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# Handedness and Chirality

Dedicated to the memory of Prof. Dr. Vladimir Prelog

(July 23 1906 – January 7 1998)

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**Abstract**

Chiral objects and figures like hands exist, by definition of chirality, in two distinguishable forms that are mirror images of each other. But, being isometric, the two forms cannot be distinguished if we take only the metric into account. For the distinction of chiral objects we need more than just a metric, we need to introduce an orientation of the space in order to define reflections and mirror images, i.e. we need coordinates.

Once we are given coordinates, the chiral objects come in pairs, for example a right and a left hand have opposite orientation, in contrast to two hands with equal orientation. *Chirality* is in fact translated as *handedness*, which means the same relational term.

In scientific articles the description of this phenomenon is often mistaken for the two-valued measure for the absolute *sense* of handedness, like right-handed and left-handed, an arbitrary attributive term that *depends upon common visual experience and common agreement*. It needs more than just the orientation and requires the concept of clockwise and counterclockwise combined with a directional sense.

## 1 The Definition of Chirality

The term chirality for the property of handedness was first introduced 1893 by Lord Kelvin ([5]) in a Robert Boyle Lecture at the Oxford University Junior Scientific Club in 1893. W. Thomson (Lord Kelvin) defined it as follows:

> I call any geometrical figure or group of points chiral and say it has chirality, if its image in a plane mirror, ideally realized, cannot be brought to coincide with itself. Two equal and similar right hands are homochirally similar. Equal

> and similar right and left hands are heterochirally similar. They are also called enantiomorphs as introduced by German writers. Any chiral object and its image in a plane mirror are heterochirally similar.

Chirality as a multilingually accessible synonym for handedness has possibly been chosen, because the latter, in colloquial usage, has also the connotation of the preference of one hand over the other (laterality). This notion had almost vanished when it was reintroduced 1958 by L.L. Whyte ([13]). Chiral objects and figures like hands exist in two distinguishable forms that are mirror images of each other. This means that a chiral object and its mirror-image do not coincide by any ideally realized series of rigid motions. Nevertheless, since the internal spatial relations are preserved upon reflection, they are isometric, which means that corresponding points in the two mirror images have the same distance, and so the two chiral objects cannot be distinguished if we take only the metric into account. The check for chirality needs more than just the metric. In order to allow to use the concept of mirror-image, we need in fact the notion of reflection, for example, by introducing coordinates or similar mathematical tools, and we need to define translations, rotations and reflections.

# 2 Right and Left

Handedness and chirality mean the same relational term. Two hands can have the same sense or be of opposite sense, more generally, chiral objects come in pairs. It is the same situation with the helicity of screws, the relative sense of dihedral angles and of coordinate systems. Therefore, the notion of sense needs discussion, in particular if we use the words "left" or "right" in addition, for example, when we call an ear a right ear. This was a problem, for example, for various philosophers, a very prominent one of them was Immanuel Kant.

In his book Enigmas of Space and Time ([8]) Robert Le Poidevin analyzes Kant's proof of the existence of absolute space on the basis of the distinction between right and left ([4]). Kant uses the existence of asymmetric solid figures of identical size and shape but of opposite sense to prove that space is fixed and absolute with a reality of its own. Kant later had quite a different view of space. In his transcendental idealism, space is indeed independent of bodies because it is not real but it is ideal beyond our comprehension (Critique of Pure Reason, 1781, and Prolegomena, 1783). Le Poidevin uses the term handedness for the property of being either right-handed or left-handed. He then argues, that handedness is not an intrinsic property of a lone hand but whether or not it is chiral, i.e. that it can or cannot be superposed on its mirror image. "Kant was right that a characteristic feature of the lone hand depends on space, but he chose the wrong feature namely their handedness rather than their chirality." This contradicts the definition of handedness = chirality. Other authors, especially in chemistry ([7]) and physics ([1]) use the term chirality for the property of being either right-handed or left-handed i.e. as an attributive term. They are even referring to the two chiralities (or helicities) possible for such an object. This contradicts the definition of chirality (cf. section 1), which clearly describes a relational term (homochirally and heterochirally equal and similar hands). It is not necessary to know the individual right or left attributes of two hands, to recognize if they have equal ( = homochiral) or opposite ( = heterochiral) chiral sense. They are either related by translation/rotation ( = isomorphic) or by reflection ( = enantiomorphic).

# 3 Mathematical Aspects

Once we are given coordinates or other ways of distinguishing a chiral object and its mirror image, then we have an orientation at hand. This means a function that is invariant under translation and rotation but changes the sign under reflection, which means that it is a relative invariant with respect to rigid motions.

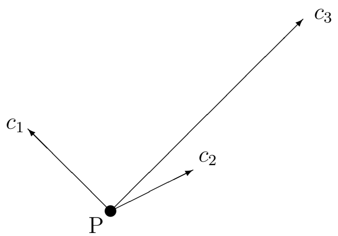
In mathematical texts ([11]) *orientation* is the choice of an equivalence class of coordinate systems, which means a sequence $(b_1, \ldots, b_n)$ of linearly independent vectors $b_i$, the basis vectors, together with all the other sequences $(c_1, \ldots, c_n)$ of basis vectors the determinant of which has the same sign as the determinant of $(b_1, \ldots, b_n)$:

$$sign(det(b_1, \ldots, b_n)) = sign(det(c_1, \ldots, c_n)),$$

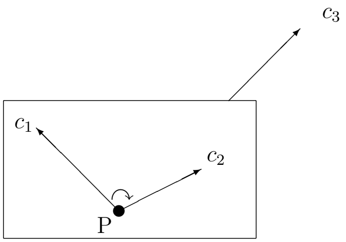
which means that the (nonzero) real numbers $det(b_1, \ldots, b_n)$ and $det(c_1, \ldots, c_n)$ are either both positive or both negative. The choice of the equivalence class of $(b_1, \ldots, b_n)$ allows to check if another basis has the *same* orientation or the *opposite* one, depending upon the *plus* or *minus* sign of the determinantal value.

Only at this point we are in a position to define what means right-handed or left-handed, in three-dimensional space.

We choose an orientation, say by selecting the equivalence class of the basis $(b_1, b_2, b_3)$. This class contains a basis of pairwise othogonal vectors, say the basis $(c_1, c_2, c_3)$. It allows to introduce a right-handed screw by using the standard argument: Assume a point $P$ in space and put the $c_i, i = 1, 2, 3$, there,




Consider the plane that carries $c_1$ and $c_2$ and look to it from that side of the plane from which the shortest way of turning $c_1$ into the direction of $c_2$ is clockwise.




Looking from this side in the direction of $c_3$ means either looking forward or looking backward i.e. to choose a directional sense. In the first case this is the direction of what is usually called a right-handed screw, and so we call this situation a right-handed orientation, the other one is called a left-handed orientation correspondingly.

So we should carefully note that this definition of "right" is a *choice*, based on an old tradition ( = choice) for clockwise[1] and for right-handed screw. This is meant by "depends upon common visual experience and common agreement" in the abstract.

# 4 Examples

In geometry the relational term orientation describes the fact that isometric figures can be pairwise of the same sense or of opposite sense. Geometric figures and real tools possess this binary property only if the individual objects don't allow symmetry operations of the second kind involving reflection or inversion, respectively. If reflection or inversion are possible automorphisms, then that particular object can be divided mentally into pairwise enantiomorphic parts and is therefore selfenantiomorphic or achiral. Pure rotational symmetry is compatible with chirality. In 1860 Louis Pasteur has coined the term dissymmetry for the absence of reflection symmetry. It is obvious that *handedness*, *chirality*, *helicity*, *orientation* and *reflection-dissymmetry* describe the *same* property of real or ideally realized objects. As it is necessary for geometric and physical properties, chirality and the equivalent terms are invariant upon reflection. Right-handed and left-handed coordinate systems cannot be defined geometrically, but it can be shown that they have opposite sense in contrast to the same sense (cf. section 3). Likewise the notion of chirality and its synonyma can be defined without specifying the individual absolute sense of the objects (cf. definition in section 1). To recognize the opposite sense of two isometric objects it isn't necessary to know which is the right-handed and which is the left-handed part. The same is true for the two possible pairs with equal sense. If, for practical purposes, one resumes to an externally defined measure like right- or left-handed, one has to refrain from it when describing properties and quantities in geometry ([6],[12]) and physics ([2]). The attributive + or − sign for pseudoscalar properties is derived by a right hand rule and therefore of conventional origin. Relevant in physical law is their equal or opposite sign.

Up to now we have compared only chiral isometric geometrical figures or rigid real objects without restricting their shape. As long as they are isometric and chiral, it is always possible to decide if two objects of arbitrary shape show equal or opposite sense of orientation. They are either related by translation/rotation or by reflection. If they are no more isometric but yet comparable in shape, a decision may be possible but depends upon common agreement. This will become the essential problem in communicating the absolute sense of handedness for an individual object (see below).

If we consider our natural environment, one could get the impression, that nature prefers one sense of handedness. In his essay, Kant gives several examples: The position of the heart on the left side of the human body, the predominant righthandedness, the left-handed coil of the bean plant, the right-handed helix of snail shells etc. One could add compounds occuring in nature with molecules of one sense of handedness like the

[1]Historically seen the generally accepted clockwise sense clocks run nowadays was chosen in the sixteenth century and kept up to now by common agreement. Seen from the northern hemisphere of the earth, the sun moves in a clockwise sense from east to west. Accordingly the shadow of a vertical pole moves clockwise in a horizontal plane. But sunclocks vertically arranged on a wall run counterclockwise. This fact might explain why mechanical clocks in the domes of Florence and Münster dating from the fifteenth century have twentyfour hours dials and run counterclockwise.

proteinogenic L-amino acids and D-sugars. They are preferentially produced and metabolized in living organisms. The action of drugs, the taste and the odor of substances can depend upon the absolute sense of handedness of their molecules. But these are all examples for an accidental predilection of nature, a predilection de facto not one dejure, that would correspond to a natural law. A matter of fact that indicates the unique origin of living organism ([9]).

That leads to the question if it is possible to communicate which absolute sense of handedness (an attributive term!) we mean by referring to an individual chiral object of any shape. In general that will not be possible. The absolute sense can be specified only by referring to an arbitrary standard that depends upon common visual experience and common agreement. Only two values exist for the measure of the absolute sense of handedness. If one has been arbitrarily chosen for instance as clockwise the other is defined as counterclockwise. They define the rotational sense in a plane and in addition, by the clock's face, what is above and below with respect to that plane thereby defining a directional sense (cf. section 3). For instance a right handed helix or screw is defined by a clockwise rotation while advancing along its axis. Similarly in a right handed cartesian coordinate system, the positive axes $x$, $y$ and $z$ are arranged in a clockwise sense, when viewed from the origin. A more popular version is: the positive directions of the $x$, $y$ and $z$ axes point along the thumb, the stretched index finger and the raised middle finger of the right hand. This palpable instruction anticipates the difficulties to communicate the absolute sense of handedness for chiral objects of any shape. In general it will not be possible to find such an attributive term. It is only feasible if there is a prominent feature in the structure that allows to assign the pattern of a tripod, a dihedral angle or a helix. The chemical structure of organic molecules for instance can be reduced to such patterns that allow to communicate their absolute configurations ([10]).

A method to specify the chiral sense of molecules as R or S has been proposed 1966 by Cahn, Ingold and Prelog using atomic numbers to classify the substituents around a specification center and then to decide if they are arranged in a clockwise (rectus) or counterclockwise (sinister) sense. A detailed and comprehensive account of this method is given by Günter Helmchen ([3]).

# 5 Summary

In this essay the notion of handedness and its multilingually accessible synonym chirality is discussed and shown to be a *relational term*. Their general meaning in mathematics and natural sciences is described. The mathematical definition of orientation reveals exactly the same notion. Symmetry considerations show that reflection-dissymmetry, the absence of any kind of reflection symmetry, is the prerequisite for chirality or handedness. In accordance with Lord Kelvin's definition, these completely equivalent terms describe a geometric-physical property that is reflection-invariant. Therefore it is not necessary to use an externally defined measure, like left- or right-handed, to define chirality or handedness. In describing geometric-physical properties it may be favourable to choose such a measure, e.g. a right-handed coordinate system, as long as the final result is independent of this choice.

The predominant righthandedness of human beings and its consequences lead to a measure for the absolute sense of handedness that depends upon common visual expe-

rience: clockwise and counterclockwise. Combined with a directional sense these two rotational senses allow to communicate the particular individual screw sense in mathematics, natural science and technique. But there are obvious restrictions. In general it will not be possible to apply it to chiral objects of any shape.

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