The genetic reactions of cyclopropane. Part I

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(Received November 6, 2006)

1. The groups of cyclopropane

In this paper we shall use freely the terminology and notation from [2]. In accord to [4, V], or [1, Corollary 5.1.3], the group \( G \leq S_6 \) of univalent substitution isomerism of cyclopropane \( \text{C}_3\text{H}_6 \) coincides up to conjugacy with the group

\[
\langle (123)(456), (14)(26)(35) \rangle,
\]

which is isomorphic to the dihedral group of order 6. Since there are chiral pairs among the derivatives of cyclopropane, the group \( G' \leq S_6 \) of stereoisomerism of cyclopropane contains \( G \) and has order 12, so it coincides up to conjugacy with the group

\[
\langle (123)(456), (14)(26)(35), (14)(25)(36) \rangle
\]

that is isomorphic to the dihedral group of order 12 — see [4, V], or [1, Corollary 5.1.4].

The structural formula (graph) of cyclopropane

![Structural formula of cyclopropane](image-url)
yields that its group $G'' \leq S_6$ of structural isomerism, up to conjugacy, coincides with the group 
\[ ((123)(456), (14)(26)(35), (14)) \]
of order 48. Moreover, $G \leq G' \leq G''$.

2. The isomers of cyclopropane and their substitution reactions

Below, for any empirical formula $\lambda \in P_6$, we list the corresponding products of cyclopropane as well as the genetic reactions among them.

Case 1. $\lambda = (6)$.
We have
\[ T_{(6);G} = T_{(6);G'} = T_{(6);G''} = \{a_{(6)}\}, \]
where $a_{(6)}$ is the only $G$- and at the same time $G'$- and $G''$-orbit of the tabloid $A^{(6)} = (\{1, 2, 3, 4, 5, 6\})$. The orbit $a_{(6)}$ represents the parent molecule of cyclopropane.

Case 2. $\lambda = (5, 1)$.
The transitivity of the group $G$ yields
\[ T_{(5,1);G} = T_{(5,1);G'} = T_{(5,1);G''} = \{a_{(5,1)}\}, \]
where $a_{(5,1)}$ is the only $G$- and at the same time $G'$- and $G''$-orbit of the tabloid $A^{(5,1)} = (\{1, 2, 3, 4, 5\}, \{6\})$.
The only possible substitution reaction between the parent substance of cyclopropane and its mono-substitution derivative is designated $a_{(5,1)} < a_{(6)}$, because $R_{1,6}A^{(5,1)} = A^{(6)}$, and hence $A^{(5,1)} < A^{(6)}$. We remind that the operation $R_{1,6}$ applied on the tabloid $A^{(5,1)}$ means “replace the ligand of type $x_2$ in position 6 by a ligand of type $x_1$”. The converse operation “replace the ligand of type $x_1$ in position 6 by a ligand of type $x_2$” represents the simple substitution reaction
\[ A^{(6)} \rightarrow A^{(5,1)}. \]

Case 3. $\lambda = (4, 2)$.
We have
\[ T_{(4,2);G} = \{a_{(4,2)}, b_{(4,2)}, c_{(4,2)}, e_{(4,2)}\}, \]
where:
- $a_{(4,2)}$ is the $G$-orbit of the tabloid $A^{(4,2)} = (\{1, 2, 3, 4\}, \{5, 6\})$,
- $b_{(4,2)}$ is the $G$-orbit of the tabloid $B^{(4,2)} = (\{1, 2, 4, 5\}, \{3, 6\})$,
- $c_{(4,2)}$ is the $G$-orbit of the tabloid $C^{(4,2)} = (\{1, 2, 4, 6\}, \{3, 5\})$,
- $e_{(4,2)}$ is the $G$-orbit of the tabloid $E^{(4,2)} = (\{1, 3, 4, 5\}, \{2, 6\})$.
Below are all inequalities between the structural formulae of di-substitution homogeneous derivatives and the structural formula of the mono-substitution derivative of cyclopropane. We have
\[ A^{(4,2)} < A^{(5,1)}, \ B^{(4,2)} < A^{(5,1)}, \]
(123)(456)C^{(4,2)} < A^{(5,1)}, E^{(4,2)} < A^{(5,1)},

because

\[ R_{1,5}A^{(4,2)} = R_{1,3}B^{(4,2)} = R_{1,1}(123)(456)C^{(4,2)} = R_{1,2}E^{(4,2)} = A^{(5,1)}. \]

Thus, we obtain the following substitution reactions

\[ A^{(5,1)} \longrightarrow A^{(4,2)}, A^{(5,1)} \longrightarrow B^{(4,2)}, \]
\[ A^{(5,1)} \longrightarrow (123)(456)C^{(4,2)}, A^{(5,1)} \longrightarrow E^{(4,2)}, \]

which mean “replace the ligand of type \( x_1 \) in position 5 of the tabloid \( A^{(5,1)} \) by a ligand of type \( x_2 \)”, “replace the ligand of type \( x_1 \) in position 3 of the tabloid \( A^{(5,1)} \) by a ligand of type \( x_2 \)”, “replace the ligand of type \( x_1 \) in position 1 of the tabloid \( A^{(5,1)} \) by a ligand of type \( x_2 \)”, and, “replace the ligand of type \( x_1 \) in position 2 of the tabloid \( A^{(5,1)} \) by a ligand of type \( x_2 \)”, respectively.

These simple substitution reactions are also designated by the inequalities

\[ a_{(4,2)} < a_{(5,1)}, \quad b_{(4,2)} < a_{(5,1)}, \quad c_{(4,2)} < a_{(5,1)}, \quad e_{(4,2)} < a_{(5,1)}. \]

The set of \( G' \)-orbits in \( T_{(4,2)} \) is

\[ T_{(4,2);G'} = \{ a_{(4,2)}, b_{(4,2)}, c_{(4,2)} \cup e_{(4,2)} \}, \]

so the \((4,2)\)-products that correspond to \( c_{(4,2)} \) and \( e_{(4,2)} \) are members of a chiral pair.

The set of \( G'' \)-orbits in \( T_{(4,2)} \) is

\[ T_{(4,2);G''} = \{ b_{(4,2)}, a_{(4,2)} \cup (c_{(4,2)} \cup e_{(4,2)}) \}, \]

hence the products that correspond to \( a_{(4,2)}, c_{(4,2)} \), and \( e_{(4,2)} \) are structurally identical, and any one of them is structurally isomorphic with the product which corresponds to \( b_{(4,2)} \).

Case 4. \( \lambda = (4,1^2) \).

In this case we have

\[ T_{(4,1^2);G} = \{ a_{(4,1^2)}, b_{(4,1^2)}, c_{(4,1^2)}, e_{(4,1^2)}, f_{(4,1^2)} \}, \]

where:

\( a_{(4,1^2)} \) is the \( G \)-orbit of the tabloid \( A^{(4,1^2)} = \{ 1,2,3,4 \}, \{ 5 \}, \{ 6 \} \),
\( b_{(4,1^2)} \) is the \( G \)-orbit of the tabloid \( B^{(4,1^2)} = \{ 1,2,3,4 \}, \{ 6 \}, \{ 5 \} \),
\( c_{(4,1^2)} \) is the \( G \)-orbit of the tabloid \( C^{(4,1^2)} = \{ 1,2,4,5 \}, \{ 3 \}, \{ 6 \} \),
\( e_{(4,1^2)} \) is the \( G \)-orbit of the tabloid \( E^{(4,1^2)} = \{ 1,2,4,6 \}, \{ 3 \}, \{ 5 \} \),
\( f_{(4,1^2)} \) is the \( G \)-orbit of the tabloid \( F^{(4,1^2)} = \{ 1,3,4,5 \}, \{ 2 \}, \{ 6 \} \).

The following inequalities hold between the di-substitution homogeneous and the di-substitution heterogeneous derivatives of cyclopropane:

\[ A^{(4,1^2)} < A^{(4,2)}, B^{(4,1^2)} < A^{(4,2)}, \]
Indeed,
\[ R_{2,6}A^{(4,1^2)} = R_{2,5}B^{(4,1^2)} = A^{(4,2)}, \]
\[ R_{2,6}C^{(4,1^2)} = B^{(4,2)}, \quad R_{2,5}E^{(4,1^2)} = C^{(4,2)}, \quad R_{2,6}F^{(4,1^2)} = E^{(4,2)}. \]

In this way we obtain the following substitution reactions
\[ A^{(4,2)} \longrightarrow A^{(4,1^2)}, \quad A^{(4,2)} \longrightarrow B^{(4,1^2)}, \]
\[ B^{(4,2)} \longrightarrow C^{(4,1^2)}, \quad C^{(4,2)} \longrightarrow E^{(4,1^2)}, \quad E^{(4,2)} \longrightarrow F^{(4,1^2)}, \]
which mean “replace the ligand of type \( x_2 \) in position 6 of the tabloid \( A^{(4,2)} \) by a ligand of type \( x_3 \)”, “replace the ligand of type \( x_2 \) in position 5 of the tabloid \( A^{(4,2)} \) by a ligand of type \( x_3 \)”, “replace the ligand of type \( x_2 \) in position 5 of the tabloid \( B^{(4,2)} \) by a ligand of type \( x_3 \)”, “replace the ligand of type \( x_2 \) in position 5 of the tabloid \( C^{(4,2)} \) by a ligand of type \( x_3 \)”, and, “replace the ligand of type \( x_2 \) in position 6 of the tabloid \( E^{(4,2)} \) by a ligand of type \( x_3 \)”, respectively. In terms of inequalities these substitution reactions can be represented as follows:
\[ a_{(4,1^2)} < a_{(4,2)}, \quad b_{(4,1^2)} < a_{(4,2)}, \]
\[ c_{(4,1^2)} < b_{(4,2)}, \quad e_{(4,1^2)} < c_{(4,2)}, \quad f_{(4,1^2)} < e_{(4,2)}. \]

Further, we obtain
\[ T_{(4,1^2);G'} = \{ a_{(4,1^2)} \cup b_{(4,1^2)}, c_{(4,1^2)}, e_{(4,1^2)} \cup f_{(4,1^2)} \}, \]
and therefore the products that correspond to the members of any one of the sets \( \{ a_{(4,1^2)}, b_{(4,1^2)} \} \), and \( \{ e_{(4,1^2)}, f_{(4,1^2)} \} \) form a chiral pair, and the product that corresponds to \( c_{(4,1^2)} \) is a dimer. Moreover,
\[ T_{(4,1^2);G''} = \{ (a_{(4,1^2)} \cup b_{(4,1^2)}) \cup (e_{(4,1^2)} \cup f_{(4,1^2)}), c_{(4,1^2)} \}. \]

Hence the four members of the above two chiral pairs are structurally identical, and each one of them is structurally isomeric to the product that corresponds to the dimer \( c_{(4,1^2)} \).

Case 5. \( \lambda = (3^2) \).

Now we have
\[ T_{(3^2);G} = \{ a_{(3^2)}, b_{(3^2)}, c_{(3^2)}, e_{(3^2)} \}, \]
where:
\[ a_{(3^2)} \] is the \( G \)-orbit of the tabloid \( A^{(3^2)} = (\{ 1, 2, 3 \}, \{ 4, 5, 6 \}) \),
\[ b_{(3^2)} \] is the \( G \)-orbit of the tabloid \( B^{(3^2)} = (\{ 1, 2, 4 \}, \{ 3, 5, 6 \}) \),
\[ c_{(3^2)} \] is the \( G \)-orbit of the tabloid \( C^{(3^2)} = (\{ 1, 2, 5 \}, \{ 3, 4, 6 \}) \),
\[ e_{(3^2)} \] is the \( G \)-orbit of the tabloid \( E^{(3^2)} = (\{ 1, 2, 6 \}, \{ 3, 4, 5 \}) \).
We have the following inequalities between the tabloids of shape \((3^2)\) and the tabloids of shape \((4,2)\):

\[
A^{(3^2)} < A^{(4,2)}, \quad B^{(3^2)} < A^{(4,2)}, \quad (132)(465)C^{(3^2)} < A^{(4,2)}, \quad (123)(456)E^{(3^2)} < A^{(4,2)},
\]

\[
B^{(3^2)} < B^{(4,2)}, \quad C^{(3^2)} < B^{(4,2)},
\]

\[
B^{(3^2)} < C^{(4,2)}, \quad E^{(3^2)} < C^{(4,2)},
\]

\[
(132)(465)C^{(3^2)} < E^{(4,2)}, \quad (132)(465)E^{(3^2)} < E^{(4,2)},
\]

because

\[
R_{1,4}A^{(3^2)} = R_{1,3}B^{(3^2)} = R_{1,2}(132)(465)C^{(3^2)} = R_{1,1}(123)(456)E^{(3^2)} = A^{(4,2)},
\]

\[
R_{1,5}B^{(3^2)} = R_{1,4}C^{(3^2)} = B^{(4,2)},
\]

\[
R_{1,6}B^{(3^2)} = R_{1,4}E^{(3^2)} = C^{(4,2)},
\]

\[
R_{1,5}(132)(465)C^{(3^2)} = R_{1,4}(132)(465)E^{(3^2)} = E^{(4,2)}.
\]

Thus, the substitution reactions among di-substitution homogeneous derivatives of cyclopropane and its tri-substitution homogeneous derivatives, can be represented as follows:

\[
a_{(3^2)} < a_{(4,2)}, \quad b_{(3^2)} < a_{(4,2)}, \quad c_{(3^2)} < a_{(4,2)}, \quad e_{(3^2)} < a_{(4,2)},
\]

\[
b_{(3^2)} < b_{(4,2)}, \quad c_{(3^2)} < b_{(4,2)},
\]

\[
b_{(3^2)} < c_{(4,2)}, \quad e_{(3^2)} < c_{(4,2)},
\]

\[
c_{(3^2)} < e_{(4,2)}, \quad e_{(3^2)} < e_{(4,2)}.
\]

The set of all \(G'\)-orbits is

\[
T_{(3^2),G'} = \{a_{(3^2)}, b_{(3^2)} \cup c_{(3^2)}, e_{(3^2)}\},
\]

so the products that correspond to the members of the set \(\{b_{(3^2)}, c_{(3^2)}\}\) form a chiral pair, and the products that correspond to \(a_{(3^2)}\) and \(e_{(3^2)}\) are dimers.

The set of all \(G''\)-orbits is

\[
T_{(3^2),G''} = \{a_{(3^2)} \cup e_{(3^2)}, (b_{(3^2)} \cup c_{(3^2)})\},
\]

and this yields structural identity of the dimers which correspond to \(a_{(3^2)}\) and \(e_{(3^2)}\), and each one of them is structurally isomeric to any member of the above chiral pair.

Case 6. \(\lambda = (3, 2, 1)\).

We have

\[
T_{(3,2,1),G} = \{a_{(3,2,1)}, b_{(3,2,1)}, c_{(3,2,1)}, e_{(3,2,1)}, f_{(3,2,1)}, h_{(3,2,1)}, k_{(3,2,1)}, \ell_{(3,2,1)}, m_{(3,2,1)}, p_{(3,2,1)}\}
\]

where:
The inequalities between the tabloids of shape (3, 2, 1) and the tabloids of shape (4, 1^2) and (3^2), respectively, are as follows:

\[ A^{(3, 2, 1)} < A^{(4, 1^2)}, \quad B^{(3, 2, 1)} < A^{(4, 1^2)}, \]

\[ (132)(465)H^{(3, 2, 1)} < A^{(4, 1^2)}, \quad (123)(456)L^{(3, 2, 1)} < A^{(4, 1^2)}, \]

\[ (132)(465)A^{(3, 2, 1)} < B^{(4, 1^2)}, \quad C^{(3, 2, 1)} < B^{(4, 1^2)}, \]

\[ (132)(465)F^{(3, 2, 1)} < B^{(4, 1^2)}, \quad (123)(456)M^{(3, 2, 1)} < B^{(4, 1^2)}, \]

\[ B^{(3, 2, 1)} < C^{(4, 1^2)}, \quad (15)(24)(36)E^{(3, 2, 1)} < C^{(4, 1^2)}, \]

\[ F^{(3, 2, 1)} < C^{(4, 1^2)}, \quad (15)(24)(36)K^{(3, 2, 1)} < C^{(4, 1^2)}, \]

\[ C^{(3, 2, 1)} < E^{(4, 1^2)}, \quad (14)(26)(35)E^{(3, 2, 1)} < E^{(4, 1^2)}, \]

\[ L^{(3, 2, 1)} < E^{(4, 1^2)}, \quad (14)(26)(35)P^{(3, 2, 1)} < E^{(4, 1^2)}, \]

\[ (132)(465)H^{(3, 2, 1)} < F^{(4, 1^2)}, \quad (15)(24)(36)K^{(3, 2, 1)} < F^{(4, 1^2)}, \]

\[ (132)(465)M^{(3, 2, 1)} < F^{(4, 1^2)}, \quad (15)(24)(36)P^{(3, 2, 1)} < F^{(4, 1^2)}, \]

and

\[ A^{(3, 2, 1)} < A^{(3^2)}, \]

\[ B^{(3, 2, 1)} < B^{(3^2)}, \quad C^{(3, 2, 1)} < B^{(3^2)}, \quad E^{(3, 2, 1)} < B^{(3^2)}, \]

\[ F^{(3, 2, 1)} < C^{(3^2)}, \quad H^{(3, 2, 1)} < C^{(3^2)}, \quad K^{(3, 2, 1)} < C^{(3^2)}, \]

\[ L^{(3, 2, 1)} < E^{(3^2)}, \quad M^{(3, 2, 1)} < E^{(3^2)}, \quad P^{(3, 2, 1)} < E^{(3^2)}, \]

because

\[ R_{1, 4}A^{(3, 2, 1)} = R_{1, 3}B^{(3, 2, 1)} = R_{1, 2}(132)(465)H^{(3, 2, 1)} = R_{1, 1}(123)(456)L^{(3, 2, 1)} = A^{(4, 1^2)}, \]

\[ R_{1, 4}(132)(465)A^{(3, 2, 1)} = R_{1, 3}C^{(3, 2, 1)} = R_{1, 2}(132)(465)F^{(3, 2, 1)} = \]

\[ R_{1, 1}(123)(456)(36)M^{(3, 2, 1)} = B^{(4, 1^2)}, \]

\[ R_{1, 5}B^{(3, 2, 1)} = R_{1, 1}(15)(24)(36)E^{(3, 2, 1)} = R_{1, 4}F^{(3, 2, 1)} = \]
\[ R_{1,2}(15)(24)(36)K^{(3,2,1)} = C^{(4,1^2)}, \]
\[ R_{1,6}C^{(3,2,1)} = R_{1,2}(14)(26)(35)E^{(3,2,1)} = R_{1,4}L^{(3,2,1)} = \]
\[ R_{1,1}(14)(26)(35)P^{(3,2,1)} = E^{(4,1^2)}, \]
\[ R_{1,5}(132)(465)H^{(3,2,1)} = R_{1,3}(15)(24)(36)K^{(3,2,1)} = R_{1,4}(132)(465)M^{(3,2,1)} = \]
\[ R_{1,1}(15)(24)(36)P^{(3,2,1)} = F^{(4,1^2)}, \]
and
\[ R_{2,6}A^{(3,2,1)} = A^{(3^2)}, \]
\[ R_{2,6}B^{(3,2,1)} = R_{2,5}C^{(3,2,1)} = R_{2,3}E^{(3,2,1)} = B^{(3^2)}, \]
\[ R_{2,6}P^{(3,2,1)} = R_{2,4}H^{(3,2,1)} = R_{2,3}K^{(3,2,1)} = C^{(3^2)}, \]
\[ R_{2,5}L^{(3,2,1)} = R_{2,4}M^{(3,2,1)} = R_{2,3}P^{(3,2,1)} = E^{(3^2)}. \]

Therefore all simple substitution reactions between \((3, 2, 1)\)-derivatives of cyclopropane and its \((4, 1^2)\)-derivatives and \((3^2)\)-derivatives, respectively, are:

\[
a_{(3,2,1)} < a_{(4,1^2)}, \quad b_{(3,2,1)} < a_{(4,1^2)},
\]
\[
h_{(3,2,1)} < a_{(4,1^2)}, \quad \ell_{(3,2,1)} < a_{(4,1^2)},
\]
\[
a_{(3,2,1)} < b_{(4,1^2)}, \quad c_{(3,2,1)} < b_{(4,1^2)},
\]
\[
f_{(3,2,1)} < b_{(4,1^2)}, \quad m_{(3,2,1)} < b_{(4,1^2)},
\]
\[
b_{(3,2,1)} < c_{(4,1^2)}, \quad e_{(3,2,1)} < c_{(4,1^2)},
\]
\[
f_{(3,2,1)} < c_{(4,1^2)}, \quad k_{(3,2,1)} < c_{(4,1^2)},
\]
\[
c_{(3,2,1)} < e_{(4,1^2)}, \quad e_{(3,2,1)} < e_{(4,1^2)},
\]
\[
\ell_{(3,2,1)} < e_{(4,1^2)}, \quad p_{(3,2,1)} < e_{(4,1^2)},
\]
\[
h_{(3,2,1)} < f_{(4,1^2)}, \quad k_{(3,2,1)} < f_{(4,1^2)},
\]
\[
m_{(3,2,1)} < f_{(4,1^2)}, \quad p_{(3,2,1)} < f_{(4,1^2)},
\]
and

\[
a_{(3,2,1)} < a_{(3^2)},
\]
\[
b_{(3,2,1)} < b_{(3^2)}, \quad c_{(3,2,1)} < b_{(3^2)}, \quad e_{(3,2,1)} < b_{(3^2)},
\]
\[
f_{(3,2,1)} < c_{(3^2)}, \quad h_{(3,2,1)} < c_{(3^2)}, \quad k_{(3,2,1)} < c_{(3^2)},
\]
\[
\ell_{(3,2,1)} < e_{(3^2)}, \quad m_{(3,2,1)} < e_{(3^2)}, \quad p_{(3,2,1)} < e_{(3^2)}.
\]

The set of \(G'\)-orbits in \(T_{(3,2,1)}\) is

\[
T_{(3,2,1);G'} = \{ a_{(3,2,1)}, b_{(3,2,1)} \cup f_{(3,2,1)}, c_{(3,2,1)} \cup h_{(3,2,1)}, e_{(3,2,1)} \cup k_{(3,2,1)}, \ell_{(3,2,1)} \cup m_{(3,2,1)}, p_{(3,2,1)} \}.\]
In particular, the products that correspond to the members of the two-element sets
\[ \{b_{(3,2,1)}, f_{(3,2,1)}\}, \{c_{(3,2,1)}, h_{(3,2,1)}\}, \{e_{(3,2,1)}, k_{(3,2,1)}\}, \{\ell_{(3,2,1)}, m_{(3,2,1)}\}, \]
form chiral pairs and the products that correspond to \(\alpha_{(3,2,1)}\) and \(p_{(3,2,1)}\) are dimers. Further, the set of \(G''\)-orbits in \(T_{(3,2,1)}\) is
\[ T_{(3,2,1); G''} = \{a_{(3,2,1)} \cup (\ell_{(3,2,1)} \cup m_{(3,2,1)}) \cup p_{(3,2,1)}, \]
\[ (b_{(3,2,1)} \cup f_{(3,2,1)}) \cup (e_{(3,2,1)} \cup k_{(3,2,1)}), (c_{(3,2,1)} \cup h_{(3,2,1)})\} \].

Hence the members of different sets below are structural isomers:
\[ \{a_{(3,2,1)}, \ell_{(3,2,1)}, m_{(3,2,1)}, p_{(3,2,1)}\}, \{b_{(3,2,1)}, f_{(3,2,1)}, e_{(3,2,1)}, k_{(3,2,1)}\}, \{c_{(3,2,1)}, h_{(3,2,1)}\}. \]

Case 7. \(\lambda = (3, 1^3)\).

Now, we have
\[ T_{(3,1^3); G} = \{a_{(3,1^3)}, \bar{a}_{(3,1^3)}, b_{(3,1^3)}, \bar{b}_{(3,1^3)}, c_{(3,1^3)}, \bar{c}_{(3,1^3)}, e_{(3,1^3)}, \bar{e}_{(3,1^3)}, f_{(3,1^3)}, \bar{f}_{(3,1^3)}, \]
\[ h_{(3,1^3)}, \bar{h}_{(3,1^3)}, k_{(3,1^3)}, \bar{k}_{(3,1^3)}, \ell_{(3,1^3)}, \bar{\ell}_{(3,1^3)}, m_{(3,1^3)}, \bar{m}_{(3,1^3)}, p_{(3,1^3)}, \bar{p}_{(3,1^3)}\}; \]
where:
\(a_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(A^{(3,1^3)} = (\{1, 2, 3\}, \{4\}, \{5\}, \{6\})\),
\(\bar{a}_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(\bar{A}^{(3,1^3)} = (\{1, 2, 3\}, \{4\}, \{6\}, \{5\})\),
\(b_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(B^{(3,1^3)} = (\{1, 2, 4\}, \{3\}, \{5\}, \{6\})\),
\(\bar{b}_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(\bar{B}^{(3,1^3)} = (\{1, 2, 4\}, \{3\}, \{6\}, \{5\})\),
\(c_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(C^{(3,1^3)} = (\{1, 2, 4\}, \{5\}, \{3\}, \{6\})\),
\(\bar{c}_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(\bar{C}^{(3,1^3)} = (\{1, 2, 4\}, \{5\}, \{6\}, \{3\})\),
\(e_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(E^{(3,1^3)} = (\{1, 2, 4\}, \{3\}, \{6\}, \{5\})\),
\(\bar{e}_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(\bar{E}^{(3,1^3)} = (\{1, 2, 4\}, \{6\}, \{5\}, \{3\})\),
\(f_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(F^{(3,1^3)} = (\{1, 2, 5\}, \{3\}, \{4\}, \{6\})\),
\(\bar{f}_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(\bar{F}^{(3,1^3)} = (\{1, 2, 5\}, \{3\}, \{6\}, \{4\})\),
\(h_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(H^{(3,1^3)} = (\{1, 2, 5\}, \{4\}, \{3\}, \{6\})\),
\(\bar{h}_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(\bar{H}^{(3,1^3)} = (\{1, 2, 5\}, \{4\}, \{6\}, \{3\})\),
\(k_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(K^{(3,1^3)} = (\{1, 2, 5\}, \{6\}, \{3\}, \{4\})\),
\(\bar{k}_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(\bar{K}^{(3,1^3)} = (\{1, 2, 5\}, \{6\}, \{4\}, \{3\})\),
\(\ell_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(L^{(3,1^3)} = (\{1, 2, 6\}, \{3\}, \{4\}, \{5\})\),
\(\bar{\ell}_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(\bar{L}^{(3,1^3)} = (\{1, 2, 6\}, \{3\}, \{5\}, \{4\})\),
\(m_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(M^{(3,1^3)} = (\{1, 2, 6\}, \{4\}, \{3\}, \{5\})\),
\(\bar{m}_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(\bar{M}^{(3,1^3)} = (\{1, 2, 6\}, \{4\}, \{5\}, \{3\})\),
\(p_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(P^{(3,1^3)} = (\{1, 2, 6\}, \{5\}, \{3\}, \{4\})\),
\(\bar{p}_{(3,1^3)}\) is the \(G\)-orbit of the tabloid \(\bar{P}^{(3,1^3)} = (\{1, 2, 6\}, \{5\}, \{4\}, \{3\})\).
The inequalities between the tabloids of shape (3, 1^3) and the tabloids of shape (3, 2, 1) are as follows:

\[ A^{(3,1^3)} < A^{(3,2,1)}, \quad A^{(3,1^3)} < A^{(3,2,1)}, \]

\[ B^{(3,1^3)} < B^{(3,2,1)}, \quad B^{(3,1^3)} < B^{(3,2,1)}, \quad C^{(3,1^3)} < B^{(3,2,1)}, \quad C^{(3,1^3)} < B^{(3,2,1)}, \]

\[ B^{(3,1^3)} < C^{(3,2,1)}, \quad B^{(3,1^3)} < C^{(3,2,1)}, \quad E^{(3,1^3)} < C^{(3,2,1)}, \quad E^{(3,1^3)} < C^{(3,2,1)}, \]

\[ C^{(3,1^3)} < E^{(3,2,1)}, \quad C^{(3,1^3)} < E^{(3,2,1)}, \quad E^{(3,1^3)} < E^{(3,2,1)}, \quad E^{(3,1^3)} < E^{(3,2,1)}, \]

\[ F^{(3,1^3)} < F^{(3,2,1)}, \quad F^{(3,1^3)} < F^{(3,2,1)}, \quad H^{(3,1^3)} < F^{(3,2,1)}, \quad H^{(3,1^3)} < F^{(3,2,1)}, \]

\[ F^{(3,1^3)} < H^{(3,2,1)}, \quad F^{(3,1^3)} < H^{(3,2,1)}, \quad K^{(3,1^3)} < H^{(3,2,1)}, \quad K^{(3,1^3)} < H^{(3,2,1)}, \]

\[ H^{(3,1^3)} < K^{(3,2,1)}, \quad H^{(3,1^3)} < K^{(3,2,1)}, \quad K^{(3,1^3)} < K^{(3,2,1)}, \quad K^{(3,1^3)} < K^{(3,2,1)}, \]

\[ L^{(3,1^3)} < L^{(3,2,1)}, \quad L^{(3,1^3)} < L^{(3,2,1)}, \quad M^{(3,1^3)} < L^{(3,2,1)}, \quad M^{(3,1^3)} < L^{(3,2,1)}, \]

\[ M^{(3,1^3)} < P^{(3,2,1)}, \quad M^{(3,1^3)} < P^{(3,2,1)}, \quad P^{(3,1^3)} < M^{(3,2,1)}, \quad P^{(3,1^3)} < M^{(3,2,1)}, \]

\[ P^{(3,1^3)} < P^{(3,2,1)}, \quad P^{(3,1^3)} < P^{(3,2,1)}, \quad \bar{P}^{(3,1^3)} < P^{(3,2,1)}, \quad \bar{P}^{(3,1^3)} < P^{(3,2,1)}, \]

because

\[ R_{2,5}R_{3,6}A^{(3,1^3)} = R_{2,5}\bar{A}^{(3,1^3)} = A^{(3,2,1)}, \]

\[ R_{2,5}R_{3,6}B^{(3,1^3)} = R_{2,5}\bar{B}^{(3,1^3)} = R_{2,3}R_{3,6}C^{(3,1^3)} = R_{2,3}\bar{C}^{(3,1^3)} = B^{(3,2,1)}, \]

\[ R_{2,6}B^{(3,1^3)} = R_{2,6}R_{3,5}B^{(3,1^3)} = R_{2,3}R_{3,5}E^{(3,1^3)} = R_{2,3}\bar{E}^{(3,1^3)} = C^{(3,2,1)}, \]

\[ R_{2,6}C^{(3,1^3)} = R_{2,6}R_{3,3}C^{(3,1^3)} = R_{2,5}E^{(3,1^3)} = R_{2,5}\bar{E}^{(3,1^3)} = E^{(3,2,1)}, \]

\[ R_{2,4}R_{3,6}F^{(3,1^3)} = R_{2,4}\bar{F}^{(3,1^3)} = R_{2,3}R_{3,6}H^{(3,1^3)} = R_{2,3}\bar{H}^{(3,1^3)} = F^{(3,2,1)}, \]

\[ R_{2,6}F^{(3,1^3)} = R_{2,6}R_{3,4}F^{(3,1^3)} = R_{2,3}R_{3,4}K^{(3,1^3)} = R_{2,3}\bar{K}^{(3,1^3)} = H^{(3,2,1)}, \]

\[ R_{2,6}H^{(3,1^3)} = R_{2,6}R_{3,3}H^{(3,1^3)} = R_{2,4}K^{(3,1^3)} = R_{2,4}\bar{K}^{(3,1^3)} = K^{(3,2,1)}, \]

\[ R_{2,4}R_{3,5}L^{(3,1^3)} = R_{2,4}L^{(3,1^3)} = R_{2,3}R_{3,5}M^{(3,1^3)} = R_{2,3}\bar{M}^{(3,1^3)} = L^{(3,2,1)}, \]

\[ R_{2,5}L^{(3,1^3)} = R_{2,5}R_{3,4}L^{(3,1^3)} = R_{2,3}R_{3,4}P^{(3,1^3)} = R_{2,3}\bar{P}^{(3,1^3)} = M^{(3,2,1)}, \]

\[ R_{2,5}M^{(3,1^3)} = R_{2,5}R_{3,3}M^{(3,1^3)} = R_{2,4}P^{(3,1^3)} = R_{2,4}R_{3,3}\bar{P}^{(3,1^3)} = P^{(3,2,1)}. \]

Thus, all substitution reactions among (3, 1^3)-derivatives and (3, 2, 1)-derivatives of cyclopropane are designated by the following inequalities:

\[ a^{(3,1^3)} < a^{(3,2,1)}, \quad \bar{a}^{(3,1^3)} < a^{(3,2,1)}, \]

\[ b^{(3,1^3)} < b^{(3,2,1)}, \quad \bar{b}^{(3,1^3)} < b^{(3,2,1)}, \quad c^{(3,1^3)} < b^{(3,2,1)}, \quad \bar{c}^{(3,1^3)} < b^{(3,2,1)}, \]

\[ b^{(3,1^3)} < c^{(3,2,1)}, \quad \bar{b}^{(3,1^3)} < c^{(3,2,1)}, \quad e^{(3,1^3)} < c^{(3,2,1)}, \quad \bar{e}^{(3,1^3)} < c^{(3,2,1)}, \]

\[ c^{(3,1^3)} < e^{(3,2,1)}, \quad \bar{c}^{(3,1^3)} < e^{(3,2,1)}, \quad e^{(3,1^3)} < e^{(3,2,1)}, \quad \bar{e}^{(3,1^3)} < e^{(3,2,1)}, \]
\( f_{(3,1^3)} < f_{(3,2,1)}, \bar{f}_{(3,1^3)} < f_{(3,2,1)}, h_{(3,1^3)} < f_{(3,2,1)}, \bar{h}_{(3,1^3)} < f_{(3,2,1)}, \)
\( f_{(3,1^3)} < h_{(3,2,1)}, \bar{f}_{(3,1^3)} < h_{(3,2,1)}, k_{(3,1^3)} < h_{(3,2,1)}, \bar{k}_{(3,1^3)} < h_{(3,2,1)}, \)
\( h_{(3,1^3)} < k_{(3,2,1)}, \bar{h}_{(3,1^3)} < k_{(3,2,1)}, k_{(3,1^3)} < k_{(3,2,1)}, \bar{k}_{(3,1^3)} < k_{(3,2,1)}, \)
\( \ell_{(3,1^3)} < \ell_{(3,2,1)}, \bar{\ell}_{(3,1^3)} < \ell_{(3,2,1)}, m_{(3,1^3)} < \ell_{(3,2,1)}, \bar{m}_{(3,1^3)} < \ell_{(3,2,1)}, \)
\( \ell_{(3,1^3)} < m_{(3,2,1)}, \bar{\ell}_{(3,1^3)} < m_{(3,2,1)}, p_{(3,1^3)} < m_{(3,2,1)}, \bar{p}_{(3,1^3)} < m_{(3,2,1)}, \)
\( m_{(3,1^3)} < p_{(3,2,1)}, \bar{m}_{(3,1^3)} < p_{(3,2,1)}, p_{(3,1^3)} < p_{(3,2,1)}, \bar{p}_{(3,1^3)} < p_{(3,2,1)}. \)

We have
\[
T_{(3,1^3);G} = \\
\{ a_{(3,1^3)} \cup \bar{a}_{(3,1^3)}, b_{(3,1^3)} \cup f_{(3,1^3)}, \bar{b}_{(3,1^3)} \cup \bar{f}_{(3,1^3)}, e_{(3,1^3)} \cup h_{(3,1^3)}, \bar{e}_{(3,1^3)} \cup \bar{h}_{(3,1^3)}, \}
\]
\[
e_{(3,1^3)} \cup k_{(3,1^3)}, \bar{e}_{(3,1^3)} \cup \bar{k}_{(3,1^3)}, \ell_{(3,1^3)} \cup \bar{\ell}_{(3,1^3)}, m_{(3,1^3)} \cup p_{(3,1^3)}, \bar{m}_{(3,1^3)} \cup \bar{p}_{(3,1^3)} \}.
\]

Therefore the members of any one of the following two-element sets are chiral pairs:
\[
\{ a_{(3,1^3)}, \bar{a}_{(3,1^3)} \}, \quad \{ b_{(3,1^3)}, f_{(3,1^3)} \}, \quad \{ \bar{b}_{(3,1^3)}, \bar{f}_{(3,1^3)} \}, \quad \{ e_{(3,1^3)}, k_{(3,1^3)} \}, \quad \{ \bar{e}_{(3,1^3)}, \bar{k}_{(3,1^3)} \}, \quad \{ c_{(3,1^3)}, h_{(3,1^3)} \},
\]
\[
\{ \bar{c}_{(3,1^3)}, \bar{h}_{(3,1^3)} \}, \quad \{ e_{(3,1^3)}, k_{(3,1^3)} \}, \quad \{ \bar{e}_{(3,1^3)}, \bar{k}_{(3,1^3)} \}, \quad \{ \ell_{(3,1^3)}, \bar{\ell}_{(3,1^3)} \},
\]
\[
\{ m_{(3,1^3)}, p_{(3,1^3)} \}, \quad \{ \bar{m}_{(3,1^3)}, \bar{p}_{(3,1^3)} \}.
\]

Further,
\[
T_{(3,1^3);G''} = \\
\{ (a_{(3,1^3)} \cup \bar{a}_{(3,1^3)}) \cup (\ell_{(3,1^3)} \cup \bar{\ell}_{(3,1^3)}) \cup (m_{(3,1^3)} \cup p_{(3,1^3)}) \cup (\bar{m}_{(3,1^3)} \cup \bar{p}_{(3,1^3)}), \}
\]
\[
(b_{(3,1^3)} \cup f_{(3,1^3)}) \cup (\bar{e}_{(3,1^3)} \cup \bar{k}_{(3,1^3)}), \quad (\bar{b}_{(3,1^3)} \cup \bar{f}_{(3,1^3)}) \cup (e_{(3,1^3)} \cup k_{(3,1^3)}), \]
\[
(c_{(3,1^3)} \cup h_{(3,1^3)}) \cup (\bar{e}_{(3,1^3)} \cup \bar{h}_{(3,1^3)}). \}
\]

Thus, the members of any one set from the list below are structurally identical as long as the members of different sets are structural isomers:
\[
\{ a_{(3,1^3)}, \bar{a}_{(3,1^3)}, \ell_{(3,1^3)}, \bar{\ell}_{(3,1^3)}, m_{(3,1^3)}, p_{(3,1^3)}, \bar{m}_{(3,1^3)}, \bar{p}_{(3,1^3)} \},
\]
\[
\{ b_{(3,1^3)}, f_{(3,1^3)}, \bar{e}_{(3,1^3)}, \bar{k}_{(3,1^3)} \}, \quad \{ \bar{b}_{(3,1^3)}, \bar{f}_{(3,1^3)}, e_{(3,1^3)}, k_{(3,1^3)} \},
\]
\[
\{ c_{(3,1^3)}, h_{(3,1^3)}, \bar{c}_{(3,1^3)}, \bar{h}_{(3,1^3)} \}.
\]

Case 8. \( \lambda = (2^3). \)

We have
\[
T_{(2^3);G} = \{ a_{(2^3)}, \bar{a}_{(2^3)}, b_{(2^3)}, \bar{b}_{(2^3)}, c_{(2^3)}, \bar{c}_{(2^3)}, e_{(2^3)}, \bar{e}_{(2^3)}, f_{(2^3)}, \bar{f}_{(2^3)}, \}
\]
\[
h_{(2^3)}, \bar{h}_{(2^3)}, k_{(2^3)}, \bar{k}_{(2^3)}, \ell_{(2^3)}, \bar{\ell}_{(2^3)}, m_{(2^3)}, \bar{m}_{(2^3)} \},
\]

where:
\( a_{(2^3)} \) is the \( G \)-orbit of the tabloid \( A_{(2^3)} = (\{1, 2\}, \{3, 4\}, \{5, 6\}) \),
\( \bar{a}_{(2^3)} \) is the \( G \)-orbit of the tabloid \( \bar{A}_{(2^3)} = (\{1, 2\}, \{3, 5\}, \{4, 6\}) \),
\( b_{(2^3)} \) is the \( G \)-orbit of the tabloid \( B^{(2^3)} = \{(1, 2), \{3, 6\}, \{4, 5\}\),
\( \bar{b}_{(2^3)} \) is the \( G \)-orbit of the tabloid \( \bar{B}^{(2^3)} = \{(1, 2), \{4, 5\}, \{3, 6\}\),
\( c_{(2^3)} \) is the \( G \)-orbit of the tabloid \( C^{(2^3)} = \{(1, 2), \{4, 6\}, \{3, 5\}\),
\( \bar{c}_{(2^3)} \) is the \( G \)-orbit of the tabloid \( \bar{C}^{(2^3)} = \{(1, 2), \{5, 6\}, \{3, 4\}\),
\( e_{(2^3)} \) is the \( G \)-orbit of the tabloid \( E^{(2^3)} = \{(1, 4), \{2, 3\}, \{5, 6\}\),
\( \bar{e}_{(2^3)} \) is the \( G \)-orbit of the tabloid \( \bar{E}^{(2^3)} = \{(1, 4), \{2, 5\}, \{3, 6\}\),
\( f_{(2^3)} \) is the \( G \)-orbit of the tabloid \( F^{(2^3)} = \{(1, 4), \{2, 3\}, \{5, 6\}\),
\( \bar{f}_{(2^3)} \) is the \( G \)-orbit of the tabloid \( \bar{F}^{(2^3)} = \{(1, 4), \{3, 5\}, \{2, 6\}\),
\( h_{(2^3)} \) is the \( G \)-orbit of the tabloid \( H^{(2^3)} = \{(1, 5), \{2, 3\}, \{4, 6\}\),
\( \bar{h}_{(2^3)} \) is the \( G \)-orbit of the tabloid \( \bar{H}^{(2^3)} = \{(1, 5), \{2, 4\}, \{3, 6\}\),
\( k_{(2^3)} \) is the \( G \)-orbit of the tabloid \( K^{(2^3)} = \{(1, 5), \{2, 6\}, \{3, 4\}\),
\( \bar{k}_{(2^3)} \) is the \( G \)-orbit of the tabloid \( \bar{K}^{(2^3)} = \{(1, 5), \{3, 6\}, \{2, 4\}\),
\( \ell_{(2^3)} \) is the \( G \)-orbit of the tabloid \( L^{(2^3)} = \{(1, 6), \{2, 3\}, \{4, 5\}\),
\( \bar{\ell}_{(2^3)} \) is the \( G \)-orbit of the tabloid \( \bar{L}^{(2^3)} = \{(1, 6), \{2, 4\}, \{3, 5\}\),
\( m_{(2^3)} \) is the \( G \)-orbit of the tabloid \( M^{(2^3)} = \{(1, 6), \{2, 5\}, \{3, 4\}\),
\( \bar{m}_{(2^3)} \) is the \( G \)-orbit of the tabloid \( \bar{M}^{(2^3)} = \{(1, 6), \{3, 4\}, \{2, 5\}\).
(123)(456)\tilde{E}^{(2^3)} < H^{(3,2,1)}, (123)(456)\overline{F}^{(2^3)} < H^{(3,2,1)},
\quad H^{(2^3)} < H^{(3,2,1)}, K^{(2^3)} < H^{(3,2,1)}, \overline{K}^{(2^3)} < H^{(3,2,1)},
\quad \overline{B}^{(2^3)} < K^{(3,2,1)}, C^{(2^3)} < K^{(3,2,1)}, \overline{C}^{(2^3)} < K^{(3,2,1)}, (15)(24)(36)E^{(2^3)} < K^{(3,2,1)},
\quad (15)(24)(36)\overline{E}^{(2^3)} < K^{(3,2,1)}, (123)(456)\overline{F}^{(2^3)} < K^{(3,2,1)},
\quad (15)(24)(36)H^{(2^3)} < K^{(3,2,1)}, \overline{H}^{(2^3)} < K^{(3,2,1)}, K^{(2^3)} < K^{(3,2,1)},
\quad A^{(2^3)} < L^{(3,2,1)}, B^{(2^3)} < L^{(3,2,1)}, C^{(2^3)} < L^{(3,2,1)}, (123)(456)H^{(2^3)} < L^{(3,2,1)},
\quad (123)(456)K^{(2^3)} < L^{(3,2,1)}, (123)(456)\overline{K}^{(2^3)} < L^{(3,2,1)},
\quad L^{(2^3)} < L^{(3,2,1)}, \overline{L}^{(2^3)} < L^{(3,2,1)}, \overline{M}^{(2^3)} < L^{(3,2,1)},
\quad \overline{A}^{(2^3)} < M^{(3,2,1)}, B^{(2^3)} < M^{(3,2,1)}, \overline{C}^{(2^3)} < M^{(3,2,1)}, (123)(456)H^{(2^3)} < M^{(3,2,1)},
\quad (123)(456)H^{(2^3)} < M^{(3,2,1)}, (16)(25)(34)K^{(2^3)} < M^{(3,2,1)},
\quad L^{(2^3)} < M^{(3,2,1)}, (16)(25)(34)\overline{L}^{(2^3)} < M^{(3,2,1)}, M^{(2^3)} < M^{(3,2,1)},
\quad \overline{B}^{(2^3)} < P^{(3,2,1)}, C^{(2^3)} < P^{(3,2,1)}, \overline{C}^{(2^3)} < P^{(3,2,1)}, (16)(25)(34)H^{(2^3)} < P^{(3,2,1)},
\quad (16)(25)(34)K^{(2^3)} < P^{(3,2,1)}, (123)(456)\overline{K}^{(2^3)} < P^{(3,2,1)},
\quad (16)(25)(34)L^{(2^3)} < P^{(3,2,1)}, \overline{L}^{(2^3)} < P^{(3,2,1)}, M^{(2^3)} < P^{(3,2,1)},

because

\begin{align*}
R_{1,3}R_{2,5}A^{(2^3)} &= R_{1,3}R_{2,4}\overline{A}^{(2^3)} = R_{1,1}R_{2,5}(123)(456)B^{(2^3)} = \\
R_{1,3}B^{(2^3)} &= R_{1,1}(123)(456)C^{(2^3)} = R_{1,2}(132)(465)\overline{C}^{(2^3)} = A^{(3,2,1)},
\quad R_{1,4}R_{2,5}A^{(2^3)} = R_{1,4}\overline{A}^{(2^3)} = R_{1,4}R_{2,3}\overline{B}^{(2^3)} = \\
R_{1,2}R_{2,5}E^{(2^3)} &= R_{1,2}R_{2,3}\overline{E}^{(2^3)} = R_{1,2}\overline{F}^{(2^3)} = \\
R_{1,1}R_{2,5}(123)(456)\overline{L}^{(2^3)} &= R_{1,1}(123)(456)\overline{L}^{(2^3)} = R_{1,1}R_{2,3}(123)(456)\overline{M}^{(2^3)} = B^{(3,2,1)},
\quad R_{1,4}R_{2,6}A^{(2^3)} = R_{1,4}B^{(2^3)} = R_{1,4}R_{2,3}C^{(2^3)} = \\
R_{1,2}R_{2,6}E^{(2^3)} &= R_{1,2}(46)(25)\overline{E}^{(2^3)} = R_{1,2}R_{2,3}F^{(2^3)} = \\
R_{1,1}R_{2,6}(123)(456)\overline{L}^{(2^3)} &= R_{1,1}R_{2,3}(123)(456)\overline{L}^{(2^3)} = \\
R_{1,1}(123)(456)\overline{M}^{(2^3)} &= C^{(3,2,1)},
\quad R_{1,4}R_{2,6}\overline{B}^{(2^3)} = R_{1,4}R_{2,5}C^{(2^3)} = R_{1,4}\overline{C}^{(2^3)} = \\
R_{1,2}(46)(25)E^{(2^3)} &= R_{1,2}R_{2,6}\overline{E}^{(2^3)} = R_{1,2}R_{2,6}F^{(2^3)} = \\
R_{1,1}(123)(456)\overline{L}^{(2^3)} &= R_{1,1}R_{2,5}(123)(456)\overline{L}^{(2^3)} = \\
R_{1,1}R_{2,6}(123)(456)\overline{M}^{(2^3)} &= E^{(3,2,1)},
\end{align*}
Therefore the substitution reactions among (2\textsuperscript{3})-clopropane are as follows:

\[ R_{1,5}A^{(2^3)} = R_{1,5}R_{2,4}\bar{A}^{(2^3)} = R_{1,5}R_{2,3}\bar{B}^{(2^3)} = \]

\[ R_{1,1}R_{2,4}(123)(456)E^{(2^3)} = R_{1,1}R_{2,3}(15)(24)(36)\bar{E}^{(2^3)} = R_{1,1}(123)(456)F^{(2^3)} = \]

\[ R_{1,2}R_{2,4}H^{(2^3)} = R_{1,2}R_{2,3}\bar{H}^{(2^3)} = R_{1,2}(15)(24)(36)K^{(2^3)} = F^{(3,2,1)}, \]

\[ R_{1,5}R_{2,6}\bar{A}^{(2^3)} = R_{1,5}B^{(2^3)} = R_{1,5}R_{2,3}\bar{C}^{(2^3)} = \]

\[ R_{1,1}R_{2,6}(123)(456)E^{(2^3)} = R_{1,1}(123)(456)E^{(2^3)} = R_{1,1}R_{2,3}(123)(456)\bar{F}^{(2^3)} = \]

\[ R_{1,2}R_{2,6}H^{(2^3)} = R_{1,2}R_{2,3}K^{(2^3)} = R_{1,2}\bar{K}^{(2^3)} = H^{(3,2,1)}, \]

\[ R_{1,5}R_{2,6}\bar{B}^{(2^3)} = R_{1,5}C^{(2^3)} = R_{1,5}R_{2,4}\bar{C}^{(2^3)} = \]

\[ R_{1,1}(15)(24)(36)E^{(2^3)} = R_{1,1}R_{2,6}(15)(24)(36)\bar{E}^{(2^3)} = R_{1,1}R_{2,4}(123)(456)\bar{F}^{(2^3)} = \]

\[ R_{1,2}(15)(24)(36)H^{(2^3)} = R_{1,2}R_{2,6}\bar{H}^{(2^3)} = R_{1,2}R_{2,4}K^{(2^3)} = K^{(3,2,1)}, \]

\[ R_{1,6}A^{(2^3)} = R_{1,6}R_{2,4}B^{(2^3)} = R_{1,6}R_{2,3}C^{(2^3)} = \]

\[ R_{1,1}R_{2,4}(123)(456)H^{(2^3)} = R_{1,1}(123)(456)K^{(2^3)} = R_{1,1}R_{2,3}(123)(456)\bar{K}^{(2^3)} = \]

\[ R_{1,2}R_{2,4}L^{(2^3)} = R_{1,2}R_{2,3}\bar{L}^{(2^3)} = R_{1,2}\bar{M}^{(2^3)} = L^{(3,2,1)}, \]

\[ R_{1,6}\bar{A}^{(2^3)} = R_{1,6}R_{2,5}B^{(2^3)} = R_{1,6}R_{2,3}\bar{C}^{(2^3)} = \]

\[ R_{1,1}R_{2,5}(123)(456)H^{(2^3)} = R_{1,1}(123)(456)\bar{H}^{(2^3)} = R_{1,1}R_{2,3}(16)(25)(34)K^{(2^3)} = \]

\[ R_{1,2}R_{2,5}L^{(2^3)} = R_{1,2}(16)(25)(34)\bar{L}^{(2^3)} = R_{1,2}R_{2,3}M^{(2^3)} = M^{(3,2,1)}, \]

\[ R_{1,6}\bar{B}^{(2^3)} = R_{1,6}R_{2,5}C^{(2^3)} = R_{1,6}R_{2,4}\bar{C}^{(2^3)} = \]

\[ R_{1,1}(16)(25)(34)H^{(2^3)} = R_{1,1}R_{2,4}(16)(25)(34)K^{(2^3)} = R_{1,1}R_{2,5}(123)(456)\bar{K}^{(2^3)} = \]

\[ R_{1,2}(16)(25)(34)\bar{L}^{(2^3)} = R_{1,2}R_{2,5}\bar{L}^{(2^3)} = R_{1,2}R_{2,4}M^{(2^3)} = P^{(3,2,1)}. \]

Therefore the substitution reactions among (2\textsuperscript{3})-products and (3,2,1)-products of cyclopropane are as follows:

\[
a_{(2^3)} < a_{(3,2,1)}, \quad \bar{a}_{(2^3)} < a_{(3,2,1)}, \quad b_{(2^3)} < a_{(3,2,1)},
\]

\[
\bar{b}_{(2^3)} < a_{(3,2,1)}, \quad c_{(2^3)} < a_{(3,2,1)}, \quad e_{(2^3)} < a_{(3,2,1)},
\]

\[
a_{(2^3)} < b_{(3,2,1)}, \quad \bar{a}_{(2^3)} < b_{(3,2,1)}, \quad \bar{b}_{(2^3)} < b_{(3,2,1)},
\]

\[
e_{(2^3)} < b_{(3,2,1)}, \quad \bar{e}_{(2^3)} < b_{(3,2,1)}, \quad f_{(2^3)} < b_{(3,2,1)},
\]

\[
\ell_{(2^3)} < b_{(3,2,1)}, \quad \bar{\ell}_{(2^3)} < b_{(3,2,1)}, \quad \bar{m}_{(2^3)} < b_{(3,2,1)},
\]

\[
a_{(2^3)} < c_{(3,2,1)}, \quad b_{(2^3)} < c_{(3,2,1)}, \quad c_{(2^3)} < c_{(3,2,1)},
\]

\[
e_{(2^3)} < c_{(3,2,1)}, \quad \bar{e}_{(2^3)} < c_{(3,2,1)}, \quad f_{(2^3)} < c_{(3,2,1)},
\]

\[
\ell_{(2^3)} < c_{(3,2,1)}, \quad \bar{\ell}_{(2^3)} < c_{(3,2,1)}, \quad m_{(2^3)} < c_{(3,2,1)},
\]
The set of all $G'$-orbits is

$$T_{(2^3);G'} = \{ a_{(2^3)} \cup \bar{a}_{(2^3)}, b_{(2^3)} \cup \bar{b}_{(2^3)}, c_{(2^3)} \cup \bar{c}_{(2^3)}, e_{(2^3)} \cup \bar{e}_{(2^3)}, f_{(2^3)} \cup \bar{f}_{(2^3)},$$

$$h_{(2^3)} \cup \ell_{(2^3)}, \bar{h}_{(2^3)} \cup \bar{\ell}_{(2^3)}, m_{(2^3)} \cup \bar{m}_{(2^3)} \}.$$  

Hence, the members of any one of the two-element sets

$$\{ a_{(2^3)}, \bar{a}_{(2^3)} \}, \{ c_{(2^3)}, \bar{c}_{(2^3)} \}, \{ f_{(2^3)}, \bar{f}_{(2^3)} \},$$

$$\{ h_{(2^3)}, \ell_{(2^3)} \}, \{ \bar{h}_{(2^3)}, m_{(2^3)} \}, \{ k_{(2^3)}, \bar{k}_{(2^3)} \}, \{ \bar{\ell}_{(2^3)} \}$$

form a chiral pair and $b_{(2^3)}, \bar{b}_{(2^3)}, e_{(2^3)}, \bar{e}_{(2^3)}$, represent dimers.

Moreover, the set of all $G''$-orbits is

$$T_{(2^3);G''} =$$
\{(a_{(23)} \cup \bar{a}_{(23)}), (c_{(23)} \cup \bar{c}_{(23)}), (h_{(23)} \cup \ell_{(23)}), (k_{(23)} \cup \bar{k}_{(23)}),
\ b_{(23)} \cup (\bar{k}_{(23)} \cup m_{(23)}), \bar{b}_{(23)} \cup (\bar{h}_{(23)} \cup \bar{m}_{(23)}), \bar{c}_{(23)} \cup (f_{(23)} \cup \bar{f}_{(23)}), \bar{e}_{(23)}\}\}

Thus, any one from the sets below gathers all formulae that represent structurally identical derivatives, so members of different sets represent structural isomers.

\{a_{(23)}, \bar{a}_{(23)}, c_{(23)}, \bar{c}_{(23)}, h_{(23)}, \ell_{(23)}, k_{(23)}, \bar{k}_{(23)}\}
\{b_{(23)}, \bar{k}_{(23)}, m_{(23)}\}, \{\bar{b}_{(23)}, \bar{h}_{(23)}, \bar{m}_{(23)}\}, \{e_{(23)}, f_{(23)}, \bar{f}_{(23)}\}, \{\bar{e}_{(23)}\}.

3. IDENTIFICATION OF THE DERIVATIVES

Now, we will describe the Lunn-Senior’s automorphism groups \(\text{Aut}''_0(T_{D,G})\) for \(D = D_0, k = 1, \ldots, 6,\) where

\[D_1 = \{(6), (5,1), \{4,2\}\}, \quad D_2 = D_1 \cup \{(4,1^2)\},\]
\[D_3 = D_2 \cup \{(3^2)\}, \quad D_4 = D_3 \cup \{(3,2,1)\},\]
\[D_5 = D_4 \cup \{(3,1^3)\}, \quad D_6 = D_5 \cup \{(2^3)\}.\]

The elements of the \(\text{Aut}''_0(T_{D,G})\)-orbits in the set \(T_{D,G}\) will represent the products of cyclopropane that can not be distinguished via substitution reactions among the elements of \(T_{D,G}\).

We remind that we will use without referring all terminology and notation from [2], especially those from the beginning of section 3. For convenience of the reader, we state explicitly once again the conditions of the main [2, Lemma 3.1], as well as the assumptions and notation introduced before [2, Lemmas 3.3 - 3.7]. We note that the correct version of [2, Corollary 3.2] can be found in [3].

Let \(U, V, \bar{V} \subset T_{D,G}\) be unions of \(G''\)-orbits, such that \(U \subset V, \bar{V} \cap U \subset \bar{V},\) and the difference \(\bar{V} \cap U\) consists of minimal elements of the partially ordered set \(V \cup \bar{V}\). Assume that \(\bar{V}\) is a barrier of \(V \cap U\) in \(V\), and the automorphism group \(\text{Aut}''_0(U)\) is a commutative 2-group. Set \(H = \{\beta \in \text{Aut}''_0(U) \mid \beta(C_{(\bar{V}; a)}) = C_{(\bar{V}; a)}, a \in V \cap U\}\). Moreover, for any pair \(X, V \subset T_{D,G}\) of sets that are unions of \(G''\)-orbits with \(X \subset V\) we denote by \(I_{V,X}\) the image of the restriction homomorphism

\[\varphi_{V,X}: \text{Aut}''_0(V) \to \text{Aut}''_0(X).\]

**Lemma 3.1.** Let the difference \(V \cap U\) be a \(G''\)-orbit that consists of several chiral pairs \(\{A, A^1\}, \{B, B^1\}, \ldots,\) and eventually, of several dimers. Suppose that: \(C_{(\bar{V}; A)} = C_{(\bar{V}; A^1)} = P, C_{(\bar{V}; B)} = C_{(\bar{V}; B^1)} = Q, \ldots\), the cones \(P, Q, \ldots\) are pairwise different, the cones of the dimers are pairwise different, and that

\[I_{V,U} = H.\]

Then there exists a decomposition

\[\text{Aut}''_0(V) = H \rtimes \langle s \rangle \times \langle t \rangle \times \cdots,\]
where \( s = (A, A^1) \), \( t = (B, B^1) \), ..., the restriction homomorphism \( \varrho_{V,U} \) has kernel \( \langle s \rangle \times \langle t \rangle \times \cdots \), and \( \text{Aut}_0''(V) \) is a commutative 2-group.

**Proof:** [3, Corollary 3.2, (i)] yields \( H \cup Hs \cup Ht \cup \ldots \cup \text{Aut}_0''(V) \). Now, let \( \alpha \in \text{Aut}_0''(V) \). Then \( \varrho_{V,U}(\alpha) \in H \), and according to [2, Lemma 3.1, (iv)], and to the fact that \( \alpha \) maps any chiral pair onto a chiral pair, we have \( \alpha(A, A^1) = \{A, A^1\} \), \( \alpha(B, B^1) = \{B, B^1\} \), ..., and conclude that \( \alpha \) leaves all dimers invariant. Therefore \( \alpha \in H \cup Hs \cup Ht \cup \ldots \cup \text{Aut}_0''(V) \). Now, we note that every pair among the automorphisms \( s, t, \ldots \) commute, \( s^2 = id \), \( t^2 = id \), ..., and each one of them commutes with the elements of \( H \). Hence the subgroup \( \langle H, s, t, \ldots \rangle \) of \( \text{Aut}_0''(V) \) is a commutative 2-group. In particular, \( \langle H, s, t, \ldots \rangle = H \cup Hs \cup Ht \cup \ldots \cup \text{Aut}_0''(V) \), and the proof is finished.

**Lemma 3.2.** Let the difference \( V \setminus U \) be a \( G'' \)-orbit that consists of two chiral pairs \( \{A, A^1\}, \{B, B^1\} \). Suppose that \( C_{>}(\bar{V}; A) = C_{>}(\bar{V}; A^1) = P \), there exists a decomposition

\[
I_{V,U} = H \times \langle w \rangle,
\]

where \( w(P) = P \) and \( w(Q) = Q \), for \( Q = C_{>}(\bar{V}; B) \), \( Q^1 = C_{>}(\bar{V}; B^1) \). Then there exists a decomposition

\[
\text{Aut}_0''(V) = H \times \langle s \rangle \times \langle wt \rangle,
\]

where \( s = (A, A^1) \), \( t = (B, B^1) \), the restriction homomorphism \( \varrho_{V,U} \) has kernel \( \langle s \rangle \), and \( \text{Aut}_0''(V) \) is a commutative 2-group.

**Proof:** Since \( w \notin H \), we obtain that the cones \( Q, Q_1, P \), are pairwise different, and then [3, Corollary 3.2, (i)] yields \( H \cup Hs \cup Hw \cup Hst \subset \text{Aut}_0''(V) \). Now, let \( \alpha \in \text{Aut}_0''(V) \) with \( \varrho_{V,U}(\alpha) \in H \) (respectively, \( \varrho_{V,U}(\alpha) \in Hw \)). Then in accordance with [2, Lemma 3.1, (iv)], \( \alpha(A, A^1) = \{A, A^1\} \), \( \alpha(B) = B \), \( \alpha(B^1) = B^1 \) (respectively, \( \alpha(A, A^1) = \{A, A^1\} \), \( \alpha(B) = B^1 \), \( \alpha(B^1) = B \) ), hence \( \varrho_{V,V\setminus U}(\alpha) = \text{id} \), or \( \varrho_{V,V\setminus U}(\alpha) = s \) (respectively, \( \varrho_{V,V\setminus U}(\alpha) = t \), or \( \varrho_{V,V\setminus U}(\alpha) = st \)). Thus, we have \( \alpha \in H \cup Hs \) (respectively, \( \alpha \in Hst \cup Hw \)). Since the automorphisms \( s \) and \( wt \) commute, \( s^2 = id \), \((wt)^2 = id \), and each one of them commutes with the elements of \( H \), then the subgroup \( \langle H, s, wt \rangle \) of \( \text{Aut}_0''(V) \) is a commutative 2-group. In particular, \( \langle H, s, wt \rangle = H \cup Hs \cup Hw \cup Hst \), and the proof is done.

**Lemma 3.3.** Let the difference \( V \setminus U \) be a \( G'' \)-orbit that consists of a chiral pair \( \{A, A^1\} \) and of several dimers. Suppose that the cones of the dimers are pairwise different, \( \text{Aut}_0''(U) \)-invariant, and that there exists a decomposition

\[
I_{V,U} = H \times \langle w \rangle
\]

with \( w(P) = P^1 \), where \( P = C_{>}(\bar{V}; A) \), \( P^1 = C_{>}(\bar{V}; A^1) \). Then there exists a decomposition

\[
\text{Aut}_0''(V) = H \times \langle ws \rangle,
\]

where \( s = (A, A^1) \), the restriction homomorphism \( \varrho_{V,U} \) is injective, and \( \text{Aut}_0''(V) \) is a commutative 2-group.

**Proof:** Straightforward generalization of [2, Lemma 3.7].
Lemma 3.4. Let the difference $V \setminus U$ be a $G''$-orbit that consists of two types of chiral pairs: several chiral pairs $\{A, A^1\}, \{B, B^1\}, \ldots$, with $C_{\geq}(V; A) = C_{\geq}(V; A^1) = P$, $C_{\geq}(V; B) = C_{\geq}(V; B^1) = Q$, two chiral pairs $\{C, C^1\}, \{E, E^1\}$, with $C_{\geq}(V; C) = C_{\geq}(V; E) = R$, $C_{\geq}(V; C^1) = C_{\geq}(V; E^1) = R^1$, and of several dimers. Let us suppose that the cones $P, Q, \ldots$, are pairwise different, and the cones $\tilde{P}, \tilde{Q}, \ldots$, of the dimers are pairwise different. If

$$I_{V, U} = H \times \langle w \rangle,$$

where $w(P) = P$, $w(Q) = Q, \ldots$, $w(\tilde{P}) = \tilde{P}$, $w(\tilde{Q}) = \tilde{Q}, \ldots$, and $w(R) = R^1$, then there exists a decomposition

$$\text{Aut}_0''(V) = H \times \langle z \rangle \times \langle t \rangle \times \cdots \times \langle wx \rangle,$$

where $s = (A, A^1), t = (B, B^1), \ldots$, $z = (C, E)(C^1, E^1), x = (C, C^1)(E, E^1)$, the restriction homomorphism $\varrho_{V, U}$ has kernel $\langle z \rangle \times \langle s \rangle \times \langle t \rangle \times \cdots$, and $\text{Aut}_0''(V)$ is a commutative $2$-group.

Proof: The relation $w \notin H$ implies $R \neq R^1$, and then each of the cones $R$ and $R^1$ is different from any of the cones $P, Q, \ldots$, $\tilde{P}, \tilde{Q}, \ldots$. [3, Corollary 3.2, (i)] yields that $H, Hs, Ht, \ldots$, $Hz, Hwx, Hwy$ are subsets of $\text{Aut}_0''(V)$. Now, let $\alpha \in \text{Aut}_0''(V)$, and let $\beta = \varrho_{V, U}(\alpha), \alpha_0 = \varrho_{V, Wx}(\alpha)$. Suppose that $\beta \in H$. In accordance with [3, Corollary 3.2, (i)] we obtain $C_{\geq}(V; a) = C_{\geq}(V; a_0(a))$ for all $a \in V \setminus U$. Hence $\alpha_0(\{A, A^1\}) = \{A, A^1\}$, $\alpha_0(\{B, B^1\}) = \{B, B^1\}, \ldots$, $\alpha_0(\{C, E\}) = \{C, E\}, \alpha_0(\{C^1, E^1\}) = \{C^1, E^1\}$, and $\alpha_0$ leaves the dimers invariant. Moreover, $\alpha_0$ maps any chiral pair onto a chiral pair, therefore $\alpha_0$ on the set $\{C, E, C^1, E^1\}$ is either id, or $z$. Thus, $\alpha \in K \cup Kz$, where $K = H \times \langle s \rangle \times \langle t \rangle \times \cdots$. Now, suppose that $\beta \in Hw$. Then $\beta(R) = R^1$, and $\beta$ leaves the cones $P, Q, \ldots$, and the cones $\tilde{P}, \tilde{Q}, \ldots$, invariant. According to [3, Corollary 3.2, (i)], we have $\alpha_0(\{A, A^1\}) = \{A, A^1\}$, $\alpha_0(\{B, B^1\}) = \{B, B^1\}, \ldots$, $\alpha_0(\{C, E\}) = \{C^1, E^1\}$, $\alpha_0(\{C^1, E^1\}) = \{C, E\}$, and $\alpha_0$ leaves the dimers invariant. Since $\alpha_0$ maps any chiral pair onto a chiral pair, we obtain that $\alpha_0$ on the set $\{C, E, C^1, E^1\}$ is either $x$, or $y = zx$. Now, we have $\alpha \in Kwx, Kwy$. Since $x, y, z, m$, commute among themselves, and each of them commutes with $K$, the proof is completed.

Lemma 3.5. Let the difference $V \setminus U$ be a $G''$-orbit that consists of several chiral pairs $\{A, A^1\}, \{B, B^1\}, \ldots$, $\{C, C^1\}, \{E, E^1\}$, with $C_{\geq}(V; C) = C_{\geq}(V; E) = R, C_{\geq}(V; C^1) = C_{\geq}(V; E^1) = R^1$, and of several dimers. Suppose that the cones of the members of the chiral pairs $\{A, A^1\}, \{B, B^1\}, \ldots$, the cones $R, R^1$, and the cones of the dimers, are all pairwise different, and suppose that

$$I_{V, U} = H \times \langle w \rangle,$$

where $w$ permutes the cones of the members of each chiral pair and leaves the cones of the dimers invariant. Then there exists a decomposition

$$\text{Aut}_0''(V) = H \times \langle z \rangle \times \langle wx \rangle,$$

where $z = (C, E)(C^1, E^1), x = (A, A^1)(B, B^1) \cdots (C, C^1)(E, E^1)$, the restriction homomorphism $\varrho_{V, U}$ has kernel $\langle z \rangle$, and $\text{Aut}_0''(V)$ is a commutative $2$-group.

Proof: [3, Corollary 3.2, (ii)] yields that $H, Hz, Hwx, Hwy$, where $y = zx$, are subsets of $\text{Aut}_0''(V)$. Now, let $\alpha \in \text{Aut}_0''(V)$, and let $\beta = \varrho_{V, U}(\alpha), \alpha_0 = \varrho_{V, Wx}(\alpha)$. 

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Suppose that $\beta \in H$. In accordance with [3, Corollary 3.2, (i)], the bijection $\alpha_0$ leaves the members of the chiral pairs $\{A, A^1\}$, $\{B, B^1\}$, ..., as well as the dimers invariant, and $\alpha_0\{C,E\} = \{C,E\}$, $\alpha_0\{C^1, E^1\} = \{C^1, E^1\}$. Moreover, $\alpha_0$ maps any chiral pair onto a chiral pair, therefore $\alpha_0$ on the set $\{C,E, C^1, E^1\}$ is either id, or $z$. Thus, $\alpha \in H \cup Hz$. Now, suppose that $\beta \in Hw$. Again, according to [3, Corollary 3.2, (i)], we have $\alpha_0(A) = A^1$, $\alpha_0(A^1) = A$, $\alpha_0(B) = B^1$, $\alpha_0(B^1) = B$, ..., $\alpha_0\{C,E\} = \{C,E\}$, $\alpha_0\{C^1, E^1\} = \{C,E\}$, and $\alpha_0$ leaves the dimers invariant. Since $\alpha_0$ maps any chiral pair onto a chiral pair, we obtain that $\alpha_0$ on the set $\{C,E, C^1, E^1\}$ is either $wx$, or $wy$. Now, we have $\alpha \in Hx \cup Hy$. Since $x$, $y$, $z$, commute, and since each of them commutes with $H$, the proof is done.

**Theorem 3.6. One has:**

(i) $\text{Aut}_0'(T_{D_1;G}) = \langle (c(4,2), e(4,2)) \rangle \simeq C_2$;

(ii) $\text{Aut}_0'(T_{D_2;G}) = \langle (a(4,12), b(4,12)), (c(4,2), e(4,2))(e(4,12), f(4,12)) \rangle \simeq C_2 \times C_2$;

(iii) $\text{Aut}_0'(T_{D_3;G}) = \langle (a(4,12), b(4,12)), (c(4,2), e(4,2))(e(4,12), f(4,12))(b(32), c(32)) \rangle \simeq C_2 \times C_2$;

(iv) $\text{Aut}_0'(T_{D_4;G}) = \langle (c(4,2), e(4,2))(a(4,12), b(4,12))(e(4,12), f(4,12))(b(32), c(32)) \rangle$;

(v) $\text{Aut}_0'(T_{D_5;G}) = \langle (c(3,13), \bar{c}(3,13))(h(3,13), \bar{h}(3,13)), (m(3,13), \bar{m}(3,13))(p(3,13), \bar{p}(3,13)),$

(vi) $\text{Aut}_0'(T_{D_6;G}) = \langle (c(3,13), \bar{c}(3,13))(h(3,13), \bar{h}(3,13)), (m(3,13), \bar{m}(3,13))(p(3,13), \bar{p}(3,13)),$

**Proof:** (i) Let us set $D'_0 = \{(6), (5,1)\}$. Section 2. Cases 2, 3, yield that $\text{Aut}_0'(T_{D_0;G})$ is the trivial group. The structure of the $(4,2)$-level as well as the inequalities among tabloids that correspond to the dominance order inequality $(4,2) < (5,1)$ are presented in Section 2, Case 3. We set $U = T_{D_0;G}$, $U_1^{(1)} = T_{D_0;G} \cup \{(c(4,2), e(4,2), a(4,2))\}$, $V = T_{D_1;G}$, where $D'_1 = \{((5,1), (4,2))\}$, and note that $\tilde{V}$ is a barrier of $T_{(4,2);G}$ in $T_{D_1;G}$. We have

$C_>(\tilde{V}; c(4,2)) = C_>(\tilde{V}; e(4,2)) = C_>(\tilde{V}; a(4,2)) = C_>(\tilde{V}; b(4,2))^\dagger = \{a(5,1)\}$. 
the dimer is 
\[ P = C_>(\bar{V}; a_{(4,1^2)}) = C_>(\bar{V}; b_{(4,1^2)}) = \{a_{(4,2)}\}, \]
\[ Q = C_>(\bar{V}; e_{(4,1^2)}) = \{c_{(4,2)}\}, \quad Q^1 = C_>(\bar{V}; f_{(4,1^2)}) = \{e_{(4,2)}\}. \]

There exists a decomposition \( Aut''_0(U_2^{(1)}) = \langle(a_{(4,1^2)}, b_{(4,1^2)})\rangle \), and by adding this dimer to the set \( U_2^{(1)} \) we get \( T_{D_2;G} \). Now, [2, Lemma 3.3] finishes the proof of (ii).

(iii) The inequalities among tableaux, which correspond to the dominance order inequality \((3^2) < (4, 2)\), as well as the description of the \((3^2)\)-level are presented in Section 2, Case 5. We set \( \bar{V} = T_{D_3;G} \), where \( D_3 = \{(4, 2), (4, 1^2), (3^2)\} \) and note that \( \bar{V} \) is a barrier of \( T_{(3^2);G} \) in \( T_{D_3;G} \).

First, we add the two dimers to \( T_{D_2;G} \) and get \( U_3^{(1)} = T_{D_2;G} \cup \{a_{(3^2)}, e_{(3^2)}\} \). The cones \( C_>(\bar{V}; a_{(3^2)}) = \{a_{(4,2)}\} \) and \( C_>(\bar{V}; e_{(3^2)}) = \{a_{(4,2)}, c_{(4,2)}, e_{(4,2)}\} \) of the dimer are \( Aut''_0(T_{D_2;G}) \) invariant, therefore [2, Lemma 3.3] yields

\[ Aut''_0(U_3^{(1)}) = \langle(a_{(4,1^2)}, b_{(4,1^2)}), (e_{(4,2)}, c_{(4,2)})(e_{(4,1^2)}, f_{(4,1^2)})\rangle. \]

Next, we supplement the set \( U_3^{(1)} \) with the chiral pair \( \{b_{(3^2)}, c_{(3^2)}\} \) and obtain \( T_{D_3;G} \). We have \( P = C_>(\bar{V}; b_{(3^2)}) = \{a_{(4,2)}, b_{(4,2), c_{(4,2)}, e_{(4,2)}}, \quad P^1 = C_>(\bar{V}; c_{(3^2)}) = \{a_{(4,2)}, b_{(4,2), e_{(4,2)}}, \quad and the group \( Aut''_0(U_3^{(1)}) \) can be decomposed as \( Aut''_0(U_3^{(1)}) = H \times \langle w \rangle \), where as usual \( H = \langle(a_{(4,1^2)}, b_{(4,1^2)})\rangle \) is the group of automorphisms that leave the cones \( P \) and \( P^1 \) invariant, and the automorphism \( w = (c_{(4,2)}, e_{(4,2)})(e_{(4,1^2)}, f_{(4,1^2)}) \) permutes \( P \) and \( P^1 \).

Now, [2, Lemma 3.4, (ii)] implies (iii).

(iv) In Section 2, Case 6, we have a description of the \((3, 2, 1)\)-level, and the inequalities among tableaux that correspond to the inequalities \((3, 2, 1) < (4, 1^2)\) and \((3, 2, 1) < (3^2)\) in the dominance order. We set \( \bar{V} = T_{D_3;G} \), where \( D_3 = \{(4, 1^2), (3^2), (3, 2, 1)\} \) and note that \( \bar{V} \) is a barrier of \( T_{(3, 2, 1);G} \) in \( T_{D_3;G} \). Let us first add the chiral pair that takes alone is a \( G'' \)-orbit: \( U_4^{(1)} = T_{D_3;G} \cup \{c_{(3, 2, 1)}, h_{(3, 2, 1)}\} \). We have \( P = C_>(\bar{V}; c_{(3, 2, 1)}) = \{b_{(4,1^2)}, e_{(4,1^2)}, b_{(3^2)}\}, \quad P^1 = C_>(\bar{V}; h_{(3, 2, 1)}) = \{a_{(4,12)}, f_{(4,12)}, c_{(32)}\} \). Among the four elements of the automorphism group \( Aut''_0(T_{D_3;G}) \) only two induce a permutation of the cones \( P \) and \( P^1 \): \( id \) and \( w = (a_{(4,1^2)}, b_{(4,1^2)})(c_{(4,2)}, e_{(4,2)})(e_{(4,1^2)}, f_{(4,1^2)})(b_{(3^2)}, c_{(3^2)}). \)
Then [3, Corollary 3.2, (i), (ii)] yields $H \leq Aut''_0(U_4^{(1)})$, and $H wr \leq Aut''_0(U_4^{(1)})$, where $H = \langle id \rangle$, and $r = (c_{(3,2,1)}, h_{(3,2,1)})$. Thus, $I_{U_4^{(1)}, T_{D_6;G}} = H \times \langle w \rangle$. Now, [2, Lemma 3.4, (i)] implies $Aut''_0(U_4^{(1)}) = H \times \langle wr \rangle$. Next we set $U_4^{(2)} = U_4^{(1)} \cup \{b_{(3,2,1)}, f_{(3,2,1)}, e_{(3,2,1)}, k_{(3,2,1)}\}$. We denote for short the corresponding cones as follows:

$$Q = C > (\bar{V}; b_{(3,2,1)}) = \{a_{(4,1^2)}, c_{(4,1^2)}, b_{(3^2)}\},$$

$$Q^1 = C > (\bar{V}; f_{(3,2,1)}) = \{b_{(4,1^2)}, c_{(4,1^2)}, c_{(3^2)}\},$$

$$R = C > (\bar{V}; e_{(3,2,1)}) = \{c_{(4,1^2)}, e_{(4,1^2)}, b_{(3^2)}\},$$

$$R^1 = C > (\bar{V}; k_{(3,2,1)}) = \{c_{(4,1^2)}, f_{(4,1^2)}, c_{(3^2)}\}.$$

We have $u(Q) = Q^1$, $u(R) = R^1$, for $u = wr$. Again, [3, Corollary 3.2, (i), (ii)] implies $H \leq Aut''_0(U_4^{(2)})$, and $H us \leqAut''_0(U_4^{(2)})$, where $s = (b_{(3,2,1)}, f_{(3,2,1)})(e_{(3,2,1)}, k_{(3,2,1)})$, so the restriction homomorphism $\theta_{U_4^{(2)}, U_4^{(1)}}$ is surjective, and in accordance with [2, Lemma 3.6, (i)] we obtain $Aut''_0(U_4^{(2)}) = H \times \langle us \rangle$. Further, we supplement $U_4^{(2)}$ with the last $G''$-orbit consisting of a chiral pair and two dimers:

$$T_{D_4;G} = U_4^{(2)} \cup \{\ell_{(3,2,1)}, m_{(3,2,1)}, a_{(3,2,1)}, p_{(3,2,1)}\}.$$

The corresponding cones are

$$X = C > (\bar{V}; \ell_{(3,2,1)}) = \{a_{(4,1^2^2)}, e_{(4,1^2^2)}, e_{(3^2)}\},$$

$$X^1 = C > (\bar{V}; m_{(3,2,1)}) = \{b_{(4,1^2)}, f_{(4,1^2)}, e_{(3^2)}\},$$

$$\tilde{P} = C > (\bar{V}; a_{(3,2,1)}) = \{a_{(4,1^2)}, b_{(4,1^2)}, a_{(3^2)}\},$$

$$\tilde{Q} = C > (\bar{V}; p_{(3,2,1)}) = \{e_{(4,1^2)}, f_{(4,1^2)}, e_{(3^2)}\}.$$
The corresponding cones are pairs and get
\begin{align*}
C_>(\bar{V} ; k_{(3,1^3)}) &= C_>(\bar{V} ; \bar{k}_{(3,1^3)}) = \{h_{(3,2,1)}, k_{(3,2,1)}\}.
\end{align*}

Now, the decomposition $\text{Aut}_0''(T_{D_{4}; G}) = H \times \langle u \rangle$, where $H = \{id\}$, the surjectivity of the corresponding restriction homomorphism, as well as $[2$, Lemma 3.6, (i)$]$, yield consecutively $\text{Aut}_0''(U_5^{(1)}) = H \times \langle v \rangle$, for $v = u(b_{(3,1^3)}, f_{(3,1^3)})(\bar{e}_{(3,1^3)}, \bar{k}_{(3,1^3)})$, and $\text{Aut}_0''(U_5^{(2)}) = H \times \langle w \rangle$, for $w = v(b_{(3,1^3)}, f_{(3,1^3)})(e_{(3,1^3)}, k_{(3,1^3)})$. Further, let us set $U_5^{(3)} = U_5^{(2)} \cup \{c_{(3,1^3)}, h_{(3,1^3)}, \bar{c}_{(3,1^3)}, \bar{h}_{(3,1^3)}\}$. We have
\begin{align*}
C_>(\bar{V} ; c_{(3,1^3)}) &= C_>(\bar{V} ; \bar{c}_{(3,1^3)}) = \{b_{(3,2,1)}, e_{(3,2,1)}\},
\end{align*}
and the automorphism $w$ permutes these two cones. According to $[2$, Lemma 3.4, (ii)$]$, we get $\text{Aut}_0''(U_5^{(3)}) = H \times \langle x \rangle \times \langle wx \rangle$, where $x = (c_{(3,1^3)}, h_{(3,1^3)})(\bar{c}_{(3,1^3)}, \bar{h}_{(3,1^3)})$, and $y(P) = P$, $y(Q) = Q$, $y(R) = R^1$. Thus the corresponding restriction homomorphism is surjective and Lemma 3.4 yields part (v).

(vi) In Section 2, Case 8, we describe the $(2^4)$-level. All inequalities between tabloids, that correspond to the inequality $(2^3) < (3, 2, 1)$, are presented there. Let $\bar{V} = T_{D_{2}; G}$, where $D_{2} = \{((3, 2), (2^3))\}$. The set $\bar{V}$ is a barrier of $T_{(2^3); G}$ in $T_{D_{6}; G}$. Let us first add to the set $T_{D_{6}; G}$ the $G''$-orbit that contains four chiral pairs: $U_6^{(1)} = T_{D_{5}; G} \cup \{a_{(2^3)}, \bar{a}_{(2^3)}, c_{(2^3)}, \bar{c}_{(2^3)}, h_{(2^3)}, \ell_{(2^3)}, k_{(2^3)}, \ell_{(2^3)}\}$. Their cones are
\begin{align*}
C_>(\bar{V} ; a_{(2^3)}) &= \{a_{(3,2,1)}, b_{(3,2,1)}, c_{(3,2,1)}, f_{(3,2,1)}, \ell_{(3,2,1)}\},
\end{align*}
\begin{align*}
C_>(\bar{V} ; \bar{a}_{(2^3)}) &= \{a_{(3,2,1)}, b_{(3,2,1)}, f_{(3,2,1)}, h_{(3,2,1)}, m_{(3,2,1)}\},
\end{align*}
\begin{align*}
C_>(\bar{V} ; c_{(2^3)}) &= \{a_{(3,2,1)}, c_{(3,2,1)}, e_{(3,2,1)}, k_{(3,2,1)}, \ell_{(3,2,1)}, p_{(3,2,1)}\},
\end{align*}
\begin{align*}
C_>(\bar{V} ; \bar{c}_{(2^3)}) &= \{a_{(3,2,1)}, e_{(3,2,1)}, h_{(3,2,1)}, k_{(3,2,1)}, m_{(3,2,1)}, p_{(3,2,1)}\},
\end{align*}
\begin{align*}
C_>(\bar{V} ; h_{(2^3)}) &= C_>(\bar{V} ; k_{(2^3)}) = \{f_{(3,2,1)}, h_{(3,2,1)}, k_{(3,2,1)}, \ell_{(3,2,1)}, m_{(3,2,1)}, p_{(3,2,1)}\},
\end{align*}
\begin{align*}
C_>(\bar{V} ; \ell_{(2^3)}) &= C_>(\bar{V} ; \ell_{(2^3)}) = \{b_{(3,2,1)}, c_{(3,2,1)}, e_{(3,2,1)}, \ell_{(3,2,1)}, m_{(3,2,1)}; p_{(3,2,1)}\}.
\end{align*}

The group $\text{Aut}_0''(T_{D_{6}; G})$ can be decomposed as $\text{Aut}_0''(T_{D_{6}; G}) = H \times \langle w \rangle$, where $w$ is its last generator, as written in (v), and $H$ is generated by all the rest. The automorphism
$w$ permutes the cones of the members of each of the added four chiral pairs. Therefore, the corresponding restriction homomorphism is surjective, and now Lemma 3.5 implies

$$\text{Aut}''_0(U_6^{(1)}) = \langle (c_{(3,1^3)}, e_{(3,1^3)})(h_{(3,1^3)}, \tilde{h}_{(3,1^3)}), (m_{(3,1^3)}, \tilde{m}_{(3,1^3)})(p_{(3,1^3)}, \tilde{p}_{(3,1^3)}),$$

$$(a_{(3,1^3)}, \tilde{a}_{(3,1^3)}), (\ell_{(3,1^3)}, \tilde{\ell}_{(3,1^3)}), (h_{(2^3)}, k_{(2^3)})(\ell_{(2^3)}, \tilde{\ell}_{(2^3)}),$$

$$(c_{(4,2)}, e_{(4,2)})(a_{(4,1^2)}, b_{(4,1^2)})(e_{(4,1^2)}, f_{(4,1^2)})(b_{(3^2)}, c_{(3^2)})(c_{(3,1^2)}, h_{(3,1^2)}),$$

$$(b_{(3,2,1)}, f_{(3,2,1)})(e_{(3,2,1)}, k_{(3,2,1)})(\ell_{(3,2,1)}, m_{(3,2,1)})(b_{(3,1^3)}, f_{(3,1^3)})(\tilde{e}_{(3,1^3)}, \tilde{k}_{(3,1^3)}),$$

$$(b_{(3,1^3)}, \tilde{f}_{(3,1^3)})(e_{(3,1^3)}, k_{(3,1^3)})(c_{(3,1^3)}, h_{(3,1^3)})(\tilde{c}_{(3,1^3)}, \tilde{h}_{(3,1^3)})(m_{(3,1^3)}, p_{(3,1^3)}),$$

$$(\tilde{m}_{(3,1^3)}, \tilde{p}_{(3,1^3)})(h_{(2^3)}, \tilde{\ell}_{(2^3)})(k_{(2^3)}, \tilde{\ell}_{(2^3)})).$$

Now, we add to $U_6^{(1)}$ consecutively the three $G''$-orbits consisting of one chiral pair and one dimer: $U_6^{(2)} = U_6^{(1)} \cup \{k_{(2^3)}, m_{(2^3)}, b_{(2^3)}\}$, $U_6^{(3)} = U_6^{(2)} \cup \{\tilde{h}_{(2^3)}, \tilde{m}_{(2^3)}, \tilde{b}_{(2^3)}\}$, $U_6^{(4)} = U_6^{(3)} \cup \{f_{(2^3)}, \tilde{f}_{(2^3)}, e_{(2^3)}\}$. Here are the corresponding cones:

$$C_>(V; k_{(2^3)}) = \{h_{(3,2,1)}, (3,2,1), p_{(3,2,1)}\},$$

$$C_>(V; m_{(2^3)}) = \{c_{(3,2,1)}, m_{(3,2,1)}, p_{(3,2,1)}\},$$

$$C_>(V; b_{(2^3)}) = \{a_{(3,2,1)}, c_{(3,2,1)}, h_{(3,2,1)}, \ell_{(3,2,1)}, m_{(3,2,1)}\},$$

$$C_>(V; \tilde{f}_{(2^3)}) = \{f_{(3,2,1)}, k_{(3,2,1)}, m_{(3,2,1)}\},$$

$$C_>(V; \tilde{m}_{(2^3)}) = \{b_{(3,2,1)}, e_{(3,2,1)}, \ell_{(3,2,1)}\},$$

$$C_>(V; \tilde{c}_{(2^3)}) = \{a_{(3,2,1)}, b_{(3,2,1)}, e_{(3,2,1)}, f_{(3,2,1)}, k_{(3,2,1)}, p_{(3,2,1)}\},$$

$$C_>(V; \tilde{e}_{(2^3)}) = \{b_{(3,2,1)}, e_{(3,2,1)}, b_{(3,2,1)}, f_{(3,2,1)}, h_{(3,2,1)}, k_{(3,2,1)}\}.$$

For any one of these $G''$-orbits, the last generator of the group $\text{Aut}''_0(U_6^{(1)})$, or its extension, permutes the cones of the members of the chiral pair and leaves the cone of the dimer invariant. Applying Lemma 3.3 three times, we obtain

$$\text{Aut}''_0(U_6^{(4)}) = \langle (c_{(3,1^3)}, e_{(3,1^3)})(h_{(3,1^3)}, \tilde{h}_{(3,1^3)}), (m_{(3,1^3)}, \tilde{m}_{(3,1^3)})(p_{(3,1^3)}, \tilde{p}_{(3,1^3)}),$$

$$(a_{(3,1^3)}, \tilde{a}_{(3,1^3)}), (\ell_{(3,1^3)}, \tilde{\ell}_{(3,1^3)}), (h_{(2^3)}, k_{(2^3)})(\ell_{(2^3)}, \tilde{\ell}_{(2^3)}),$$

$$(c_{(4,2)}, e_{(4,2)})(a_{(4,1^2)}, b_{(4,1^2)})(e_{(4,1^2)}, f_{(4,1^2)})(b_{(3^2)}, c_{(3^2)})(c_{(3,1^2)}, h_{(3,1^2)}),$$

$$(b_{(3,2,1)}, f_{(3,2,1)})(e_{(3,2,1)}, k_{(3,2,1)})(\ell_{(3,2,1)}, m_{(3,2,1)})(b_{(3,1^3)}, f_{(3,1^3)})(\tilde{e}_{(3,1^3)}, \tilde{k}_{(3,1^3)}),$$

$$(b_{(3,1^3)}, \tilde{f}_{(3,1^3)})(e_{(3,1^3)}, k_{(3,1^3)})(c_{(3,1^3)}, h_{(3,1^3)})(\tilde{c}_{(3,1^3)}, \tilde{h}_{(3,1^3)})(m_{(3,1^3)}, p_{(3,1^3)}),$$

$$(\tilde{m}_{(3,1^3)}, \tilde{p}_{(3,1^3)})(h_{(2^3)}, \tilde{\ell}_{(2^3)})(k_{(2^3)}, \tilde{\ell}_{(2^3)})(\tilde{f}_{(2^3)}, e_{(2^3)}).$$

We have $C_>(V; \tilde{e}_{(2^3)}) = C_>(V; e_{(2^3)})$, so the cone of the dimer $\tilde{e}_{(2^3)}$ is $\text{Aut}''_0(U_6^{(4)})$-invariant, and, in compliance with [2, Lemma 3.3], we get part (vi).

**Corollary 3.7.** The chiral pairs $\{a_{(4,1^2)}, b_{(4,1^2)}\}$, $\{e_{(4,1^2)}, f_{(4,1^2)}\}$ can be distinguished via substitution reactions among the elements of $T_{D_2}G$.

**Proof:** Lun-Senior's group $\text{Aut}''_0(T_{D_2}G)$ does not contain automorphism that maps the members of one of the chiral pairs onto the members of the other.
Remark 3.8. The members of the chiral pair \( \{a(4,1^2), b(4,1^2)\} \) can be obtained via substitution reactions by one and the same dimer \( a(4,2) \), whereas each of the members of the chiral pair \( \{c(4,2), e(4,2)\} \) produces via substitution reaction exactly one member of the chiral pair \( \{e(4,1^2), f(4,1^2)\} \), and the two members of the latter can be obtained in this way.

Corollary 3.9. The two dimers \( a(3^2) \) and \( e(3^2) \), can be distinguished via substitution reactions among the elements of \( T_{D_3} \).

Proof: The group \( Aut_0''(T_{D_3}) \) does not contain automorphism that maps one of the dimers onto the other.

Remark 3.10. The dimer \( a(3^2) \) can be produced via substitution reactions by exactly one \((4,2)\)-product whereas the dimer \( e(3^2) \) can be produced by three \((4,2)\)-products.

Since the group \( Aut_0''(T_{D_3}) \) contains only the identity and the chiral automorphism, we obtain the following two corollaries:

Corollary 3.11. The two dimers \( a(3,2,1) \) and \( p(3,2,1) \), can be distinguished via substitution reactions among the elements of \( T_{D_4} \).

Remark 3.12. The dimer \( a(3,2,1) \) can be produced via substitution reactions by the dimer \( a(3^2) \) which has the property that it can produce exactly one \((3,2,1)\)-product. On the other hand, the dimer \( p(3,2,1) \) can be produced by the dimer \( e(3^2) \) which has the property that it can produce three \((3,2,1)\)-products.

Corollary 3.13. Any two chiral pairs from

\[
\{b(3,2,1), f(3,2,1)\}, \ {e(3,2,1), h(3,2,1)\}, \ {e(3,2,1), k(3,2,1)\},
\]

can be distinguished via substitution reactions among the elements of \( T_{D_4} \).

Remark 3.14. Any member of the chiral pair \( \{b(3,2,1), f(3,2,1)\} \) can be produced via substitution reactions by one member of the chiral pair \( \{a(4,1^2), b(4,1^2)\} \), any member of the chiral pair \( \{c(3,2,1), h(3,2,1)\} \) can be produced via substitution reactions by one member of each chiral pair \( \{a(4,1^2), b(4,1^2)\}, \ {e(4,1^2), f(4,1^2)\}, \ {e(3,2,1), k(3,2,1)\} \) can be produced via substitution reactions by one member of the chiral pair \( \{e(4,1^2), f(4,1^2)\} \). In the end, it is enough to note that Corollary 3.7 holds.

Corollary 3.15. The chiral pairs \( \{c(3,1^3), h(3,1^3)\}, \ \{\bar{c}(3,1^3), \bar{h}(3,1^3)\} \) can not be distinguished via substitution reactions among the elements of \( T_{D_6} \).

Corollary 3.16. The chiral pairs \( \{m(3,1^3), p(3,1^3)\}, \ \{\bar{m}(3,1^3), \bar{p}(3,1^3)\} \) can not be distinguished via substitution reactions among the elements of \( T_{D_6} \).

Proofs: It is enough to note that Lunn-Senior’s group \( Aut_0''(T_{D_6}) \) contains the automorphisms \( (c(3,1^3), \bar{c}(3,1^3))(h(3,1^3), \bar{h}(3,1^3)) \), and \( (m(3,1^3), \bar{m}(3,1^3))(p(3,1^3), \bar{p}(3,1^3)) \), respectively.

Corollary 3.17. (i) The chiral pairs

\[
\{a(3,1^3), \bar{a}(3,1^3)\}, \ {\ell}(3,1^3), \bar{\ell}(3,1^3)\},
\]

can be distinguished via substitution reactions among the elements of \( T_{D_5} \).
(ii) any chiral pair from (3.18) and any one from the chiral pairs

\[ \{m_{(3,1^3)}, p_{(3,1^3)}\}, \{\bar{m}_{(3,1^3)}, \bar{p}_{(3,1^3)}\}; \]

can be distinguished via substitution reactions among the elements of \( T_{D_5;G} \).

**Proof:** In both parts (i) and (ii) Lunn-Senior’s group \( \text{Aut}_0(T_{D_5;G}) \) does not contain an automorphism that works.

**Remark 3.19.** The members of the chiral pair \( \{a_{(3,1^3)}, \bar{a}_{(3,1^3)}\} \) can be produced via substitution reactions by exactly one \((3,2,1)\)-product – the dimer \( a_{(3,2,1)} \), any member of the chiral pair \( \{\ell_{(3,1^3)}, \bar{\ell}_{(3,1^3)}\} \) can be produced via substitution reactions only by both members of the chiral pair \( \{\ell_{(3,2,1)}, m_{(3,2,1)}\} \), and any member of the chiral pair \( \{m_{(3,1^3)}, p_{(3,1^3)}\} \) (respectively, \( \{\bar{m}_{(3,1^3)}, \bar{p}_{(3,1^3)}\} \)) can be produced via substitution reactions by one member of the chiral pair \( \{\ell_{(3,2,1)}, m_{(3,2,1)}\} \), and by the dimer \( p_{(3,2,1)} \).

The two corollaries below can be proved in the same way.

**Corollary 3.20.** The two chiral pairs \( \{b_{(3,1^3)}, f_{(3,1^3)}\}, \{e_{(3,1^3)}, \bar{k}_{(3,1^3)}\} \) can be distinguished via substitution reactions among the elements of \( T_{D_5;G} \).

**Corollary 3.21.** The two chiral pairs \( \{b_{(3,1^3)}, f_{(3,1^3)}\}, \{e_{(3,1^3)}, k_{(3,1^3)}\} \) can be distinguished via substitution reactions among the elements of \( T_{D_5;G} \).

**Remark 3.22.** The members of the chiral pair \( \{b_{(3,2,1)}, f_{(3,2,1)}\} \) (respectively, the chiral pair \( \{e_{(3,2,1)}, k_{(3,2,1)}\} \)) produce the members of the chiral pair \( \{b_{(3,1^3)}, f_{(3,1^3)}\} \) as well as the members of the chiral pair \( \{b_{(3,1^3)}, f_{(3,1^3)}\} \) (respectively, \( \{e_{(3,1^3)}, k_{(3,1^3)}\} \)) as well as \( \{e_{(3,1^3)}, e_{(3,1^3)}\} \). Moreover, the members of \( \{b_{(3,2,1)}, f_{(3,2,1)}\} \) do not produce neither the members of \( \{e_{(3,1^3)}, k_{(3,1^3)}\} \) nor the members of \( \{e_{(3,1^3)}, k_{(3,1^3)}\} \), and similarly for \( \{e_{(3,2,1)}, k_{(3,2,1)}\} \). In the end we note that in accord to Corollary 3.13 the two chiral pairs \( \{b_{(3,2,1)}, f_{(3,2,1)}\} \) and \( \{e_{(3,2,1)}, k_{(3,2,1)}\} \) are distinguishable via substitution reactions among the elements of \( T_{D_5;G} \), and hence among the elements of \( T_{D_5;G} \).

**Acknowledgement**

This work is partially supported by Grant MI-1503/2005 of the Bulgarian Foundation of Scientific Research.

**References**