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Note on the Minimal Wiener Index of Connected Graphs With n Vertices and Radius r

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Abstract

The Wiener index is defined as the sum of distances between all unordered pairs of its vertices. In [1], the authors claim to obtain the minimal Wiener index among connected graphs on n vertices and radius r. We easily show that the conclusion is not correct by examples. Therefore this question still remains open.

Introduction

For a connected graph G with vertex set $V(G)$, the distance $d_G(u,v)$ between two vertices u and v in G is the length of a shortest $u-v$ path. The Wiener index is a graph invariant defined as $W(G) = \sum_{\{u,v\}\subseteq V(G)} d_G(u,v)$. The Laplacian matrix is defined as $L(G) = D(G) - A(G)$, where $D(G)$ and $A(G)$ are the diagonal matrix of vertex degrees and adjacency matrix, respectively. The characteristic polynomial of L is

$$det(\lambda I - L(G)) = \sum_{k=0}^{n} (-1)^k \, c_k \, \lambda^{n-k} \ .$$

For trees, E. J. Farrell [2] obtained that c_{n-2} is equal to the Wiener index $W(G)$. In [1], $C(a_1,\ldots,a_{d-1})$ is defined as a caterpillar obtained from a path P_d with vertices $v_0, v_1, \ldots, v_d$ by attaching a_i pendent edges to vertex v_i, $i = 1,\ldots,d-1$. Then in [1] is considered $C_{n,d} = C(0,\ldots,0,a_{\lfloor \frac{d}{2} \rfloor},0,\ldots,0)$.

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Main results

Theorem 1. [1] *Among connected graphs on n vertices and radius r, the caterpillar $C_{n,2r-1}$ has minimal coefficient c_k, for every $k = 0, 1, \ldots, n$.*

Corollary 2. [1] *Among connected graphs on n vertices and radius r, the caterpillar $C_{n,2r-1}$ has minimal Wiener index.*

Remark 3: Corollary 2 is only true for trees, since for a tree T, $c_{n-2}(T) = W(T)$. If G is not a tree, then $c_{n-2}(G)$ is not necessarily equal to $W(G)$.

Example 4 shows a typographical problem in Theorem 1, i. e., the condition $r \geq 2$, is not included.

Example 4: K_n and S_n are the complete graph and star graph with n vertices. These have radius 1, and S_n is the graph $C_{n,1}$. By direct calculation, $W(K_n) = \frac{n(n-1)}{2} < (n-2)^2 = W(C_{n,1})$.

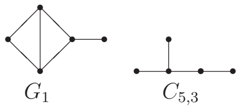

Figure 1: A counterexample with $n = 5$ and $r = 2$

When the condition $r \geq 2$ is included in Theorem 1, it is also easy to find counterexamples to Corollary 2.

Example 5: Let G_1 be the graph in Figure 1. Then G_1 and $C_{5,3}$ are two connected graphs with 5 vertices and radius 2. By direct calculations, $W(G_1) = 15 < 18 = W(C_{5,3})$.

In summary, the following problem remains open:

Question 6: Which graph does attain the minimal Wiener index among connected graphs with n vertices and radius r ?

References

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[2] E. J. Farrell, An introduction to matching polynomials, *J. Comb. Theory B* **27** (1979) 75–86.