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ON RELATION BETWEEN ENERGY AND LAPLACIAN ENERGY *

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Abstract

Let G be a simple graph with n vertices and m edges, with ordinary spectrum λ_i, $i = 1, 2, \cdots, n$, and with Laplacian spectrum μ_i, $i = 1, 2, \cdots, n$. The energy and the Laplacian energy of the graph G are defined as $E(G) = \sum_{i=1}^{n} |\lambda_i|$ and $LE(G) = \sum_{i=1}^{n} \left|\mu_i - \frac{2m}{n}\right|$, respectively. In [9] the authors provided numerous examples for the inequality $E(G) \leq LE(G)$ and conjectured that it holds for all graphs. In this paper we show that the conjecture does not hold.

1 Introduction

Let G be a simple undirected graph possessing n vertices and m edges. Let A be the symmetric $(0, 1)-$adjacency matrix of G and $D = diag(d_1, d_2, \ldots d_n)$ be the diagonal matrix of vertex degrees. The Laplacian matrix of G is $L = D - A$. Let λ_1, λ_2,... λ_n be the adjacency spectrum of G, and let μ_1, μ_2,... μ_n be the Laplacian spectrum of G.

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The energy $E(G)$ of a graph G is defined as [2]

$$E(G) = \sum_{i=1}^{n} |\lambda_i|. \tag{1}$$

This quantity has a clear connection to chemical problems [3–5] and has recently been much investigated (see [6, 7, 10, 11, 13, 15] and the references cited therein). The Laplacian energy $LE(G)$ of a graph G has been defined [8] as

$$LE(G) = \sum_{i=1}^{n} \left| \mu_i - \frac{2m}{n} \right| . \tag{2}$$

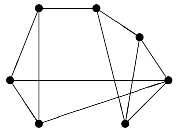
For recent investigations of this quantity see [14].

The quantities $E(G)$ and $LE(G)$ were found to have a number of analogous properties [8]. It is easy to see that if the graph G is regular, then $E(G) = LE(G)$ [8]. And there are non-regular graphs with the same property [9]. In [9] the authors also provided numerous examples for the inequality $E(G) \leq LE(G)$ and conjectured that it holds for all graphs. In this paper we show that the conjecture does not hold by counterexamples. Finally, we show that there exist infinite examples satisfying $E(G) > LE(G)$.

2 Counterexamples

1

We now provide an example showing that the conjecture in [9] does not hold. Consider the following graph G_{388} of order 7. (G_{388} is the graph $11 - 388$ in [1]). It was shown in [1] that the adjacency spectra of G_{388} is $\{3.17741, 1.73205, 0.67836,$ $-1, -1, -1.73205, -1.85577\}$. By direct calculation we obtain $E(G_{388}) \approx 11.17564$ $q > 11.1756$. And also by direct calculation we obtain the characteristic polynomial of Laplacian matrix of G_{388} is $\phi(x) = x^7 - 22x^6 + 196x^5 - 900x^4 + 2228x^3 - 2784x^2 +$ $1344x$. So its Laplacian spectrum is $\{4+\sqrt{2},\ 3+\sqrt{3},\ 4,\ 4,\ 4-\sqrt{2},\ 3-\sqrt{3},\ 0\}$. It is then immediate to verify that $LE(G_{388}) \approx 11.14967 < 11.1497$. Thus $LE(G_{388}) <$ $E(G_{388})$ holds. In fact, the graph G_{388} is the minimal counterexample.



G_{388}

2

Let $G_1 \cup G_2$ denote the graph consisting of two (disconnected) components G_1 and G_2, and let kG denote the graph consisting of k ($k > 0$ be integer) copies of the graph G. As usual, by C_n, K_n, P_n denoted the n-vertex cycle, the n-vertex complete graph and the n-vertex path, respectively. For the graph energy the equalities

$$E(G_1 \cup G_2) = E(G_1) + E(G_2), \tag{3}$$

$$E(kG) = kE(G).$$

are always satisfied. And it is easy to verify that

$$LE(kG) = kLE(G).$$

So we have $E(kG_{388}) > LE(kG_{388})$, for all $k > 0$ be an integer.

Consider now the graph $G = G_{388} \cup K_3$. Its Laplacian spectrum is the union of the Laplacian spectra of G_{388} and K_3, viz., $\{4+\sqrt{2},\ 3+\sqrt{3},\ 4,\ 4,\ 3,\ 3,\ 4-\sqrt{2},\ 3-\sqrt{3},\ 0,\ 0\}$. Therefore, by Eq. (2) $LE(G) = 14.6925 < E(G) = E(G_{388}) + E(K_3) = 15.1756$. And then $E(kG) > LE(kG)$, for all $k > 0$ be an integer.

In an analogous manner one can verify that the graph $G = G_{388} \cup K_4$ satisfying $E(G) > LE(G)$, and $E(kG) > LE(kG)$, for all $k > 0$ be an integer.

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