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DISTANCE EQUIENERGETIC GRAPHS

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Abstract

The distance energy $E_D(G)$ of a graph G is defined as the sum of the absolute values of the eigenvalues of the distance matrix of G. The graphs G_1 and G_2 are said to be distance equienergetic (D-equienergetic) if $E_D(G_1) = E_D(G_2)$. In this paper we obtain the eigenvalues of the distance matrix of the join of two graphs whose diameter is less than or equal to 2, and construct pairs of non D-cospectral, D-equienergetic graphs on n vertices for all $n \geq 9$.

INTRODUCTION

In this paper we are concerned with simple graphs, that is graphs without loops, multiple edges or directed edges. Let G be such graph on n vertices and m edges. Let its vertices be labelled as $v_1, v_2, \ldots, v_n$. The distance between the vertices v_i and v_j, denoted by d_{ij} , is the length of the shortest path between them. The diameter of a graph G, denoted by $diam(G)$, is the maximum distance between any pair of vertices of G [4,12].

The distance matrix of a graph G is an $n \times n$ matrix $D(G) = [d_{ij}]$. The characteristic polynomial of $D(G)$ is defined as $\psi(G : \mu) = \det(\mu I - D(G))$, where I is the identity matrix of order n . The eigenvalues of the distance matrix $D(G)$, denoted by $\mu_1, \mu_2, \ldots, \mu_n$, are said to be the distance or D-eigenvalues of G and their collection is called the distance or D-spectrum of G . Two non-isomorphic graphs are said to be D-cospectral if they have same D-spectra [4,5,6]. Since the distance matrix is symmetric, its eigenvalues are real and can be ordered as $\mu_1 \geq \mu_2 \geq \cdots \geq \mu_n$.

The characteristic polynomial and the eigenvalues of the distance matrix of a graph were considered in [7–9,13,14,29].

The distance energy $E_D(G)$ of a graph G is defined as

$$E_D(G) = \sum_{i=1}^{n} |\mu_i| \ . \tag{1}$$

Eq. (1) was recently introduced by Indulal et al. [15] and was conceived in full analogy to the ordinary graph energy $E(G)$ defined as [10,11]

$$E(G) = \sum_{i=1}^{n} |\lambda_i| \tag{2}$$

where $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of the adjacency matrix of G [6].

Several bounds for the distance energy of a graph are obtained in [15,20,21].

The graphs G_1 and G_2 are said to be equienergetic if $E(G_1) = E(G_2)$. Numerous results on (non-isomorphic) equienergetic graphs can be found in [1–3,16–19,23–28].

The connected graphs G_1 and G_2 are said to be D-equienergetic (distance equienergetic) if $E_D(G_1) = E_D(G_2)$. For obvious reason D-cospectral graphs are D- equienergetic. Therefore we are interested in non D-cospectral, D-equienergetic graphs having equal number of vertices. Indulal et al. [15] constructed pairs of D-equienergetic graphs on n vertices for $n \equiv 1 \pmod 3$ and for $n \equiv 0 \pmod 6$. Ramane, Revankar, Gutman and Walikar [22] proved that if G_1 and G_2 are r-regular graphs on n vertices and $diam(G_i) \leq 2$, $i = 1, 2$, then $E_D(L^k(G_1)) = E_D(L^k(G_2))$ for $k \geq 1$, where $L^k(G)$ is the k-th iterated line graph of G.

In this paper we obtain the characteristic polynomial of the distance matrix of the join of two regular graphs whose diameter is less than or equal to 2 and thereby construct pairs of non D-cospectral, D-equienergetic graphs on n vertices for all $n \geq 9$.

ON THE JOIN OF GRAPHS

Definition [6, 12]. The join of two graphs G_1 and G_2, denoted by $G_1 \nabla G_2$, is a graph obtained from G_1 and G_2 by joining each vertex of G_1 to all vertices of G_2.

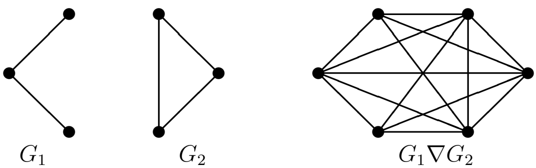


Fig. 1

Theorem 1. Let G_i be an r_i-regular graph on n_i vertices and $diam(G_i) \leq 2$, $i = 1, 2$. Then the characteristic polynomial of the distance matrix of $G_1 \nabla G_2$ is

$$\psi(G_1 \nabla G_2 : \mu) = \frac{[(\mu - 2n_1 + 2 + r_1)(\mu - 2n_2 + 2 + r_2) - n_1\, n_2]}{(\mu - 2n_1 + 2 + r_1)(\mu - 2n_2 + 2 + r_2)} \psi(G_1 : \mu) \psi(G_2 : \mu) \, . \tag{3}$$

Proof.

$$\begin{aligned} \psi(G_1 \nabla G_2 : \mu) &= \det(\mu I - D(G_1 \nabla G_2)) \\ &= \begin{vmatrix} \mu I_{n_1} - D(G_1) & -J_{n_1 \times n_2} \\ -J_{n_2 \times n_1} & \mu I_{n_2} - D(G_2) \end{vmatrix} \end{aligned} \tag{4}$$

where J is a matrix whose all entries are equal to unity.

The determinant (4) can be written as

$$\begin{vmatrix} \mu & -d_{12} & \dots & -d_{1n_1} & -1 & -1 & \dots & -1 \\ -d_{21} & \mu & \dots & -d_{2n_1} & -1 & -1 & \dots & -1 \\ \vdots & & \vdots & & & & \vdots & \\ -d_{n_11} & -d_{n_12} & \dots & \mu & -1 & -1 & \dots & -1 \\ -1 & -1 & \dots & -1 & \mu & -d'_{12} & \dots & -d'_{1n_2} \\ -1 & -1 & \dots & -1 & -d'_{21} & \mu & \dots & -d'_{2n_2} \\ \vdots & & \vdots & & & & \vdots & \\ -1 & -1 & \dots & -1 & -d'_{n_21} & -d'_{n_22} & \dots & \mu \end{vmatrix} \tag{5}$$

where d_{ij} is the distance between the vertices v_i and v_j in G_1 and d'_{ij} is the distance between the vertices u_i and u_j in G_2. In G_i, every vertex is at distance one from r_i vertices and at distance two from remaining $n_i - 1 - r_i$ vertices. Therefore

$$\sum_{j=1}^{n_1} d_{ij} = 2n_1 - r_1 - 2 \qquad \text{for} \quad i = 1, 2, \dots, n_1 \tag{6}$$

and

$$\sum_{j=1}^{n_2} d'_{ij} = 2n_2 - r_2 - 2 \qquad \text{for} \quad i = 1, 2, \dots, n_2 \ . \tag{7}$$

We now perform the number of transformations that leave the value of the determinant (5) unchanged.

Subtract the row (n_1+1) from the rows $(n_1+2), (n_1+3), \dots, (n_1+n_2)$ of (5) to obtain (8):

$$\begin{vmatrix} \mu & -d_{12} & \dots & -d_{1n_1} & -1 & -1 & \dots & -1 \\ -d_{21} & \mu & \dots & -d_{2n_1} & -1 & -1 & \dots & -1 \\ \vdots & & \vdots & & & & \vdots & \\ -d_{n_11} & -d_{n_12} & \dots & \mu & -1 & -1 & \dots & -1 \\ -1 & -1 & \dots & -1 & \mu & -d'_{12} & \dots & -d'_{1n_2} \\ 0 & 0 & \dots & 0 & -d'_{21} - \mu & \mu + d'_{12} & \dots & -d'_{2n_2} + d'_{1n_2} \\ \vdots & & \vdots & & & & \vdots & \\ 0 & 0 & \dots & 0 & -d'_{n_21} - \mu & -d'_{n_22} + d'_{12} & \dots & \mu + d'_{1n_2} \end{vmatrix} . \tag{8}$$

Adding the columns $(n_1+2), (n_1+3), \dots, (n_1+n_2)$ to the column (n_1+1) of (8), using Eq. (7), and noting that $d'_{ij} = d'_{ji}$ we arrive at the determinant (9):

$$\begin{vmatrix} \mu & -d_{12} & \dots & -d_{1n_1} & -n_2 & -1 & \dots & -1 \\ -d_{21} & \mu & \dots & -d_{2n_1} & -n_2 & -1 & \dots & -1 \\ \vdots & & \vdots & & & & \vdots & \\ -d_{n_11} & -d_{n_12} & \dots & \mu & -n_2 & -1 & \dots & -1 \\ -1 & -1 & \dots & -1 & \mu - 2n_2 + 2 + r_2 & -d'_{12} & \dots & -d'_{1n_2} \\ 0 & 0 & \dots & 0 & 0 & \mu + d'_{12} & \dots & -d'_{2n_2} + d'_{1n_2} \\ \vdots & & \vdots & & & & \vdots & \\ 0 & 0 & \dots & 0 & 0 & -d'_{n_22} + d'_{12} & \dots & \mu + d'_{1n_2} \end{vmatrix} \quad (9)$$

which evidently is equal to (10):

$$\begin{vmatrix} \mu & -d_{12} & \dots & -d_{1n_1} & -n_2 \\ -d_{21} & \mu & \dots & -d_{2n_1} & -n_2 \\ \vdots & & \vdots & & \\ -d_{n_11} & -d_{n_12} & \dots & \mu & -n_2 \\ -1 & -1 & \dots & -1 & \mu - 2n_2 + 2 + r_2 \end{vmatrix} |B| \quad (10)$$

where

$$|B| = \begin{vmatrix} \mu + d'_{12} & -d'_{23} + d'_{13} & \dots & -d'_{2n_2} + d'_{1n_2} \\ -d'_{32} + d'_{12} & \mu + d'_{13} & \dots & -d'_{3n_2} + d'_{1n_2} \\ \vdots & & \vdots & \\ -d'_{n_22} + d'_{12} & -d'_{n_23} + d'_{13} & \dots & \mu + d'_{1n_2} \end{vmatrix} . \quad (11)$$

In (10) the determinant is of order $(n_1 + 1)$. Subtract the first row from the rows $2, 3, \dots, n_1$, to obtain (12):

$$\begin{vmatrix} \mu & -d_{12} & \dots & -d_{1n_1} & -n_2 \\ -d_{21} - \mu & \mu + d_{12} & \dots & -d_{2n_1} + d_{1n_1} & 0 \\ \vdots & & \vdots & & \\ -d_{n_11} - \mu & -d_{n_12} + d_{12} & \dots & \mu + d_{1n_1} & 0 \\ -1 & -1 & \dots & -1 & \mu - 2n_2 + 2 + r_2 \ . \end{vmatrix} |B| \ . \quad (12)$$

Adding columns $2, 3, \dots, n_1$ to the first column of (12) and using Eq. (6) we get (13):

$$\begin{vmatrix} \mu - 2n_1 + 2 + r_1 & -d_{12} & \dots & -d_{1n_1} & -n_2 \\ 0 & \mu + d_{12} & \dots & -d_{2n_1} + d_{1n_1} & 0 \\ \vdots & & \vdots & & \\ 0 & -d_{n_12} + d_{12} & \dots & \mu + d_{1n_1} & 0 \\ -n_1 & -1 & \dots & -1 & \mu - 2n_2 + 2 + r_2 \end{vmatrix} |B| \ . \quad (13)$$

Expand it along the first column to obtain (14):

$$\{(\mu - 2n_1 + 2 + r_1)\,\Delta_1 - (-1)^{n_1}\, n_1\, \Delta_2\}\,|B| \quad (14)$$

where

$$\Delta_1 := \begin{vmatrix} \mu + d_{12} & -d_{23} + d_{13} & \dots & -d_{2n_1} + d_{1n_1} & 0 \\ -d_{32} + d_{12} & \mu + d_{13} & \dots & -d_{3n_1} + d_{1n_1} & 0 \\ \vdots & & \vdots & & \\ -d_{n_1 2} + d_{12} & -d_{n_1 3} + d_{13} & \dots & \mu + d_{1n_1} & 0 \\ -1 & -1 & \dots & -1 & \mu - 2n_2 + 2 + r_2 \end{vmatrix}$$

and

$$\Delta_2 = \begin{vmatrix} -d_{12} & -d_{13} & \dots & -d_{1n_1} & -n_2 \\ \mu + d_{12} & -d_{23} + d_{13} & \dots & -d_{2n_1} + d_{1n_1} & 0 \\ -d_{32} + d_{12} & \mu + d_{13} & \dots & -d_{3n_1} + d_{1n_1} & 0 \\ \vdots & & \vdots & & \\ -d_{n_1 2} + d_{12} & -d_{n_1 3} + d_{13} & \dots & \mu + d_{1n_1} & 0 \end{vmatrix} .$$

The expression (14) can be rewritten as

$$\begin{aligned} & \{(\mu - 2n_1 + 2 + r_1)(\mu - 2n_2 + 2 + r_2)|A| - n_1 n_2 |A|\}|B| \\ = \; & |A||B|\{(\mu - 2n_1 + 2 + r_1)(\mu - 2n_2 + 2 + r_2) - n_1 n_2\} \end{aligned} \tag{15}$$

where

$$|A| = \begin{vmatrix} \mu + d_{12} & -d_{23} + d_{13} & \dots & -d_{2n_1} + d_{1n_1} \\ -d_{32} + d_{12} & \mu + d_{13} & \dots & -d_{3n_1} + d_{1n_1} \\ \vdots & & \vdots & \\ -d_{n_1 2} + d_{12} & -d_{n_1 3} + d_{13} & \dots & \mu + d_{1n_1} \end{vmatrix} . \tag{16}$$

The determinant (16) can be written as

$$\begin{aligned} |A| \; &= \; \frac{1}{(\mu - 2n_1 + 2 + r_1)} \times \\ & \times \begin{vmatrix} \mu - 2n_1 + 2 + r_1 & -d_{12} & -d_{13} & \dots & -d_{1n_1} \\ 0 & \mu + d_{12} & -d_{23} + d_{13} & \dots & -d_{2n_1} + d_{1n_1} \\ 0 & -d_{32} + d_{12} & \mu + d_{13} & \dots & -d_{3n_1} + d_{1n_1} \\ \vdots & & & \vdots & \\ 0 & -d_{n_1 2} + d_{12} & -d_{n_1 3} + d_{13} & \dots & \mu + d_{1n_1} \end{vmatrix} . \end{aligned} \tag{17}$$

From Eq. (6) the sum of the i-th row in (17) is $\mu + d_{i1}$ for $i = 2, 3, \dots, n_1$. Therefore, by subtracting the columns $2, 3, \dots, n_1$ of (17) from the first column, we obtain (18):

$$|A| = \frac{1}{(\mu - 2n_1 + 2 + r_1)} \times$$

$$\times \begin{vmatrix} \mu & -d_{12} & -d_{13} & \dots & -d_{1n_1} \\ -\mu - d_{21} & \mu + d_{12} & -d_{23} + d_{13} & \dots & -d_{2n_1} + d_{1n_1} \\ -\mu - d_{31} & -d_{32} + d_{12} & \mu + d_{13} & \dots & -d_{3n_1} + d_{1n_1} \\ \vdots & & & \vdots & \\ -\mu - d_{n_1 1} & -d_{n_1 2} + d_{12} & -d_{n_1 3} + d_{13} & \dots & \mu + d_{1n_1} \end{vmatrix} . \qquad (18)$$

Add the first row of (18) to the rows $2, 3, \dots, n_1$ to obtain (19):

$$|A| = \frac{1}{(\mu - 2n_1 + 2 + r_1)} \begin{vmatrix} \mu & -d_{12} & -d_{13} & \dots & -d_{1n_1} \\ -d_{21} & \mu & -d_{23} & \dots & -d_{2n_1} \\ -d_{31} & -d_{32} & \mu & \dots & -d_{3n_1} \\ \vdots & & & \vdots & \\ -d_{n_1 1} & -d_{n_1 2} & -d_{n_1 3} & \dots & \mu \end{vmatrix}$$

$$= \frac{1}{(\mu - 2n_1 + 2 + r_1)} \psi(G_1 : \mu) . \qquad (19)$$

In a similar manner we can show that from (11) follows

$$|B| = \frac{1}{(\mu - 2n_2 + 2 + r_2)} \psi(G_2 : \mu) . \qquad (20)$$

Substituting (19) and (20) back into (15) yields Eq. (3). ■

Theorem 2. Let G_i be an r_i-regular graph on n_i vertices and $diam(G_i) \leq 2$, $i = 1, 2$. Then

$$\begin{aligned} E_D(G_1 \nabla G_2) &= E_D(G_1) + E_D(G_2), \qquad \text{if} \quad XY \geq n_1 n_2 \\ &= E_D(G_1) + E_D(G_2) - (X + Y) + \sqrt{(X+Y)^2 - 4(XY - n_1 n_2)}, \\ & \qquad \qquad \text{if} \quad XY < n_1 n_2 \end{aligned}$$

where $X = 2n_1 - 2 - r_1$ and $Y = 2n_2 - 2 - r_2$.

Proof. If G_i is an r_i-regular graph on n_i vertices and $diam(G_i) \leq 2$, $i = 1, 2$, then from Theorem 1,

$$\begin{aligned} \psi(G_1 \nabla G_2 : \mu) &= \frac{[(\mu - 2n_1 + 2 + r_1)(\mu - 2n_2 + 2 + r_2) - n_1 n_2]}{(\mu - 2n_1 + 2 + r_1)(\mu - 2n_2 + 2 + r_2)} \psi(G_1 : \mu) \psi(G_2 : \mu) \\ &= \frac{[(\mu - X)(\mu - Y) - n_1 n_2]}{(\mu - X)(\mu - Y)} \psi(G_1 : \mu) \psi(G_2 : \mu) \end{aligned}$$

which gives that

$$(\mu - X)(\mu - Y)\psi(G_1 \nabla G_2 : \mu) = (\mu - X)(\mu - Y)\psi(G_1 : \mu)\psi(G_2 : \mu) \ .$$

Let

$$P_1(\mu) = (\mu - X)(\mu - Y)\,\psi(G_1 \nabla G_2 : \mu)$$

and

$$P_2(\mu) = (\mu - X)(\mu - Y)\,\psi(G_1 : \mu)\,\psi(G_2 : \mu) \ .$$

The roots of the equation $P_1(\mu) = 0$ are X, Y and the D-eigenvalues of $G_1 \nabla G_2$. Therefore the sum of the absolute values of the roots of $P_1(\mu) = 0$ is

$$X + Y + E_D(G_1 \nabla G_2) \ . \tag{21}$$

The roots of $P_2(\mu) = 0$ are the D-eigenvalues of G_1 and G_2 and

$$\frac{1}{2}\left(X + Y \pm \sqrt{(X+Y)^2 - 4(XY - n_1\,n_2)}\right) \ .$$

Therefore the sum of the absolute values of the roots of $P_2(\mu) = 0$ is

$$\begin{aligned} E_D(G_1) + E_D(G_2) \quad &+ \quad \left|\frac{1}{2}\left[X + Y + \sqrt{(X+Y)^2 - 4(XY - n_1\,n_2)}\right]\right| \\ &+ \quad \left|\frac{1}{2}\left[X + Y - \sqrt{(X+Y)^2 - 4(XY - n_1\,n_2)}\right]\right| \ . \end{aligned} \tag{22}$$

Since $P_1(\mu) = P_2(\mu)$, equating Eqs. (21) and (22) we get

$$\begin{aligned} E_D(G_1 \nabla G_2) \quad &= \quad E_D(G_1) + E_D(G_2) - (X + Y) \\ &+ \quad \left|\frac{1}{2}\left[X + Y + \sqrt{(X+Y)^2 - 4(XY - n_1\,n_2)}\right]\right| \\ &+ \quad \left|\frac{1}{2}\left[X + Y - \sqrt{(X+Y)^2 - 4(XY - n_1\,n_2)}\right]\right| \ . \end{aligned} \tag{23}$$

<u>Case 1:</u> If $XY \geq n_1\,n_2$, then Eq. (23) reduces to

$$E_D(G_1 \nabla G_2) = E_D(G_1) + E_D(G_2) \ .$$

<u>Case 2:</u> If $XY < n_1\,n_2$, then Eq. (23) reduces to

$$E_D(G_1 \nabla G_2) = E_D(G_1) + E_D(G_2) - (X + Y) + \sqrt{(X+Y)^2 - 4(XY - n_1\,n_2)} \ .$$

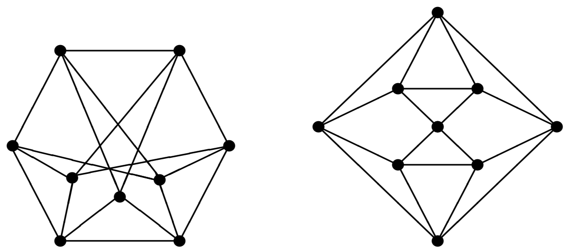
This completes the proof. ■

Corollary 2.1. If H_1 and H_2 are non D-cospectral, D-equienergetic regular graphs on n vertices and of same degree and $diam(H_i) \leq 2$, $i = 1, 2$, then for any regular graph G with $diam(G) \leq 2$, $E_D(H_1 \nabla G) = E_D(H_2 \nabla G)$. ■

CONSTRUCTION OF DISTANCE EQUIENERGETIC GRAPHS

Theorem 3. There exist pairs of non D-cospectral, D-equienergetic graphs on n vertices for all $n \geq 9$.

Proof. Consider the graphs H_a and H_b as shown in Fig. 2.



H_a H_b

Fig. 2

By direct computation,

$$\psi(H_a : \mu) = (\mu - 12)(\mu + 3)^4 \, \mu^4 \tag{24}$$

and

$$\psi(H_b : \mu) = (\mu - 12)(\mu + 4)(\mu + 3)^2 \, (\mu + 1)^2 \, \mu^3 \, . \tag{25}$$

Both H_a and H_b are regular graphs on 9 vertices and of degree 4. Also $diam(H_i) \leq 2$, $i = a, b$, and $E_D(H_a) = 24 = E_D(H_b)$.

Let H be any r-regular graph on $p \geq 1$ vertices and $diam(H) \leq 2$. Then by Theorem 2

$$E_D(H_a \nabla H) = E_D(H_b \nabla H) = 24 + E_D(H) \qquad \text{if } \; 5p \geq 8 + 4r$$

and

$$E_D(H_a \nabla H) = E_D(H_b \nabla H) = E_D(H) + 14 - 2p + r$$
$$+ \sqrt{(10+2p-r)^2 - 12(5p-8-4r)} \qquad \text{if } \ 5p < 8+4r \ .$$

Thus from both cases, $H_a \nabla H$ and $H_b \nabla H$ are D-equienergetic. By Eqs. (24) and (25), H_a and H_b are non D-cospectral, so from Theorem 1, $H_a \nabla H$ and $H_b \nabla H$ are also non D-cospectral. Further $H_a \nabla H$ and $H_b \nabla H$ possesses equal number of vertices $n = 9 + p$, $p = 1, 2, \ldots$.

That the theorem holds also for $n = 9$ is directly verified from Eqs. (24) and (25). ■

Let K_p be the complete graph on p vertices. It is regular of degree $p-1$ and $diam(K_p) = 1$. The adjacency matrix and the distance matrix of K_p are same. Therefore $E_D(K_p) = E(K_p) = 2(p-1)$ [6,10]. Using this in Theorem 2 we have following result.

Theorem 4. If H_a and H_b are the graphs as shown in Fig. 2, then

$$E_D(H_a \nabla K_p) = E_D(H_b \nabla K_p) = 2(p+11) \qquad \text{if } \ p \geq 4$$

and

$$E_D(H_a \nabla K_p) = E_D(H_b \nabla K_p) = p + 11 + \sqrt{(p+11)^2 - 12(p-4)} \qquad \text{if } p < 4 \ .$$ ■

Conclusion. From Corollary 2.1 it is easy to construct a pair of non D-cospectral, D-equienergetic graphs. In particular from Theorem 3 and Theorem 4, it is easy to construct a pair of non D-cospectral, D-equienergetic n-vertex graphs for all $n \geq 9$.

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