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THE EDGE VERSIONS OF DETOUR INDEX

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Abstract

The detour index is equal to the sum of distances between all pairs of vertices of the connected graph on the longest path between corresponding vertices. The edge-detour index is conceived the same way as the sum of distances between all pairs of edges of the connected graph on the longest path between corresponding edges. In this paper, several possible distances between edges of a graph are considered. According to them, the corresponding edge-detour indices have been defined. Also, the combinatorial expressions of these two edge-detour indices of some familiar graphs have been computed.

1. Introduction

The detour matrix is one of the particularly important distance matrices which are based on the topological distance for vertices in a graph. It was introduces into the mathematical literature in 1969 by Frank Harary [1] and it was discussed in 1990 by Buckley and Harary [2]. The detour matrix was introduced into the chemical literature in 1994 under the name *"the maximum path matrix of a molecular graph"* [3-7] and theoretical graph theory contribution to finding the some interest in chemistry [8-16]. During these works, the ordinary (vertex) version of detour index has been defined for a connected graph G as follows:

$$D(G) = \sum_{\{u,v\} \subseteq V(G)} d_l(u,v|G) \qquad (1)$$

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where $d_l(u,v|G)$ denotes the distance between the vertices u and v on the longest path, and where the other details are explained below. In [17-21], some work has been done on detour index.

The edge version of the Detour index has not been considered until now. In analogy with Eq. (1), the edge-Detour index needs to be defined as

$$D_e(G) = \sum_{\{e,f\}\subseteq E(G)} d_l(e,f|G) \qquad (2)$$

where $d_l(e,f|G)$ denotes the distance between the edges e and f of the graph G on the longest path. In order that formula (2) is meaningful, we have to specify what $d_l(e,f|G)$ actually is. In [22], the distance between the edges on the shortest path is defined for introducing the edge-Wiener index. In follow, we show that the "distance between edges" can be defined in several non-equivalent ways on the longest path. Therefore, we will have several non-equivalent edge-detour indices.

2. Definitions and Properties

We first recall general definitions.

Let S be any set, the distance in S is a mapping $d: S\times S \to R$, such that for $a,b,c \in S$,

1°- $d(a,b) \geq 0$

2°- $d(a,b) = 0 \Leftrightarrow a = b$

3°- $d(a,b) = d(b,a)$

4°- $d(a,b) + d(b,c) \geq d(a,c)$

Let G be a connected graph, and $V(G), E(G)$ are the set of vertices and the set of edges, respectively. And in this paper, we suppose G is connected.

Definition 2-1. The detour distance between the vertices $a,b \in V(G)$ is equal to the length of (=number of edges in) a longest path connecting a and b. This distance will be denoted by

$d_l(a,b)$ or $d_l(a,b|G)$, of course $d_l(a,a)=0$. It is well known (and easy to verify) that d_l satisfies the conditions 1^o-4^o.

The original Detour index of a connected graph G is equal to the sum of distances between all pairs of vertices of G on the longest path, which is shown in Eq. (1).

In analogy with (1) we conceive the edge version of the Detour index, denoted by D_e, as the sum of distances between all pairs of edges of G.

However, for this we need to define the distance between edges on the longest paths.

Let $L(G)$ be the line graph of G.

Definition 2-2. The distance between the edges $e,f \in E(G)$ is equal to the distance between the vertices e,f on the longest path in the line graph of G. We denote this distance by $d_{l0}(e,f)$ or $d_{l0}(e,f|G)$, of course $d_{l0}(a,a)=0$. Thus,

$$d_{l0}(e,f|G)=d_l(e,f|L(G)) \qquad (3)$$

Using definition (2-2), we have:

$$D_{e0}(G)=\frac{1}{2}\sum_{e,f\in E(G)} d_{l0}(e,f)$$

From which it immediately follows $D_{e0}(G)=D(L(G))$.

The fact that the distance between edges defines via Definition (2-2) satisfies the conditions 1^o-4^o is evident from the relation (3).

3. Some alternative approaches and results

Let $e,f \in E(G)$ and let $e=(u,v), f=(x,y)$.

In [22], the distances between edges for the introduce of the edge-Wiener index are as follows:

$$d_3(e,f)=\begin{cases} d_1(e,f)+1 & ,e\neq f \\ d_1(e,f) & ,e=f \end{cases} \text{ and } d_4(e,f)=\begin{cases} d_2(e,f) & ,e\neq f \\ 0 & ,e=f \end{cases}$$

where $d_1(e,f)=\min\{d_s(u,x),d_s(u,y),d_s(v,x),d_s(v,y)\}$ and

$d_2(e,f)=\max\{d_s(u,x),d_s(u,y),d_s(v,x),d_s(v,y)\}$ such that distance d_s is the distance between vertices on the shortest path. The mathematical quantities d_1 and d_2 defined on the shortest path according to distance d_s and they are not distance because they do not satisfy in all of conditions 1^o-4^o and they are named the like-distances. And for all $e,f\in E(G)$, $d_3(e,f)=d_0(e,f)=d_s(e,f|L(G))$.

Therefore, the edge-Wiener indices are defined according to distances d_3 and d_4, as follow:

$W(L(G))=W_{e0}(G)=W_{e1}(G)+\frac{1}{2}m(m-1)$, and $W_{e4}(G)=W_{e2}(G)-m$

where, W_{e1} and W_{e2} are like-edge-Wiener indices according to like-distances d_1 and d_2.

Now, we define the new like-distances according to longest path between edges.

Definition 3-1. $d_5(e,f)=\min\{d_l(u,x),d_l(u,y),d_l(v,x),d_l(v,y)|\forall i,j\in V(G);d_l(i,j)>0\}$

Definition 3-2. $d_6(e,f)=\max\{d_l(u,x),d_l(u,y),d_l(v,x),d_l(v,y)|\forall i,j\in V(G);d_l(i,j)>0\}$

Two above mathematical quantity are not distance, it is easy to check. We named d_5,d_6 like-distance such as d_1,d_2. Therefore, the edge-Detour indices based on d_5,d_6 have been named like-edge-Detour indices.

Definition 3-3. We define the like-edge-Detour indices according to like-distances d_5 and d_6. They are:

$D_{e5}(G)=\sum_{\{e,f\}\subseteq E(G)} d_5(e,f)$ and $D_{e6}(G)=\sum_{\{e,f\}\subseteq E(G)} d_6(e,f)$.

Definition 3-4. For every $e,f\in E(G)$, we define: $d_7(e,f)=\begin{cases} d_5(e,f)+1 & e\neq f \\ 0 & e=f \end{cases}$

Lemma 3-5. d_7 satisfies the conditions 1^o-4^o and it is a true distance.

Proof. The fact that conditions 1^o-3^o are obeyed is evident. In order to verify that also 4^o holds, consider two cases:

If e, f and g appear on a cycle, the graph G satisfies the condition 4^o.

i) If two edge appear on a cycle, we will have two sub-cases:

a) If e, f appear on longest cycle, it is easy to see that the condition 4^0 is true.

b) If e, g appear on longest cycle, we have two sub-cases in this case, such as the following Figure. (R is equal to longest path and r is not the longest path.)

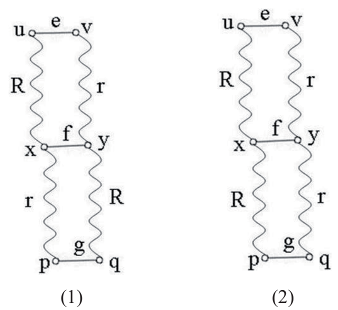

Figure 1.

In the first sub-case which has been shown in Figure 1 part (1), we have:

$d_7(e,f) = d_5(e,f)+1 = d_l(u,x)+1$, $\quad d_7(f,g) = d_5(f,g)+1 = d_l(y,q)+1$ and $d_7(e,g) = d_5(e,g)+1 = d_l(u,q)+1 = d_l(u,x)+d_l(y,q)+2$. Then,

$d_7(e,f)+d_l(f,g) = d_l(e,g)$. Therefore, the condition 4^0 is true. For figure 1, part (2), the proof is similar as the above. ■

Definition 3-6. For every $e, f \in E(G)$, we define: $d_8(e,f) = \begin{cases} d_6(e,f) & e \neq f \\ 0 & e = f \end{cases}$.

Lemma 3-7. $d_8(e,f)$ for any $e, f \in E(G)$ is a distance.

Proof. The fact that conditions 1^o-3^o are obeyed is evident. In order to verify that also 4^o holds, consider two cases:

i) If e, f and g appear on a cycle, the graph G satisfies the condition 4^o .

ii) If two edge appear on a cycle, , we will have two sub-cases:

a) If e, f appear on longest cycle, it is easy to see that the condition 4^0 is true.

b) If e, g appear on longest cycle, then in this case we have two sub-cases which have been shown in Figure 1. We prove the first sub-case which has been shown in Figure 1 part (1). Since

$d_8(e,f) = d_l(v,y)$, $d_8(f,g) = d_l(x,p)$ and $d_8(e,g) = d_l(v,p)$. We have

$d_8(e,f) + d_8(f,g) = d_l(v,y) + d_l(x,p) = d_l(v,p) + 1 > d_l(v,p) = d_8(e,g)$. Then the condition 4^0 holds. For figure 1, part (2), the proof is similar as the above. ■

Now, we define the edge-Detour indices according to distances d_7 and d_8.

Definition 3-8. There are three edge detour indices which are:

$$D_{e7}(G) = \sum_{\{e,f\} \subseteq E(G)} d_7(e,f|G),\ D_{e0}(G) = \sum_{\{e,f\} \subseteq E(L(G))} d_{lo}(e,f|L(G)) \text{ and } D_{e8}(G) = \sum_{\{e,f\} \subseteq E(G)} d_8(e,f).$$

Corollary 3-9. Let m be the number of edges in $E(G)$ and $e_i \in E(G), (1 \le i \le m)$, be all edges of G and n_i be the length of longest cycle which consists of e_i, then we have:

$$D_{e7}(G) = D_{e5}(G) + \frac{1}{2}m(m-1) \text{ and } D_{e8}(G) = D_{e6}(G) - \sum_{i=1}^{m}(n_i - 1).$$

Proof. Let m be the number of edges in $E(G)$ and $e_i \in E(G), (1 \le i \le m)$, be all edges of G and n_i be the length of longest cycle which consists of e_i. Then,

$$D_{e5}(G) = \frac{1}{2}\sum_{e,f \in E(G)} d_5(e,f) = \frac{1}{2}\sum_{\substack{e,f \in E(G)\\ e \neq f}} d_5(e,f) = \frac{1}{2}\sum_{\substack{e,f \in E(G)\\ e \neq f}} [d_7(e,f) - 1]$$

$$= \frac{1}{2}\sum_{e,f \in E(G)} d_7(e,f) - \binom{|E(G)|}{2}$$

$$= D_{e7}(G) - \binom{|E(G)|}{2} = D_{e7}(G) - \binom{m}{2}.$$

And $D_{e8}(G) = \frac{1}{2}\sum_{\substack{e,f \in E(G)\\ e \neq f}} d_6(e,f) = D_{e6}(G) - \sum_{\substack{e,f \in E(G)\\ e = f}} d_6(e,f) = D_{e6}(G) - \sum_{i=1}^{m}(n_i - 1).$ ■

Theorem 3-10. For acyclic graphs, $D_{e7} = W_{e0} = D_{e0}$ and $D_{e8} = W_{e4}$ and for cyclic graphs, they may be different.

Proof. There is unique path between vertices in acyclic graphs. Therefore, the shortest and longest paths are equal to between vertices. Also, there are $d_5 = d_1 = d_{l0}$ and $d_6 = d_2$ in acyclic graphs. So, the conditions $D_{e5} = W_{e1}$ and $D_{e6} = W_{e2}$ satisfy in acyclic graphs. And due to this fact that there is not cycle in acyclic graphs, $D_{e7} = W_{e0} = D_{e0}$ and $D_{e8} = W_{e4}$.

In cyclic graphs, there are different paths with different length. Therefore, W_{e0}, W_{e4}, D_{e0}, D_{e7} and D_{e8} may be different numbers in cyclic graphs. ■

Theorem 3-11. Let T be a tree on n vertices. Then:

$$D_{e0}(T) = D_{e7}(T) = D(T) - \binom{n}{2}. \qquad (5)$$

Proof. Buckley investigated the below relation between the Wiener index of a tree on n vertices and of its line graph [23].

$$W_{e0}(T) = W(L(T)) = W(T) - \binom{n}{2}$$

And according to theorem (3-10), the condition (5) is satisfied. ■

Theorem 3-12. The relation between edge-Detour indices for acyclic graphs with m edges is:

$$D_{e8}(G) = D_{e7}(G) + \binom{m}{2}.$$

Proof. There exist an unique path between edges and vertices in acyclic graphs. Therefore, $d_5 = d_2$ and $d_6 = d_1$. So, according to definition of d_2 and d_1, we have for every

$e, f \in E(G)$ *and* $e \neq f$: $d_2(e,f) = d_1(e,f) + 2$ and $d_2(e,e) = d_1(e,e) + 1$

Therefore, we have for an acyclic graph G with m edges:

$$W_{e2}(G) = \sum_{\{e,f\} \subseteq E(G)} d_2(e,f) = \sum_{\substack{\{e,f\} \subseteq E(G) \\ e \neq f}} (d_1(e,f) + 2) + \sum_{e \in E(G)} (d_1(e,e) + 1) = W_{e1}(G) + 2\binom{m}{2} + m.$$

Thus,

$$W_{e4}(G) = W_{e2}(G) - m = W_{e1}(G) + 2\binom{m}{2} + m - m = W_{e0}(G) - \binom{m}{2} + 2\binom{m}{2}$$
$$= W_{e0}(G) + \binom{m}{2}$$

According to theorem (3-10), we have

$$D_{e8}(G) = D_{e7}(G) + \binom{m}{2}.$$

And the proof is completed. ■

4. The edge-Detour indices of some familiar graphs

4-1. Computations of $D_{e0}(G)$, $D_{e7}(G)$ and $D_{e8}(G)$ for Paths and Stars

In [22], the $W_{e0}(G)$ and $W_{e4}(G)$ are computed for paths and stars as follow:

$$W_{e0}(P_n) = \frac{1}{6}n(n-1)(n-2)\,;\ W_{e0}(S_n) = \frac{1}{2}(n-1)(n-2)\,.$$

$$W_{e4}(P_n) = \frac{1}{6}n(n-1)(n-2) + \binom{n-1}{2} = \frac{1}{6}(n-1)(n-2)(n+3)\,;$$

$$W_{e4}(S_n) = \frac{1}{2}(n-1)(n-2) + \frac{1}{2}(n-1)(n-2) = (n-1)(n-6).$$

Then, according to Theorem (3-10, 3-11 and 3-12), we have:

$$D_{e0}(P_n) = D_{e7}(P_n) = \frac{1}{6}n(n-1)(n-2)\,;\ D_{e0}(S_n) = D_{e7}(S_n) = \frac{1}{2}(n-1)(n-2)\,.$$

$$D_{e8}(P_n) = \frac{1}{6}n(n-1)(n-2) + \binom{n-1}{2} = \frac{1}{6}(n-1)(n-2)(n+3)\,;$$

$$D_{e8}(S_n) = \frac{1}{2}(n-1)(n-2) + \frac{1}{2}(n-1)(n-2) = (n-1)(n-6).$$

4-2. Computations of $D_{e0}(G)$, $D_{e7}(G)$ and $D_{e8}(G)$ for C_n

For a cycle C_n, we have the following Theorem:

Theorem 4-2-1. Let C_n be a cycle with n vertices and $m=n$ edges. Then, we have:

$$D_{e7}(C_n)=n\binom{m}{2}-m(n-2)-W_{e0}(C_n) \text{ and}$$

$$D_{e8}(C_n)=\begin{cases}(n+2)\binom{m}{2}-2m-W_{e4}(C_n) & If \quad n \quad is \quad even\\(n+2)\binom{m}{2}-3m-W_{e4}(C_n) & If \quad n \quad is \quad odd\end{cases}.$$

Proof. Suppose C_n is a cycle with n vertices and $m=n$ edges. We define:

$$A=\left\{\{e,f\}\subseteq E(C_n)\middle| e=f \quad or \quad e \ is \ adjacent \ to \ f\right\}$$

According to the definition of d_1 *and* d_5, we have:

$$\forall\{e,f\}\in A \quad d_1(e,f)=d_5(e,f)=0 \text{ and } \forall\{e,f\}\notin A \quad d_1(e,f)+d_5(e,f)=n-2$$

Therefore,

$$D_{e5}(C_n)=\sum_{\{e,f\}\subseteq E(C_n)}d_5(e,f)=\sum_{\substack{\{e,f\}\subseteq E(C_n)\\ \{e,f\}\notin A}}(n-d_1(e,f)-2)=(n-2)(\binom{m}{2}-m)-W_{e1}(C_n)$$

Then,

$$D_{e7}(C_n)=D_{e5}(C_n)+\binom{m}{2}=(n-1)\binom{m}{2}-m(n-2)-W_{e1}(C_n)=n\binom{m}{2}-m(n-2)-W_{e0}(C_n).$$

For the second part of conclusion, we define the following sets:

$$A_1=\{\{e,f\}\subseteq E(C_n)|e=f\}, \qquad A_2=\left\{\{e,f\}\subseteq E(C_n)\middle| \ e \ is \ adjacent \ to \ f\right\} \qquad \text{and}$$

$$B=\begin{cases}\left\{\{e,f\}\subseteq E(C_n)\middle| If \ e=uv, \ f=xy: \ d_l(u,x)=d_l(v,y)\right\} & If \ n \ is \ even\\ \left\{\{e,f\}\subseteq E(C_n)\middle| If \ e=uv, \ f=xy: \ d_l(u,x)=d_l(u,y)\right\} & If \ n \ is \ odd\end{cases}$$

Now, according to the definition of d_2 *and* d_6, we have:

$$d_2(e,f)+d_6(e,f)=\begin{cases} n & \{e,f\}\in A_1 \\ n+1 & \{e,f\}\in A_2\cup B \\ n+2 & \{e,f\}\notin A_1\cup A_2\cup B \end{cases}$$

(Let e be a fix edge in C_n. We denote the subset of B which consist of the edge e with B_e.

Then, we have: $\forall e\in E(C_n)\ \ |B_e|=\begin{cases}1 & if\ n\ is\ even\\ 2 & if\ n\ is\ odd\end{cases})$

Therefore,

$$D_{e6}(C_n)=\sum_{\{e,f\}\subseteq E(C_n)} d_6(e,f)=\sum_{\{e,f\}\in A_1}(n-d_2(e,f))+\sum_{\{e,f\}\in A_2\cup B}(n-d_2(e,f)+1)+\sum_{\{e,f\}\in A_1\cup A_2\cup B}(n-d_2(e,f)+2)$$

$$=\begin{cases} nm+(n+1)(2m)+(n+2)(\binom{m}{2}-2m)-W_{e2}(C_n) & if\ n\ is\ even\\ nm+(n+1)(3m)+(n+2)(\binom{m}{2}-3m)-W_{e2}(C_n) & if\ n\ is\ odd\end{cases}$$

$$=\begin{cases} (n+2)\binom{m}{2}+nm-2m-W_{e2}(C_n) & if\ n\ is\ even\\ (n+2)\binom{m}{2}+nm-3m-W_{e2}(C_n) & if\ n\ is\ odd\end{cases}$$

$$D_{e8}(C_n)=D_{e6}(C_n)-m(n-1)=\begin{cases} (n+2)\binom{m}{2}-m-W_{e2}(C_n) & if\ n\ is\ even\\ (n+2)\binom{m}{2}-2m-W_{e2}(C_n) & if\ n\ is\ odd\end{cases}$$

$$=\begin{cases} (n+2)\binom{m}{2}-2m-W_{e4}(C_n) & if\ n\ is\ even\\ (n+2)\binom{m}{2}-3m-W_{e4}(C_n) & if\ n\ is\ odd\end{cases}$$

And the proof is completed. ■

According to Theorem (4-2-1) we can compute the $D_{e7}(G)$ and $D_{e8}(G)$ for C_n.

If n is even, then we have : $W_{e0}(C_n)=\frac{1}{8}n^3$ and $W_{e4}(C_n)=\frac{1}{8}n(n^2+4n-8)$.

Therefore,

$$D_{e7}(C_n) = n\binom{m}{2} - m(n-2) - W_{e0}(C_n) = \frac{1}{2}n^2(n-1) - n(n-2) - \frac{1}{8}n^3 = \frac{3}{8}n^3 - \frac{3}{2}n^2 + 2n$$ and

$$D_{e8}(C_n) = (n+2)\binom{m}{2} - 2m - W_{e4}(C_n) = \frac{1}{2}n(n-1)(n+2) - 2n - \frac{1}{8}n(n^2+4n-8) = \frac{3}{8}n^3 - 2n.$$

$$D_{e0}(C_n) = \frac{3}{8}n^3 - \frac{3}{2}n^2 + \frac{3}{2}n$$

If n is odd, then

$W_{e0}(C_n) = \frac{1}{8}n(n^2-1)$ and $W_{e4}(C_n) = \frac{1}{8}n(n^2+4n-5)$.

Therefore,

$$D_{e7}(C_n) = n\binom{m}{2} - m(n-2) - W_{e0}(C_n) = \frac{1}{2}n^2(n-1) - n(n-2) - \frac{1}{8}n(n^2-1) = \frac{3}{8}n^3 - \frac{3}{2}n^2 + \frac{17}{8}n$$ and

$$D_{e8}(C_n) = (n+2)\binom{m}{2} - 3m - W_{e4}(C_n) = \frac{1}{2}n(n-1)(n+2) - 3n - \frac{1}{8}n(n^2+4n-5) = \frac{3}{8}n^3 - \frac{27}{8}n.$$

$$D_{e0}(C_n) = \frac{3}{8}n^3 - \frac{3}{2}n^2 + \frac{9}{8}n$$

4-3. Computations of $D_{e7}(G)$ and $D_{e8}(G)$ for K_n

We suppose the K_n is a complete graph, and fix the edge $e \in E(K_n)$. The longest cycle in K_n is with length n. Then,

$$D_{e5}(K_n) = \frac{1}{2}\sum_{e\in E(K_n)}[(\frac{1}{2}n(n-1) - 2n + 3)(n-3) + (n-2)(2n-4)] = \frac{1}{8}n(n-1)(n^3 - 4n^2 + 5n - 2);$$

$$D_{e7}(K_n) = D_{e5}(K_n) + \frac{1}{2}m(m-1) = \frac{1}{8}n^5 - \frac{1}{2}n^4 + \frac{7}{8}n^3 - n^2 + \frac{1}{2}n.$$

$$D_{e6}(K_n) = (n-1)\binom{m}{2} + m(n-1);$$

$$D_{e8}(K_n) = D_{e6}(K_n) - m(n-1) = (n-1)\binom{m}{2} = \frac{1}{8}(n-1)^2 n(n^2 - n - 2).$$

4-4. Computations of $D_{e7}(G)$ and $D_{e8}(G)$ for $K_{a,b}$:

Suppose $K_{a,b}$ is a complete bipartite graph and $a \le b$ and $e \in E(K_{a,b})$ is a fix edge. The longest cycle in $K_{a,b}$ is with length $2a$. Then,

$$D_{e5}(K_{a,b}) = \frac{ab}{2}[(ab-a-b+1)(2a-3)+(2a-2)(a+b-2)] = \frac{1}{2}ab(2a^2b-3ab-a+b+1);$$

$$D_{e7}(K_{a,b}) = \frac{ab}{2}[(ab-a-b+1)(2a-3)+(2a-2)(a+b-2)] + \binom{m}{2} = a^3b^2 - a^2b^2 - \frac{1}{2}a^2b + \frac{1}{2}ab^2.$$

$$D_{e6}(K_{a,b}) = (2a-1)\binom{ab}{2} + ab(2a-1);$$

$$D_{e8}(K_{a,b}) = (2a-1)\binom{ab}{2} = \frac{1}{2}(2a-1)ab(ab-1).$$

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