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BOOK REVIEW

A Combinatorial Approach to Matrix Theory and Its Applications

by

Richard. A. Brualdi and Dragoš Cvetković

CRC Press, Boca Raton, 2008, XV+267 pp. ISBN 978-1-4020-8223-4

This book is aimed at presenting the theory of matrices from a somewhat unusual point of view. Namely, it develops the entire matrix theory by employing combinatorial, primarily graph–theoretic, concepts and tools. Graphs are used to illuminate many of the basic results of matrix theory, and to help understanding the true meaning of several of its definitions and procedures.

Each author has earlier published a book on similar matter:

- D. Cvetković, *Combionatorial Matrix Theory with Applications to Electrical Engineering, Chemistry and Physics*, Naučna knjiga, Belgrade, 1980, 2nd ed. 1987 (in Serbian).
- R. A. Brualdi and H. J. Ryser, *Combinatorial Matrix Theory*, Cambridge Univ. Press., Cambridge, 1991;

The present book is a kind of combination of these two earlier books. Much material in it is obtained by translating, reworking, and updating parts of Cvetković's Serbian–language book.

The Brualdi–Cvetković book is divided into Preface, Dedication, ten chapters (1. Introduction, 2. Basic Matrix Operations, 3. Powers of Matrices, 4. Determinants, 5. Matrix Inverses, 6. Systems of Linear Equations, 7. Spectrum of a Matrix, 8. Nonnegative Matrices, 9. Additional Topics, 10. Applications), each ending with a list of problems and exercises, followed by Coda, Answers and Hints, Bibliography (quoting 83 bibliographic items), and Index.

The fact on which the entire consideration is based is that both a matrix and a weighted (di)graph are just different representations of one and the same mathematical object. Although this detail is well known and fairly obvious, in the majority of existing approaches to matrix theory and linear algebra it is simply ignored. The Brualdi–Cvetković book offers numerous convincing examples for the cases when the graph–based approach is more appropriate and easier to learn and understand. Of these we mention here the considerations related to the Perron–Frobenius theorem (Chapter 8), and the combinatorial proofs of the Cayley–Hamilton theorem and of the Jordan canonical form of a matrix (Chapter 7). In solving systems of linear equations in which many of the coefficients are zero (pertaining to sparse matrices), the flow–graph–based approaches – the Coates formula etc. – are superior to standard algebraic procedures, and are much used in engineering applications (Chapters 6 and 10).

The readers of *MATCH Communications in Mathematical and in Computer Chemistry* may be interested to know that in Chapter 10 there is a section devoted to chemistry. In it we read on Hückel Molecular Orbital Theory, Two Examples: Linear Polyenes and Annulenes, Stability of Molecules, and Alternant Hydrocarbons and Their Graphs.

In the Preface the authors point out that their book is not written as a first course in linear algebra, and that it could be used in special courses of matrix theory for students who know the basics of vector spaces. They maintain that this book could be used in undergraduate seminars or for self–study. This reviewer fully agrees with their opinion. The book certainly deserves to be procured by major mathematical libraries. It may well be that some of the readers of *MATCH Communications in Mathematical and in Computer Chemistry* will find it not only useful, but also stimulating.

Ivan Gutman