S_n^{2+}, f(S_n, S_n^{4+}, S_n^{2+}), S_n^{2+}) \\
 &\cap f(S_n^{2+}, S_n, f(S_n^{4+}, S_n^{2+}, S_n^{2+})) = \{S_n^{2+}\} \neq \emptyset.
 \end{aligned}$$

Vanadium is a chemical element with the symbol V and atomic number 23. The element is found only in chemically combined form in nature. Andrés Manuel del Río discovered vanadium in 1801 by analyzing a new lead-bearing mineral.

Vanadium forms a number of different ions including $V, V^{2+}, V^{3+}, VO^{2+}$ and VO_2^+ . The oxidation states of these species are 0, +2, +3, +4 and +5, respectively. The standard reduction potentials of all corresponding half reactions are:



All combinational probability for the set $S = \{V, V^{2+}, V^{3+}, VO^{2+}, VO_2^+\}$ to do without energy in acidic media can be displayed as following table. When the reactants are added in appropriate stoichiometric ratios. For example vanadium (V) reacts with VO^{2+} as follows:



\oplus	V	V^{2+}	V^{3+}	VO^{2+}	VO_2^+
V	V	V, V^{2+}	V^{2+}	V^{2+}, V^{3+}	V^{3+}
V^{2+}	V, V^{2+}	V^{2+}	V^{2+}, V^{3+}	V^{3+}	V^{3+}, VO^{2+}
V^{3+}	V^{2+}	V^{2+}, V^{3+}	V^{3+}	V^{3+}, VO^{2+}	VO^{2+}
VO^{2+}	V^{2+}, V^{3+}	V^{3+}	V^{3+}, VO^{2+}	VO^{2+}	VO^{2+}, VO_2^+
VO_2^+	V^{3+}	V^{3+}, VO^{2+}	VO^{2+}	VO^{2+}, VO_2^+	VO_2^+

Consider $S = \{V, V^{2+}, V^{3+}, VO^{2+}, VO_2^+\}$ and the following ternary hyperoperation f

obtained from \oplus , i.e.,

$f(V, -, -)$	V	V^{2+}	V^{3+}	VO^{2+}	VO_2^+
V	V	V, V^{2+}	V, V^{2+}	V, V^{2+}	V^{2+}
V^{2+}	V, V^{2+}	V, V^{2+}	V^{2+}, V	V^{2+}	V^{2+}, V^{3+}
V^{3+}	V, V^{2+}	V, V^{2+}	V^{2+}	V^{2+}, V^{3+}	V^{3+}
VO^{2+}	V, V^{2+}	V^{2+}	V^{2+}, V^{3+}	V^{3+}	V^{2+}, V^{3+}
VO_2^+	V^{2+}	V^{2+}, V^{3+}	V^{3+}	V^{2+}, V^{3+}	V^{3+}

$f(V^{2+}, -, -)$	V	V^{2+}	V^{3+}	VO^{2+}	VO_2^+
V	V, V^{2+}	V, V^{2+}	V^{2+}	V^{2+}, V^{3+}	V^{2+}, V^{3+}
V^{2+}	V, V^{2+}	V^{2+}	V^{2+}, V^{3+}	V^{2+}, V^{3+}	V^{2+}, V^{3+}
V^{3+}	V^{2+}	V^{2+}, V^{3+}	V^{2+}, V^{3+}	V^{2+}, V^{3+}	V^{3+}
VO^{2+}	V^{2+}, V^{3+}	V^{2+}, V^{3+}	V^{2+}, V^{3+}	V^{3+}	VO^{2+}, V^{3+}
VO_2^+	V^{2+}, V^{3+}	V^{2+}, V^{3+}	V^{3+}	VO^{2+}, V^{3+}	V^{3+}, VO_2^+

$f(V^{3+}, -, -)$	V	V^{2+}	V^{3+}	VO^{2+}	VO_2^+
V	V^{2+}	V^{2+}, V^{3+}	V^{2+}, V^{3+}	V^{2+}, V^{3+}	V^{3+}
V^{2+}	V^{2+}, V^{3+}	V^{2+}, V^{3+}	V^{2+}, V^{3+}	V^{3+}	V^{3+}, VO^{2+}
V^{3+}	V^{2+}, V^{3+}	V^{2+}, V^{3+}	V^{3+}	V^{3+}, VO^{2+}	VO^{2+}
VO^{2+}	V^{2+}, V^{3+}	V^{3+}	V^{3+}, VO^{2+}	VO^{2+}	VO^{2+}, V^{3+}
VO_2^+	V^{3+}	V^{3+}, VO^{2+}	VO^{2+}	VO^{2+}, V^{3+}	VO_2^+

$f(VO^{2+}, -, -)$	V	V^{2+}	V^{3+}	VO^{2+}	VO_2^+
V	V^{2+}, V^{3+}	V^{2+}, V^{3+}	V^{3+}	VO^{2+}, V^{3+}	VO^{2+}, V^{3+}
V^{2+}	V^{2+}, V^{3+}	V^{3+}	VO^{2+}, V^{3+}	VO^{2+}, V^{3+}	V^{3+}, VO^{2+}
V^{3+}	V^{2+}, V^{3+}	VO^{2+}, V^{3+}	VO^{2+}, V^{3+}	V^{3+}, VO^{2+}	VO^{2+}
VO^{2+}	VO^{2+}, V^{3+}	VO^{2+}, V^{3+}	V^{3+}, VO^{2+}	VO^{2+}	VO^{2+}, VO_2^+
VO_2^+	VO^{2+}, V^{3+}	V^{3+}, VO^{2+}	VO^{2+}	VO^{2+}, VO_2^+	VO_2^+

$f(VO_2^+, -, -)$	V	V^{2+}	V^{3+}	VO^{2+}	VO_2^+
V	V^{3+}	V^{3+}, VO^{2+}	V^{3+}, VO^{2+}	VO^{2+}, V^{3+}	VO^{2+}
V^{2+}	V^{3+}, VO^{2+}	V^{3+}, VO^{2+}	VO^{2+}, V^{3+}	VO^{2+}	V^{3+}, VO^{2+}
V^{3+}	V^{3+}, VO^{2+}	V^{3+}, VO^{3+}	VO^{2+}	VO_2^+, VO^{2+}	VO^{2+}, VO_2^+
VO^{2+}	V^{3+}, VO^{2+}	VO^{2+}	VO_2^+, VO^{2+}	VO^{2+}, VO_2^+	VO^{2+}, VO_2^+
VO_2^+	VO^{2+}	V^{3+}, VO^{2+}	VO^{2+}, VO_2^+	VO^{2+}, VO_2^+	VO_2^+

Then (S, f) is a ternary hypergroupoid. The hyperstructures $(\{V, V^{2+}\}, f)$, $(\{V^{2+}, V^{3+}\}, f)$, $(\{V^{3+}, VO^{2+}\}, f)$ and $(\{VO^{2+}, VO_2^+\}, f)$ are ternary hypergroups.

4 Conclusion

In ordinary algebras (group, ring, module etc.), the composition of two elements is an element. But in real life, there are many phenomena where the composition of two

elements is a set of elements. Algebraic hyperstructure theory is one of the useful tools to interpret them. In this paper, we observed that dismutation reactions are excellent examples of ternary algebraic hyperstructures.

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References

- [1] P. Corsini, V. Leoreanu, *Applications of Hyperstructure Theory*, Kluwer, Dordrecht, 2003.
- [2] B. Davvaz, A brief survey of applications of generalized algebraic structures and Santilli–Vougiouklis hyperstructures, Proceedings of the Third International Conference on Lie–Admissible Treatment of Irreversible Processes (ICLATIP-3), Kathmandu University, Nepal, April (2011) 91–110.
- [3] B. Davvaz, Fuzzy hyperideals in ternary semihyperrings, *Iran J. Fuzzy Systems* **6** (2009) 21–36.
- [4] B. Davvaz, A. Dehghan Nezhad, A. Benvidi, Chain reactions as experimental examples of ternary algebraic hyperstructures, *MATCH Commun. Math. Comput. Chem.* **65** (2011) 491–499.
- [5] B. Davvaz, A. Dehghan Nezhad, A. Benvidi, Chemical hyperalgebra: Dismutation reactions, *MATCH Commun. Math. Comput. Chem.* **67** (2012) 55–63 .
- [6] B. Davvaz, V. Leoreanu–Fotea, Binary relations on ternary hypergroups, *Commun. Algebra* **38** (2010) 3621–3636.
- [7] B. Davvaz, V. Leoreanu–Fotea, *Hyperring Theory and Applications*, International Academic Press, Palm Harbour, 2007.
- [8] B. Davvaz, R. M. Santilli, T. Vougiouklis, Studies of multivalued hyperstructures for the characterization of matter–antimatter systems and their extension, *Algebras Groups Geometries* **28** (2011) 105–116.
- [9] B. Davvaz, T. Vougiouklis, n -Ary hypergroups, *Iranian J. Sci. Techn.* **A30** (2006) 165–174.
- [10] B. Davvaz, J. Zhan, Y. Yin, Fuzzy H_v -ideals in Γ - H_v -rings, *Comput. Math. Appl.* **61** (2011) 690–698.

- [11] A. Delavar Khalafi, B. Davvaz, Algebraic hyper-structures associated to convex analysis and applications, *Filomat* **26** (2012) 55–65.
- [12] J. Gadolin, *K. Sv. Vet. Acad. Handl.* (1788) 186–197.
- [13] I. Gandzha, J. Kadeisvily, *New Sciences for a New Era — Mathematical, Physical and Chemical Discoveries of Ruggero Maria Santilli*, Institute for Basic Research, Palm Harbor, Florida, 2010.
- [14] M. Ghadiri, B. Davvaz, R. Nekouian, H_ν -Semigroup Structure on F_2 -Offspring of a Gene Pool, *Int. J. Biomath.*, in press.
- [15] Š. Hošková, J. Chvalina, P. Račková, Transposition hypergroups of Fredholm integral operators and related hyperstructures. Part I, *J. Basic Sci.* **4** (2008) 43–54.
- [16] Š. Hošková, J. Chvalina, P. Račková, Transposition hypergroups of Fredholm integral operators and related hyperstructures. Part II, *J. Basic Sci.* **4** (2008) 55–60.
- [17] F. Marty, Sur une generalization de la notion de groupe, 8th Congress Math. Scandenesaves, Stockholm 1934, 45-49.
- [18] R. M. Santilli, *Isodual Theory of Antimatter with Application to Antigravity, Grand Unification and the Spacetime Machine*, Springer, Dordrecht, 2006.
- [19] R. M. Santilli, *Hadronic Mathematics, Mechanics and Chemistry*, Vols. I–V, International Academic Press, Palm Harbour, 2008.
- [20] R. M. Santilli, T. Vougiouklis, Isotopies, genotopies, hyperstructures and their applications, in: T. Vougiouklis (Ed.), *New Frontiers in Hyperstructures*, Hadronic Press, Palm Harbor, 1996.
- [21] T. Vougiouklis, The Santilli’s theory ‘invasion’ in hyperstructures, Proceedings of the Third International Conference on Lie–Admissible Treatment of Irreversible Processes (ICLATIP-3), Kathmandu University, Nepal, April (2011) 69–90.
- [22] T. Vougiouklis, On some representations of hypergroups, *Ann. Sci. Univ. “Blaise Pascal”, Clermont* **26** (1991) 21–29.
- [23] T. Vougiouklis, The fundamental relation in hyperrings. The general hyperfield, Algebraic hyperstructures and applications (Xanthi, 1990), World Sci. Publishing, Teaneck, 1991, pp. 203-211.
- [24] T. Vougiouklis, *Hyperstructures and Their Representations*, Hadronic Press, Palm Harbour, 1994.