MATCH Communications in Mathematical and in Computer Chemistry

## **BOOK REVIEW**

## Graph Spectra for Complex Networks

by

Piet Van Mieghem

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This book is devoted to the mathematical background of complex networks that are crucially important to humanity nowadays. It is oriented towards mathematicians interested in algebraic graph theory, as well as scientists who apply graph spectra to different complex networks (e.g., social, information, technological, and biological networks). Graphs are used as a representation of these networks and various properties of them are dealt with in the spectral domain.

Complex networks are networks with non-trivial topological features that often occur among real-world networks. Complex networks, of relevance for the present book, are such as a network of researchers, an email network, the world wide web, electrical power grid, the human brain network, etc. The interesting fact is that these networks, although totally different, have a number of mutually coinciding properties.

As the author points out in the Preface, the book is written in an unusual articlestyle, inspired by great scholars of the past such as Gauss, Titchmarsh, and Hardy & Wright. The book focuses on general theory that may be applied to all graphs and much less to special classes of graphs, and does not contain exercises. Multigraphs, directed graph, and weighted graphs are omitted.

The book includes a Preface, Introduction and the core material, that is divided into two parts. Part I (chapters 2–7) is the main part, containing results on graph spectra. Part II (chapters 8–10) outlines textbook data from linear algebra and the theory of polynomials, that have been used in Part I. The book ends with a moderately long list of references.

Chapter 1 is an introduction to complex networks. Their graph representation is described and the main classes of graphs of importance for these networks are introduced.

Chapters 2 and 3 include elementary data of the matrix theory for graphs and the properties of the eigenvalues of the adjacency matrix, respectively. In Chapter 4 the author presents results on eigenvalues of the Laplacian of a graph. Since the Laplacian spectrum is rich, this chapter abounds with many interesting concepts such as the complexity of a graph, removal of a node, vertex connectivity, edge connectivity, etc. In Chapter 5 the adjacency spectra of different types of graphs are computed (e. g., small-world graphs, complete multi-partite graphs, and a chain of cliques). Chapter 6 is an extension of Chapter 5 and discusses density functions of large graphs. In Chapter 7 many experimental results are presented. An insight into the nature of complex networks is given, by applying the spectral knowledge from the previous chapters.

Part II includes three chapters, in which the eigensystem of a matrix, polynomials with real coefficients, and orthogonal polynomials are discussed, respectively.

Some mistakes need to be mentioned. For instance, on page 73 it is stated that a component of a graph G is a connected subgraph of G. On page 38 we read that only the adjacency spectrum of a tree is symmetric around  $\lambda = 0$ . These benign errors reveal the haste in which the book might have been written.

From the point of view of mathematical chemistry, it may be interesting and somewhat surprising that in the chapter "7. Spectra of complex networks", in the section "7.8. Spectral graph metrics" there is an entire subsection "7.8.2. Graph energy" (pp. 201–203). There are also results on Wiener and Kirchhoff indices (subsections and 4.1.3 and 7.8.3, respectively).

The theory in this book is supported by extensive explanations, details and examples. The book may be beneficial both for graduate students and researchers, mathematicians and scientists. Therefore, it deserves to be procured by any decent science library.

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