The genetic reactions of ethane

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ABSTRACT

A complete description of the genetic reactions among the derivatives of ethane C\textsubscript{2}H\textsubscript{6} by using Lunn-Senior’s mathematical model of isomerism in organic chemistry is presented. Lunn-Senior’s automorphism group (the structural group of the mathematical structure underlying the model) is introduced, found for some important substructures, and used for a comprehensive study of all pairs of derivatives that can and that can not be distinguished via substitution reactions.

1. Rationale

Lunn-Senior’s theory is based on a division of the molecule under question into skeleton and a set of \(d\) univalent substituents, and this division is fixed during a particular consideration. If we also fix a numbering \(1, 2, \ldots, d\) of the valences of the substituents, then a distribution of \(\lambda_1\) in number ligands of type \(x_1\), \(\lambda_2\) in number ligands of type \(x_2\), and so on, \(\lambda_1 + \lambda_2 + \cdots = d\), is uniquely determined via a dissection of the integer-valued interval \([1, d] = \{1, 2, \ldots, d\}\) into disjoined subsets \(A_1, A_2, \ldots\), where \(A_i\) is the set of numbers of valences of the substituents of type \(x_i\), \(i = 1, 2, \ldots\). We have \(|A_i| = \lambda_i\) and may suppose that \(\lambda_1 \geq \lambda_2 \geq \cdots\), so \(\lambda = (\lambda_1, \lambda_2, \ldots)\) is a partition of the number \(d\). Then the dissection \(A = (A_1, A_2, \ldots)\) of the set \([1, d]\) is called tabloid with \(d\) nodes. Thus, the partition \(\lambda\) represents the empirical formula \(x_1^{\lambda_1}x_2^{\lambda_2}\ldots\) and the tabloid \(A\) represents the structural formula of the ligands. The substitution isomerism, stereoisomerism, and structural isomerism define three equivalence relations on the set \(T_d\) of all tabloids (structural formulae) \(A\), and on its subsets \(T_\lambda \subset T_d\) consisting of all tabloids (structural formulae) \(A\) of shape \(\lambda\) (with empirical formula \(x_1^{\lambda_1}x_2^{\lambda_2}\ldots\)).

The fundamental assumption of Lunn and Senior in [4] is (up to a slight change of the terminology) that the classes of these equivalence relations are produced as orbits of three permutation groups \(G, G', G''\), respectively, which are subgroups of the symmetric group \(S_d\) and inherit the natural action \(\zeta(A) = (\zeta(A_1), \zeta(A_2), \ldots)\) of \(S_d\) on
the set $T_d$. Since the symmetric group $S_d$ is $d$-transitive, we can identify the factor-set $S_d/T_d$ with the set $P_d$ of all partitions $\lambda$ of $d$. If $\varphi:T_d \to P_d$ is the canonical projection that assigns to each tabloid $A$ its shape $\lambda$, $\varphi(A) = \lambda$, $\lambda_1 = |A_1|$, $\lambda_2 = |A_2|$, $\ldots$, then $\varphi^{-1}(\lambda) = T_\lambda$.

Further, Lunn and Senior assume that $G = G'$ or $G'$ contains $G$ as a (normal) subgroup of index 2, and the last statement is true if and only if there are chiral pairs among the stereoisomers of the molecule under consideration; the chiral pairs are represented by the $G'$-orbits in $T_d$ that contain two $G'$-orbits and the dimers are represented by the $G'$-orbits in $T_d$ that contain one $G'$-orbit. Moreover, every structural isomer is represented by a $G''$-orbit in $T_d$, and any such orbit is a union of $G'$-orbits. As a consequence of the latter we obtain that the group $G'$ is a subgroup of $G''$.

Let $T_{d;G}$ ($T_{d;G'}$, $T_{d;G''}$) be the set of all $G'$-orbits ($G'$-orbits, $G''$-orbits, respectively) in the set $T_d$.

The canonical projection $\varphi$ factors out to a canonical projection $\varphi_G:T_{d;G} \to P_d$ and for any $\lambda \in P_d$ the inverse image $\varphi_G^{-1}(\lambda)$ can be identified with the factor-set $G\backslash T_\lambda$. Similarly we get the canonical projections $\varphi_{G'}:T_{d;G'} \to P_d$ and $\varphi_{G''}:T_{d;G''} \to P_d$, and the inverse images $T_{\lambda;G'}$ and $T_{\lambda;G''}$. The elements of the set $T_{d;G}$ ($T_{d;G'}$, $T_{d;G''}$) represent the univalent substitution isomers (stereoisomers, structural isomers, respectively). The above projections allow us to say that a univalent substitution isomer (stereoisomer, structural isomer) has empirical formula $x_1^\lambda_1 x_2^\lambda_2 \ldots$ if for the corresponding $G'$-orbit $a \in T_{d;G'}$ ($G'$-orbit $a' \in T_{d;G'}$, $G''$-orbit $a'' \in T_{d;G''}$, respectively) one has $\varphi_{G'}(a) = \lambda$ (or $\varphi_{G'}(a') = \lambda$, $\varphi_{G''}(a'') = \lambda$, respectively). Equivalently, $a \in T_{\lambda;G}$ ($a' \in T_{\lambda;G'}$, $a'' \in T_{\lambda;G''}$, respectively).

Because of the inclusions $G \leq G' \leq G''$, the $G$-orbit $a = O_G(A)$, $A \in T_d$, which represents an univalent substitution isomer produces canonically a $G'$-orbit $a' = O_{G'}(A)$ that represents a stereoisomer, and a $G''$-orbit $a'' = O_{G''}(A)$ that represents a structural isomer. Moreover, any $G'$-orbit $a' = O_{G'}(A)$ produces canonically the $G''$-orbit $a'' = O_{G''}(A)$. Thus, we get a diagram

$$
\begin{array}{ccc}
\psi & \leftarrow & \psi' \\
\downarrow & & \downarrow \\
T_{d;G'} & \to & T_{d;G''}
\end{array}
$$

of canonical surjective maps $\psi$, $\psi'$, $\psi''$, which is commutative, that is, $\psi' \circ \psi = \psi''$. This diagram allows us to say that two univalent substitution isomers are stereoisomERICally (structurally) identical if for the corresponding $G'$-orbits $a$ and $b$ one has $\psi(a) = \psi(b)$ ($\psi''(a) = \psi''(b)$, respectively). Similarly, we say that two stereoisomers are structurally identical if for the corresponding $G'$-orbits $a'$ and $b'$ one has $\psi'(a') = \psi'(b')$. The commutativity of the above diagram yields that if two univalent substitution isomers are stereoisomerically identical, then they also are structurally identical.

Since any $G'$-orbit ($G''$-orbit) is a union of $G$-orbits, we can identify it with a subset of the corresponding $T_{\lambda;G}$. In particular, any chiral pair can be identified with a pair of elements in this $T_{\lambda;G}$.

The model reflects the substitution (genetic) reactions among the derivatives via introducing a partial order on the set $T_{d;G}$ in the following way:

(a) on the level of empirical formulae a simple substitution reaction has the form

$$
x_1^{\mu_1} \ldots x_i^{\mu_i} \ldots x_j^{\mu_j} \ldots \to x_1^{\lambda_1} \ldots x_i^{\lambda_i} \ldots x_j^{\lambda_j} \ldots,
$$

(1.1)
where $\lambda, \mu \in P_d$, and $\mu_1 = \lambda_1, \ldots, \mu_i = \lambda_i + 1, \ldots, \mu_j = \lambda_j - 1, \ldots, \mu_d = \lambda_d$, that is, a replacement of a ligand of type $x_i$ by a ligand of type $x_j$, and in this case we write $\mu = \rho_{i,j} \lambda$ and $\lambda < \mu$;

(b) on the level of structural formulae the simple substitution reaction (1.1) has the form

$$B = (B_1, B_2, \ldots B_i \ldots B_j \ldots) \rightarrow A = (A_1, A_2, \ldots A_i \ldots A_j \ldots),$$

where $B$ can be obtained from $A$ by moving an element $s \in A_j$ to the set $A_i$, and in this case we write $B = R_{i,s}A$ and $A < B$; formula (1.2) carries the following instruction:

“replace the ligand of type $x_i$ of $B$ in position $s$ by a ligand of type $x_j$, and, as a result, obtain $A$”;

(c) in general: we write $\lambda \leq \mu$ if $\lambda$ can be obtained from $\mu$ via a finite sequence of simple replacements of the type (a) — this is the famous dominance order on the set $P_d$; we write $A \leq B$ if $B$ can be obtained from $A$ via a finite sequence of simple movements of type (b) — this a partial order on the set $T_d$; an equivalent definition of the latter partial order is: $A \leq B$ if $A_1 \cup A_2 \ldots \cup A_k \subseteq B_1 \cup B_2 \ldots \cup B_k$ for all $k = 1, 2, \ldots d$;

(d) we factor out the partial order from (c) and get a partial order on the set $T_{d|G}$ of $G$-orbits via the rule: $a \leq b$ if there exist $A \in a$ and $B \in b$ such that $A \leq B$.

The relation $a < b$ means that the product that corresponds to $a$ can be obtained from the product that corresponds to $b$ via several consecutive simple substitution reactions. Therefore the partially ordered set $T_{d|G}$ represents the univalent substitution isomers and all possible genetic relations among them.

An important problem that can be (at least partially) solved within the framework of Lunn-Senior’s model is: given an univalent substitution isomer how to find (the $G$-orbit of) the structural formula that corresponds to this isomer. The classical Körner relations show that this problem can be solved completely for the di-substitution and tri-substitution homogeneous derivatives of benzene (with empirical formulae $C_6^6 x_1^4 x_2^2$ and $C_6^6 x_1^3 x_2^3$), thus identifying them as para, ortho, meta compound, and asymmetrical, vicinal, symmetrical compound, respectively. On the other hand, the pairs of structurally identical di-substitution homogeneous and di-substitution heterogeneous derivatives of ethene (with empirical formulae $C_2^2 x_1^3 x_2^2$ and $C_2^2 x_1^2 x_2 x_3$) can not be identified with their structural formulae using only substitution reactions. The intrinsic reason of this phenomenon is the existence of an automorphism of the mathematical structure that underlies the model, which maps one of the members of the pair onto the other. The formal definitions are as follows. Several univalent substitution isomers with substituents empirical formula $x_1^{\lambda_1} x_2^{\lambda_2} \ldots$ are said to be indistinguishable via substitution reactions if for any pair of elements of the corresponding set of structural formulae \{a, b, \ldots\} $\subset T_{\lambda;G}$, say, $a$, $b$, there exists an automorphism $\alpha: T_{d;G} \rightarrow T_{d;G}$ of the partially ordered set $T_{d;G}$, such that: (i) $\alpha(T_{\mu;G}) = T_{\mu;G}$ for any $\mu \in P_d$, (ii) $\alpha$ maps any chiral pair onto a chiral pair, (iii) $c$ and $\alpha(c)$ are structurally identical for any $c \in T_{d;G}$, and (iv) $\alpha(a) = b$. Otherwise, they are called distinguishable via substitution reactions.

Unfortunately, the chemists have synthesized enough products with a given substituents empirical formula only for small number of $\lambda \in P_d$ — usually they know the mono-substitution ($\lambda = (d - 1, 1)$), di-substitution ($\lambda = (d - 2, 2), (d - 2, 1^2)$), tri-substitution ($\lambda = (d - 3, 3), (d - 3, 2, 1), (d - 3, 1^3)$), etc. derivatives. Thus, most of the time we have to consider not the whole partially ordered set $T_{d;G}$ alone, but some parts of it — the subsets of the form $T_{D;G} = \cup_{\mu \in D} T_{\mu;G}$, where $D \subset P_d$, with the induced partial order.
Similarly, we set \( T_{D;G'} = \bigcup_{\mu \in D} T_{\mu;G'} \), \( T_{D;G''} = \bigcup_{\mu \in D} T_{\mu;G''} \), and let

\[
\psi_D \quad \psi_D'' \\
T_{D;G'} \quad T_{D;G''} \\
\psi_D'
\]

be the corresponding commutative diagram build by the surjective restrictions \( \psi_D, \psi_D', \psi_D'' \). For a fixed \( D \subset P_d \) the above definitions can be generalized in the following way. Several univalent substitution isomers with substituents empirical formula \( x_1^{\lambda_1}x_2^{\lambda_2} \ldots \), \( \lambda \in D \), are said to be indistinguishable via substitution reactions among the elements of \( T_{D;G} \) if for any pair of elements of the corresponding set of structural formulae \( \{a, b, \ldots\} \subset T_{\lambda;G} \), say \( a, b \), there exists an automorphism \( \alpha: T_{D;G} \to T_{D;G} \) of the partially ordered set \( T_{D;G} \), such that: (i)\( \alpha(T_{\mu;G}) = T_{\mu;G} \) for any \( \mu \in D \), (ii) \( \alpha \) maps any chiral pair onto a chiral pair, (iii) \( c \) and \( \alpha(c) \) are structurally identical for any \( c \in T_{D;G} \), and (iv) \( \alpha(a) = b \). Otherwise, they are called distinguishable via substitution reactions among the elements of \( T_{D;G} \).

Let \( Aut(T_{D;G}) \) be group of the automorphisms of the partially ordered set \( T_{D;G} \) (that is, bijections \( u: T_{D;G} \to T_{D;G} \) such that the inequalities \( a \leq b \) and \( u(a) \leq u(b) \) are equivalent for any pair \( a, b \in T_{D;G} \)). The elements of \( Aut(T_{D;G}) \) which satisfy the condition (i) form a subgroup \( Aut_0(T_{D;G}) \leq Aut(T_{D;G}) \), and those of them that obey the condition (ii) form a subgroup \( Aut_0'(T_{D;G}) \leq Aut_0(T_{D;G}) \). The elements of the latter are called chiral automorphisms of the partially ordered set \( T_{D;G} \). In order to take into account also the structural isomerism we introduce the following group of automorphisms:

\[
Aut_0''(T_{D;G}) = \{ \alpha \in Aut_0(T_{D;G}) \mid \psi_D'' \circ \alpha = \psi_D'' \}
\]

The condition \( \psi_D'' \circ \alpha = \psi_D'' \) is equivalent to (iii). The group \( Aut_0''(T_{D;G}) \) is called Lunn-Senior’s automorphism group and its elements are said to be Lunn-Senior’s automorphisms of \( T_{D;G} \).

More generally, let \( U \subset T_{d;G} \) be a union of \( G'' \)-orbits. For any \( \mu \in P_d \) we set \( U_\mu = U \cap T_{\mu;G} \). We define Lunn-Senior’s automorphism group \( Aut_0''(U) \) to be the set of all automorphisms \( \alpha \) of the partially ordered set \( U \), such that \( \alpha(U_\mu) = U_\mu \) for any \( \mu \in P_d \), the automorphism \( \alpha \) maps any chiral pair in \( U \) onto a chiral pair, and any \( G'' \)-orbit onto itself.

The group \( Aut(T_{D;G}) \) acts naturally on the set \( T_{D;G} \) and all of its subgroups inherit that action. The fact that the elements of a set of univalent substitution isomers with empirical formula \( x_1^{\lambda_1}x_2^{\lambda_2} \ldots \), \( \lambda \in D \), are indistinguishable via substitution reactions among the elements of \( T_{D;G} \) is equivalent to the statement that the corresponding set of structural formulae \( \{a, b, \ldots\} \subset T_{\lambda;G} \) is contained in a single \( Aut_0''(T_{D;G}) \)-orbit in \( T_{D;G} \).

In case there are chiral pairs among the derivatives of the molecule under question we have \( |G' : G| = 2 \), and then any element \( \tau \in G' \setminus G \) produces an involution \( \hat{\tau} \in Aut_0''(T_{D;G}) \) (so called chiral involution) which permutes the members of the chiral pairs and leaves the dimers invariant. In this case Lunn-Senior’s automorphism group \( Aut_0''(T_{D;G}) \) contains the cyclic group \( \langle \hat{\tau} \rangle \) of order 2, so its order is even, and, in particular, \( Aut_0''(T_{D;G}) \) is not trivial. We note especially the particular case when \( D \) has a single element \( \lambda \). Then the set \( T_{\lambda;G} \) is trivially ordered (that is, \( a \leq b \) is equivalent to \( a = b \)), so Lunn-Senior’s automorphism group \( Aut_0''(T_{\lambda;G}) \) consists of all permutations of the set \( T_{\lambda;G} \) that map any chiral pair onto a chiral pair and any \( G'' \)-orbit onto itself.
Remark 1.3. The relation of indistinguishability defined here implies the relation with the same name defined in [3], but does not coincide with it. Nevertheless, all statements proved there are true, if the words indistinguishable and distinguishable have the present meaning.

Section 2 contains a meticulous listing of all $G$-orbits, $G'$-orbits, and $G''$-orbits in $T_\lambda$, as well as their elements, for all $\lambda \in P_6$. Equivalently, we list all univalent substitution isomers, stereoisomers, and structural isomers of ethane with fixed empirical formula $\lambda$ for all $\lambda \in P_6$, as well as all different distributions of ligands that produce the same isomer. Moreover, we find all inequalities among the elements of $T_{6;G}$, that is, all substitution reactions among the products of ethane.

In Section 3 we prove several ad hoc lemmas and theorems 3.8, 3.41, 3.44, that describe the Lunn-Senior automorphism groups $\text{Aut}_0''(T_{D;G})$ for various subsets $D \subset P_6$. Corollaries 3.39, 3.40, 3.42, 3.45, of these theorems present structurally identical products of ethane that can and that can not be distinguished via substitution reactions among the elements of certain $T_{D;G}$.

2. The genetic reactions

Let $G$, $G'$, and $G''$ be the Lunn-Senior’s groups of substitution isomerism, stereoisomerism, and structural isomerism of ethane, respectively. We know from [4, section V], or [2, corollaries 1.4.3, 1.4.4] that the group $G \leq S_6$ coincides up to conjugation in the symmetric group $S_6$ with the group $\langle (123), (456), (14)(25)(36) \rangle$ of order 18, and that then the group $G'$ coincides with the group $\langle (123), (456), (14)(25)(36), (12)(45) \rangle$ of order 36. The graph which represents the structural formula of ethane shows that the group $G''$ coincides with the (unique up to conjugation in $S_6$) group $\langle (123), (456), (14)(25)(36), (12), (45) \rangle$ of order 72.

Below we list the elements of the sets $T_{\lambda;G}$, $T_{\lambda;G'}$, $T_{\lambda;G''}$ for all $\lambda \in P_6$ as well as all inequalities among the elements of $T_{\lambda;G}$. We also list all elements $A$ of each particular $G$-orbit $a \in T_{\lambda;G}$ because we think it would be of use of the chemists to have ready to hand all equivalent structural formulae $A$ (that is, all equivalent positions of the ligands) which represent the substitution isomer that corresponds to $a$.

Case 1. $\lambda = (6)$. We have

$$T_{(6);G} = T_{(6);G'} = T_{(6);G''} = \{a_{(6)}\},$$

where $a_{(6)}$ is the only $G$- and at the same time $G'$- and $G''$-orbit

$$\{(\{1, 2, 3, 4, 5, 6\})\}

of the tabloid $A^{(6)} = (\{1, 2, 3, 4, 5, 6\})$. The orbit $a_{(6)}$ represents the parent molecule of ethane.

Case 2. $\lambda = (5, 1)$. We have

$$T_{(5,1);G} = T_{(5,1);G'} = T_{(5,1);G''} = \{a_{(5,1)}\},$$

(2.2)
where $a_{(5,1)}$ is the only $G$- and at the same time $G'$- and $G''$-orbit
\[
\{(\{1, 2, 3, 4, 5\}, \{6\}), (\{1, 2, 3, 5, 6\}, \{4\}), (\{1, 2, 3, 4, 6\}, \{5\}),
\]
\[
(\{1, 2, 4, 5, 6\}, \{3\}), (\{2, 3, 4, 5, 6\}, \{1\}), (\{1, 3, 4, 5, 6\}, \{2\})\},
\]
of the tabloid $A^{(5,1)} = (\{1, 2, 3, 4, 5\}, \{6\})$.
The only possible substitution reaction between the parent substance of ethane and its mono-substitution derivative is designated $a_{(5,1)} < a_{(6)}$, because $R_{1,6}A^{(5,1)} = A^{(6)}$, and hence $A^{(5,1)} < A^{(6)}$. The operation $R_{1,6}$ performed on $A^{(5,1)}$ means ”replace the ligand of type $x_2$ in position 6 by a ligand of type $x_1$”. The converse operation ”replace the ligand of type $x_1$ in position 6 by a ligand of type $x_2$” is the essence of the simple substitution reaction
\[
A^{(6)} \longrightarrow A^{(5,1)}.
\]

Case 3. $\lambda = (4, 2)$.
Now, we have
\[
T_{(4,2);G} = T_{(4,2);G'} = T_{(4,2);G''} = \{a_{(4,2)}, b_{(4,2)}\},
\]
where $a_{(4,2)}$ is the $G$-, $G'$-, and $G''$-orbit
\[
\{(\{1, 2, 3, 4\}, \{5, 6\}), (\{1, 2, 3, 5\}, \{4, 6\}), (\{1, 2, 3, 6\}, \{4, 5\}),
\]
\[
(\{1, 4, 5, 6\}, \{2, 3\}), (\{2, 4, 5, 6\}, \{1, 3\}), (\{3, 4, 5, 6\}, \{1, 2\})\},
\]
of the tabloid $A^{(4,2)} = (\{1, 2, 3, 4\}, \{5, 6\})$;

$b_{(4,2)}$ is the $G$-, $G'$-, and $G''$-orbit
\[
\{(\{1, 2, 4, 5\}, \{3, 6\}), (\{2, 3, 4, 5\}, \{1, 6\}), (\{1, 3, 4, 5\}, \{2, 6\}),
\]
\[
(\{1, 2, 5, 6\}, \{3, 4\}), (\{1, 2, 4, 6\}, \{3, 5\}), (\{2, 3, 5, 6\}, \{1, 4\}),
\]
\[
(\{2, 3, 4, 6\}, \{1, 5\}), (\{1, 3, 5, 6\}, \{2, 4\}), (\{1, 3, 4, 6\}, \{2, 5\})\},
\]
of the tabloid $B^{(4,2)} = (\{1, 2, 4, 5\}, \{3, 6\})$.
All inequalities between the structural formulae of the di-substitution homogeneous and mono-substitution derivatives of ethane are
\[
A^{(4,2)} < A^{(5,1)}, B^{(4,2)} < A^{(5,1)},
\]
because
\[
R_{1,5}A^{(4,2)} = A^{(5,1)}, R_{1,3}B^{(4,2)} = A^{(5,1)}.
\]
Thus, we get the simple substitution reactions
\[
A^{(5,1)} \longrightarrow A^{(4,2)}, A^{(5,1)} \longrightarrow B^{(4,2)},
\]
which mean ”replace the ligand of type $x_1$ in position 5 of the tabloid $A^{(5,1)}$ by a ligand of type $x_2$”, and ”replace the ligand of type $x_1$ in position 3 of the tabloid $A^{(5,1)}$ by a
ligand of type $x_2^-$, respectively. The existence of these simple substitution reactions is designated as follows:

$$a_{(4,2)} < a_{(5,1)}, \ b_{(4,2)} < a_{(5,1)}.$$ 

Case 4. $\lambda = (4,1^2)$.

We have

$$T_{(4,1^2)}; G = \{a_{(4,1^2)}, b_{(4,1^2)}, c_{(4,1^2)}\},$$

where:

$a_{(4,1^2)}$ is the $G$-orbit

$$\{(\{1, 2, 3, 4\}, \{5\}, \{6\}), (\{1, 2, 3, 5\}, \{6\}, \{4\}), (\{1, 2, 3, 6\}, \{4\}, \{5\}), (\{1, 4, 5, 6\}, \{2\}, \{3\}), (\{2, 4, 5, 6\}, \{3\}, \{1\}), (\{3, 4, 5, 6\}, \{1\}, \{2\})\}$$

of the tabloid $A^{(4,1^2)} = (\{1, 2, 3, 4\}, \{5\}, \{6\})$;

$b_{(4,1^2)}$ is the $G$-orbit

$$\{(\{1, 2, 3, 4\}, \{6\}, \{5\}), (\{1, 2, 3, 5\}, \{4\}, \{6\}), (\{1, 2, 3, 6\}, \{5\}, \{4\}), (\{2, 4, 5, 6\}, \{1\}, \{3\}), (\{3, 4, 5, 6\}, \{2\}, \{1\}), (\{1, 4, 5, 6\}, \{3\}, \{2\})\}$$

of the tabloid $B^{(4,1^2)} = (\{1, 2, 3, 4\}, \{6\}, \{5\})$;

$c_{(4,1^2)}$ is the $G$-orbit

$$\{(\{1, 2, 4, 5\}, \{3\}, \{6\}), (\{2, 3, 4, 5\}, \{1\}, \{6\}), (\{1, 3, 4, 5\}, \{2\}, \{6\}), (\{1, 2, 4, 6\}, \{3\}, \{5\}), (\{2, 3, 5, 6\}, \{1\}, \{4\}), (\{2, 3, 4, 6\}, \{1\}, \{5\}), (\{1, 3, 5, 6\}, \{2\}, \{4\}), (\{1, 3, 4, 6\}, \{2\}, \{5\}), (\{1, 2, 4, 5\}, \{6\}, \{3\}), (\{1, 2, 5, 6\}, \{4\}, \{3\}), (\{1, 2, 4, 6\}, \{5\}, \{3\}), (\{2, 3, 4, 5\}, \{6\}, \{1\}), (\{1, 3, 4, 5\}, \{6\}, \{2\}), (\{2, 3, 5, 6\}, \{4\}, \{1\}), (\{1, 3, 5, 6\}, \{4\}, \{2\}), (\{2, 3, 4, 6\}, \{5\}, \{1\}), (\{1, 3, 4, 6\}, \{5\}, \{2\})\}$$

of the tabloid $C^{(4,1^2)} = (\{1, 2, 4, 5\}, \{3\}, \{6\})$.

We have the following inequalities between the di-substitution heterogeneous and the di-substitution homogeneous derivatives of ethane

$$A^{(4,1^2)} < A^{(4,2)}, \ B^{(4,1^2)} < A^{(4,2)}, \ C^{(4,1^2)} < B^{(4,2)},$$

because

$$R_{2,6}A^{(4,1^2)} = R_{2,5}B^{(4,1^2)} = A^{(4,2)}, \ R_{2,6}C^{(4,1^2)} = B^{(4,2)}.$$ 

Thus, we obtain the following simple substitution reactions:

$$A^{(4,2)} \rightarrow A^{(4,1^2)}, \ A^{(4,2)} \rightarrow B^{(4,1^2)}, \ B^{(4,2)} \rightarrow C^{(4,1^2)},$$

which mean “replace the ligand of type $x_2$ in position 6 of the tabloid $A^{(4,2)}$ by a ligand of type $x_3$”, ”replace the ligand of type $x_2$ in position 5 of the tabloid $A^{(4,2)}$ by a ligand
of type \( x_3 \)”, and “replace the ligand of type \( x_2 \) in position 6 of the tabloid \( B^{(4,2)} \) by a ligand of type \( x_3 \)”, respectively. The existence of these simple substitution reactions is designated as follows:

\[
a_{(4,1^2)} < a_{(4,2)}, \quad b_{(4,1^2)} < a_{(4,2)}, \quad c_{(4,1^2)} < b_{(4,2)}.\]

Below, in general, we omit similar remarks and present only the corresponding replacements by using the operators \( R_{i,s} \) and write down the inequalities that show the existence of substitution reactions.

The set of \( G' \)-orbits in \( T_{(4,1^2)} \) is

\[
T_{(4,1^2);G'} = \{a_{(4,1^2)} \cup b_{(4,1^2)}, c_{(4,1^2)}\}. \tag{2.5}
\]

In particular, the products that correspond to the formulae \( a_{(4,1^2)} \) and \( b_{(4,1^2)} \) form a chiral pair and the product that corresponds to \( c_{(4,1^2)} \) is a dimer. Since the sets of \( G' \)-orbits and \( G'' \)-orbits coincide,

\[
T_{(4,1^2);G''} = \{a_{(4,1^2)} \cup b_{(4,1^2)}, c_{(4,1^2)}\}, \tag{2.6}
\]

we obtain that the product that corresponds to \( c_{(4,1^2)} \) and any of the products which correspond to the members of the chiral pair \( \{a_{(4,1^2)}, b_{(4,1^2)}\} \) are structural isomers.

Case 5. \( \lambda = (3^2) \).

In this case we have

\[
T_{(3^2);G} = T_{(3^2);G'} = T_{(3^2);G''} = \{a_{(3^2)}, b_{(3^2)}\}, \tag{2.7}
\]

where:

\( a_{(3^2)} \) is the \( G' \)-, \( G'' \)-, and \( G''' \)-orbit

\[
\{(\{1, 2, 3\}, \{4, 5, 6\}), (\{4, 5, 6\}, \{1, 2, 3\})\},
\]

of the tabloid \( A^{(3^2)} = (\{1, 2, 3\}, \{4, 5, 6\}) \);

\( b_{(3^2)} \) is the \( G' \)-, \( G'' \)-, and \( G''' \)-orbit

\[
\{(\{1, 2, 4\}, \{3, 5, 6\}), (\{2, 3, 4\}, \{1, 5, 6\}), (\{1, 3, 4\}, \{2, 5, 6\}), (\{1, 2, 5\}, \{3, 4, 6\}), (\{1, 2, 6\}, \{3, 4, 5\}), (\{2, 3, 5\}, \{1, 4, 6\}), (\{2, 3, 6\}, \{1, 4, 5\}), (\{1, 3, 5\}, \{2, 4, 6\}), (\{1, 3, 6\}, \{2, 4, 5\}), (\{1, 4, 5\}, \{2, 3, 6\}), (\{1, 5, 6\}, \{2, 3, 4\}), (\{1, 4, 6\}, \{2, 3, 5\}), (\{2, 4, 5\}, \{1, 3, 6\}), (\{3, 4, 5\}, \{1, 2, 6\}), (\{2, 5, 6\}, \{1, 3, 4\}), (\{3, 5, 6\}, \{1, 2, 4\}), (\{2, 4, 6\}, \{1, 3, 5\}), (\{3, 4, 6\}, \{1, 2, 5\})\}
\]

of the tabloid \( B^{(3^2)} = (\{1, 2, 4\}, \{3, 5, 6\}) \).

We have the inequalities

\[
A^{(3^2)} < A^{(4,2)}, \quad B^{(3^2)} < A^{(4,2)}, \quad B^{(3^2)} < B^{(4,2)},
\]
because
\[ R_{1,4}A^{(3^2)} = R_{1,3}B^{(3^2)} = A^{(4,2)}, \quad R_{1,5}B^{(3^2)} = B^{(4,2)}. \]
Therefore
\[ a_{(3^2)} < a_{(4,2)}, \quad b_{(3^2)} < a_{(4,2)}, \quad b_{(3^2)} < b_{(4,2)}. \]

Case 6. \( \lambda = (3, 2, 1) \).
We have
\[ T_{(3,2,1);G} = \{ a_{(3,2,1)}, b_{(3,2,1)}, c_{(3,2,1)}, e_{(3,2,1)} \}, \] (2.8)
where:
- \( a_{(3,2,1)} \) is the \( G \)-orbit
  \[ \{ (\{1, 2, 3\}, \{4, 5\}, \{6\}), (\{1, 2, 3\}, \{5, 6\}, \{4\}), (\{1, 2, 3\}, \{4, 6\}, \{5\}), (\{4, 5, 6\}, \{1, 2\}, \{3\}) \} \]
of the tabloid \( A^{(3,2,1)} = (\{1, 2, 3\}, \{4, 5\}, \{6\}) \);
- \( b_{(3,2,1)} \) is the \( G \)-orbit
  \[ \{ (\{1, 2, 4\}, \{3, 5\}, \{6\}), (\{2, 3, 4\}, \{1, 5\}, \{6\}), (\{1, 3, 4\}, \{2, 5\}, \{6\}), (\{1, 2, 5\}, \{3, 6\}, \{4\}), (\{1, 2, 6\}, \{3, 4\}, \{5\}), (\{2, 3, 5\}, \{1, 6\}, \{4\}), (\{2, 3, 6\}, \{1, 4\}, \{5\}), (\{1, 3, 5\}, \{2, 6\}, \{4\}), (\{1, 3, 6\}, \{2, 4\}, \{5\}), (\{1, 4, 5\}, \{2, 6\}, \{3\}), (\{1, 5, 6\}, \{2, 4\}, \{3\}), (\{2, 4, 5\}, \{3, 6\}, \{1\}), (\{3, 4, 5\}, \{1, 6\}, \{2\}), (\{2, 5, 6\}, \{3, 4\}, \{1\}), (\{3, 5, 6\}, \{1, 4\}, \{2\}), (\{2, 4, 6\}, \{3, 5\}, \{1\}), (\{3, 4, 6\}, \{1, 5\}, \{2\}) \} \]
of the tabloid \( B^{(3,2,1)} = (\{1, 2, 4\}, \{3, 5\}, \{6\}) \);
- \( c_{(3,2,1)} \) is the \( G \)-orbit
  \[ \{ (\{1, 2, 4\}, \{3, 6\}, \{5\}), (\{2, 3, 4\}, \{1, 6\}, \{5\}), (\{1, 3, 4\}, \{2, 6\}, \{5\}), (\{1, 2, 5\}, \{3, 4\}, \{6\}), (\{1, 2, 6\}, \{3, 5\}, \{4\}), (\{2, 3, 5\}, \{1, 4\}, \{6\}), (\{2, 3, 6\}, \{1, 5\}, \{4\}), (\{1, 3, 5\}, \{2, 4\}, \{6\}), (\{1, 3, 6\}, \{2, 5\}, \{4\}), (\{1, 4, 5\}, \{3, 6\}, \{2\}), (\{2, 4, 5\}, \{1, 6\}, \{3\}), (\{3, 4, 5\}, \{2, 6\}, \{1\}), (\{1, 5, 6\}, \{3, 4\}, \{2\}), (\{1, 4, 6\}, \{3, 5\}, \{2\}), (\{2, 5, 6\}, \{1, 4\}, \{3\}), (\{2, 4, 6\}, \{1, 5\}, \{3\}), (\{3, 5, 6\}, \{2, 4\}, \{1\}), (\{3, 4, 6\}, \{2, 5\}, \{1\}) \} \]
of the tabloid \( C^{(3,2,1)} = (\{1, 2, 4\}, \{3, 6\}, \{5\}) \);
- \( e_{(3,2,1)} \) is the \( G \)-orbit
  \[ \{ (\{1, 2, 4\}, \{5, 6\}, \{3\}), (\{2, 3, 4\}, \{5, 6\}, \{1\}), (\{1, 3, 4\}, \{5, 6\}, \{2\}), (\{1, 2, 5\}, \{4, 6\}, \{3\}), (\{1, 2, 6\}, \{4, 5\}, \{3\}), (\{2, 3, 5\}, \{4, 6\}, \{1\}), (\{2, 3, 6\}, \{4, 5\}, \{1\}), (\{1, 3, 5\}, \{4, 6\}, \{2\}), (\{1, 3, 6\}, \{4, 5\}, \{2\}) \} \]
and (3)
so, in particular, the products that correspond to formulae

The set of (3)
Therefore all simple substitution reactions between (3)

B

R

A

B

C

A

B

R

B

A

(3)

A

B

A

(3)

R

B

R

B

R

A

B

R

B

R

Therefore all simple substitution reactions between (3, 2, 1)-derivatives of ethane and its (4, 1²) -derivatives and (3²) -derivatives, respectively, are:

\[ a_{(3, 2, 1)} < a_{(4, 1²)}, \quad b_{(3, 2, 1)} < a_{(4, 1²)}, \]

\[ a_{(3, 2, 1)} < b_{(4, 1²)}, \quad c_{(3, 2, 1)} < b_{(4, 1²)}, \]

\[ b_{(3, 2, 1)} < c_{(4, 1²)}, \quad c_{(3, 2, 1)} < c_{(4, 1²)}, \quad e_{(3, 2, 1)} < c_{(4, 1²)}, \]

and

\[ a_{(3, 2, 1)} < a_{(3²)}, \]

\[ b_{(3, 2, 1)} < b_{(3²)}, \quad c_{(3, 2, 1)} < b_{(3²)}, \quad e_{(3, 2, 1)} < b_{(3²)}. \]

The set of \( G' \)-orbits in \( T_{(3, 2, 1)} \) is

\[ T_{(3, 2, 1); G'} = \{ a_{(3, 2, 1)}, b_{(3, 2, 1)} \cup c_{(3, 2, 1)}, c_{(3, 2, 1)} \}, \]  \hspace{1cm} (2.9)

so, in particular, the products that correspond to formulae \( b_{(3, 2, 1)} \) and \( c_{(3, 2, 1)} \) form a chiral pair and the products that correspond to \( a_{(3, 2, 1)} \) and \( e_{(3, 2, 1)} \) are dimers.
Since the set $T_{(3,2,1); G''}$ of $G''$-orbits in $T_{(3,2,1)}$ coincides with the set of $G'$-orbits there, we obtain that the members of different sets below are structural isomers:

$$\{a_{(3,2,1)}, b_{(3,2,1)}, c_{(3,2,1)}, e_{(3,2,1)}\}.$$  \tag{2.10}

Case 7. $\lambda = (3,1^3)$.

Now, we have

$$T_{(3,1^3); G'} = \{a_{(3,1^3)}, b_{(3,1^3)}, c_{(3,1^3)}, e_{(3,1^3)}, f_{(3,1^3)}, h_{(3,1^3)}, k_{(3,1^3)}, \ell_{(3,1^3)}\},$$  \tag{2.11}

where:

- $a_{(3,1^3)}$ is the $G$-orbit
  
  $$\{(\{1,2,3\}, \{4\}, \{5\}, \{6\}), (\{1,2,3\}, \{5\}, \{6\}, \{4\}), (\{1,2,3\}, \{6\}, \{4\}, \{5\}),$$
  $$\{4,5,6\}, \{1\}, \{2\}, \{3\}\}$$

- $b_{(3,1^3)}$ is the $G$-orbit
  
  $$\{(\{1,2,3\}, \{4\}, \{6\}, \{5\}), (\{1,2,3\}, \{5\}, \{4\}, \{6\}), (\{1,2,3\}, \{6\}, \{5\}, \{4\}),$$
  $$\{4,5,6\}, \{1\}, \{3\}, \{2\}\}$$

- $c_{(3,1^3)}$ is the $G$-orbit
  
  $$\{(\{1,2,4\}, \{5\}, \{3\}, \{6\}), (\{2,3,4\}, \{5\}, \{1\}, \{6\}), (\{1,3,4\}, \{5\}, \{2\}, \{6\}),$$
  $$\{1,2,5\}, \{6\}, \{3\}, \{4\}\}$$

- $e_{(3,1^3)}$ is the $G$-orbit
  
  $$\{(\{1,2,4\}, \{6\}, \{3\}, \{5\}), (\{2,3,4\}, \{6\}, \{1\}, \{5\}), (\{1,3,4\}, \{6\}, \{2\}, \{5\}),$$
  $$\{1,2,5\}, \{4\}, \{3\}, \{6\}\}$$

The tabloids $A^{(3,1^3)} = (\{1,2,3\}, \{4\}, \{5\}, \{6\})$ and $B^{(3,1^3)} = (\{1,2,3\}, \{4\}, \{6\}, \{5\})$ are the $G$-orbits of the sets $\{a_{(3,1^3)}, b_{(3,1^3)}, c_{(3,1^3)}, e_{(3,1^3)}\}$, respectively. The tabloid $C^{(3,1^3)} = (\{1,2,4\}, \{5\}, \{3\}, \{6\})$ is the $G$-orbit of the set $\{a_{(3,1^3)}, b_{(3,1^3)}, c_{(3,1^3)}, e_{(3,1^3)}\}$. Therefore, we have structural isomers for the members of these sets.
of the tabloid $E^{(3,1^3)} = (\{1, 2, 4\}, \{6\}, \{3\}, \{5\})$;  
$f_{(3, 1^3)}$ is the $G$-orbit  
\[
\{(1, 1, 2, 4, 5), (2, 3, 4, 6, 5), (6, 1, 3, 4, 5), (5, 6, 2)\},
\{(1, 2, 5, 6, 4, 3), (1, 2, 6, 4, 5, 3), (2, 3, 5, 6, 4, 1)\},
\{(2, 3, 6, 4, 5, 1), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(3, 5, 6, 1, 2, 4), (2, 4, 6, 3, 1, 5), (3, 4, 6, 1, 2, 5)\},
\]
\]
of the tabloid $F^{(3,1^3)} = (\{1, 2, 4\}, \{5\}, \{6\}, \{3\})$;  
$h_{(3, 1^3)}$ is the $G$-orbit  
\[
\{(1, 1, 2, 4, 5), (2, 3, 4, 6, 5), (6, 1, 3, 4, 5), (5, 6, 2)\},
\{(1, 2, 5, 6, 4, 3), (1, 2, 6, 4, 5, 3), (2, 3, 5, 6, 4, 1)\},
\{(2, 3, 6, 4, 5, 1), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(3, 5, 6, 1, 2, 4), (2, 4, 6, 3, 1, 5), (3, 4, 6, 1, 2, 5)\},
\]
\]
of the tabloid $H^{(3,1^3)} = (\{1, 2, 4\}, \{6\}, \{5\}, \{3\})$;  
k_{(3, 1^3)}$ is the $G$-orbit  
\[
\{(1, 1, 2, 4, 5), (2, 3, 4, 6, 5), (6, 1, 3, 4, 5), (5, 6, 2)\},
\{(1, 2, 5, 6, 4, 3), (1, 2, 6, 4, 5, 3), (2, 3, 5, 6, 4, 1)\},
\{(2, 3, 6, 4, 5, 1), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(3, 5, 6, 1, 2, 4), (2, 4, 6, 3, 1, 5), (3, 4, 6, 1, 2, 5)\},
\]
\]
of the tabloid $K^{(3,1^3)} = (\{1, 2, 4\}, \{3\}, \{5\}, \{6\})$;  
$\ell_{(3, 1^3)}$ is the $G$-orbit  
\[
\{(1, 1, 2, 4, 5), (2, 3, 4, 6, 5), (6, 1, 3, 4, 5), (5, 6, 2)\},
\{(1, 2, 5, 6, 4, 3), (1, 2, 6, 4, 5, 3), (2, 3, 5, 6, 4, 1)\},
\{(2, 3, 6, 4, 5, 1), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(3, 5, 6, 1, 2, 4), (2, 4, 6, 3, 1, 5), (3, 4, 6, 1, 2, 5)\},
\]
\]
of the tabloid $\ell_{(3,1^3)} = (\{1, 2, 4\}, \{3\}, \{5\}, \{6\})$;  
$\ell_{(3, 1^3)}$ is the $G$-orbit  
\[
\{(1, 1, 2, 4, 5), (2, 3, 4, 6, 5), (6, 1, 3, 4, 5), (5, 6, 2)\},
\{(1, 2, 5, 6, 4, 3), (1, 2, 6, 4, 5, 3), (2, 3, 5, 6, 4, 1)\},
\{(2, 3, 6, 4, 5, 1), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(1, 4, 5, 2, 3, 6), (1, 1, 3, 5, 6, 4, 2), (1, 1, 3, 6, 4, 5, 2)\},
\{(3, 5, 6, 1, 2, 4), (2, 4, 6, 3, 1, 5), (3, 4, 6, 1, 2, 5)\},
\]
\]

where:

Case 8.

so the members of different chiral pairs (2.13) are structural isomers.

Therefore the products that correspond to the members of each of the sets of the tabloid \( L^{(3,1^3)} = \{(1, 2, 4), \{3\}, \{6\}, \{5\}\} \),

We have the following inequalities between the tabloids of shape \((3, 2, 1)\) and the tabloids of shape \((3, 1^3)\):

\[
A^{(3,1^3)} < A^{(3,2,1)}, \quad B^{(3,1^3)} < A^{(3,2,1)},
\]

\[
C^{(3,1^3)} < B^{(3,2,1)}, \quad F^{(3,1^3)} < B^{(3,2,1)}, \quad K^{(3,1^3)} < B^{(3,2,1)}, \quad L^{(3,1^3)} < B^{(3,2,1)},
\]

\[
E^{(3,1^3)} < C^{(3,2,1)}, \quad H^{(3,1^3)} < C^{(3,2,1)},
\]

\[
K^{(3,1^3)} < C^{(3,2,1)}, \quad L^{(3,1^3)} < C^{(3,2,1)},
\]

\[
C^{(3,1^3)} < E^{(3,2,1)}, \quad E^{(3,1^3)} < E^{(3,2,1)}, \quad F^{(3,1^3)} < E^{(3,2,1)}, \quad H^{(3,1^3)} < E^{(3,2,1)},
\]

because

\[
R_{2,5} R_{3,6} A^{(3,1^3)} = R_{2,5} B^{(3,1^3)} = A^{(3,2,1)},
\]

\[
R_{2,3} R_{3,6} C^{(3,1^3)} = R_{2,3} F^{(3,1^3)} = R_{2,5} R_{3,6} K^{(3,1^3)} = R_{2,5} L^{(3,1^3)} = B^{(3,2,1)},
\]

\[
R_{2,3} R_{3,5} E^{(3,1^3)} = R_{2,5} H^{(3,1^3)} = R_{2,6} K^{(3,1^3)} = R_{2,6} R_{3,5} L^{(3,1^3)} = C^{(3,2,1)},
\]

\[
R_{2,5} C^{(3,1^3)} = R_{2,5} E^{(3,1^3)} = R_{2,6} R_{3,3} F^{(3,1^3)} = R_{2,5} R_{3,3} H^{(3,1^3)} = E^{(3,2,1)}.
\]

Thus, we get all simple substitution reactions between \((3, 1^3)\)- and \((3, 2, 1)\)-derivatives of ethane:

\[
a_{(3,1^3)} < a_{(3,2,1)}, \quad b_{(3,1^3)} < a_{(3,2,1)},
\]

\[
c_{(3,1^3)} < b_{(3,2,1)}, \quad f_{(3,1^3)} < b_{(3,2,1)}, \quad k_{(3,1^3)} < b_{(3,2,1)}, \quad \ell_{(3,1^3)} < b_{(3,2,1)},
\]

\[
e_{(3,1^3)} < c_{(3,2,1)}, \quad h_{(3,1^3)} < c_{(3,2,1)}, \quad k_{(3,1^3)} < c_{(3,2,1)}, \quad \ell_{(3,1^3)} < c_{(3,2,1)},
\]

\[
c_{(3,1^3)} < e_{(3,2,1)}, \quad e_{(3,1^3)} < e_{(3,2,1)}, \quad f_{(3,1^3)} < e_{(3,2,1)}, \quad h_{(3,1^3)} < e_{(3,2,1)}.
\]

The set of all \( G' \)-orbits is

\[
T_{(3,1^3);G'} = \{a_{(3,1^3)} \cup b_{(3,1^3)} \cup c_{(3,1^3)} \cup e_{(3,1^3)}, f_{(3,1^3)} \cup h_{(3,1^3)} \cup k_{(3,1^3)} \cup \ell_{(3,1^3)}\}. \quad (2.12)
\]

Therefore the products that correspond to the members of each of the sets

\[
\{a_{(3,1^3)}, b_{(3,1^3)}\}, \quad \{c_{(3,1^3)}, e_{(3,1^3)}\}, \quad \{f_{(3,1^3)}, h_{(3,1^3)}\}, \quad \{k_{(3,1^3)}, \ell_{(3,1^3)}\}\]

(2.13)

form chiral pairs; moreover, the set of all \( G'' \)-orbits coincides with the set of all \( G' \)-orbits, so the members of different chiral pairs (2.13) are structural isomers.

Case 8. \( \lambda = (2^3) \).

We have

\[
T_{(2^3);G} = \{a_{(2^3)}, b_{(2^3)}, c_{(2^3)}, e_{(2^3)}, f_{(2^3)}, h_{(2^3)}\}, \quad (2.14)
\]

where:
$a_{(2^3)}$ is the $G$-orbit

$B(2^3) = \{(1,2), \{3,4\}, \{5,6\}\}$

$b_{(2^3)}$ is the $G$-orbit

$C(2^3) = \{(1,4), \{2,3\}, \{5,6\}\}$

c_{(2^3)}$ is the $G$-orbit

$D(2^3) = \{(1,2), \{3,4\}, \{5,6\}\}$
of the tabloid $E^{(2^3)} = \{(1, 4),\{2, 6\},\{3, 5\}\};$

$f_{(2^3)}$ is the $G$-orbit

$$\{(1, 4),\{2, 5\},\{3, 6\}\}, \{(2, 4),\{3, 5\},\{1, 6\}\}, \{(3, 4),\{1, 5\},\{2, 6\}\},$$

$$\{(1, 5),\{2, 6\},\{3, 4\}\}, \{(1, 6),\{2, 4\},\{3, 5\}\}, \{(2, 5),\{3, 6\},\{1, 4\}\},$$

$$\{(2, 6),\{3, 4\},\{1, 5\}\}, \{(3, 5),\{1, 6\},\{2, 4\}\}, \{(3, 6),\{1, 4\},\{2, 5\}\}$$

of the tabloid $F^{(2^3)} = \{(1, 4),\{2, 5\},\{3, 6\}\};$

$h_{(2^3)}$ is the $G$-orbit

$$\{(1, 4),\{3, 6\},\{2, 5\}\}, \{(2, 4),\{1, 6\},\{3, 5\}\}, \{(3, 4),\{2, 6\},\{1, 5\}\},$$

$$\{(1, 5),\{3, 4\},\{2, 6\}\}, \{(1, 6),\{3, 5\},\{2, 4\}\}, \{(2, 5),\{1, 4\},\{3, 6\}\},$$

$$\{(2, 6),\{1, 5\},\{3, 4\}\}, \{(3, 5),\{2, 4\},\{1, 6\}\}, \{(3, 6),\{2, 5\},\{1, 4\}\},$$

of the tabloid $H^{(2^3)} = \{(1, 4),\{3, 6\},\{2, 5\}\}.$

The inequalities between the tabloids of shape $(2^3)$ and those of shape $(3, 2, 1)$ are as follows:

$$A^{(2^3)} < A^{(3,2,1)}, \quad B^{(2^3)} < A^{(3,2,1)},$$

$$A^{(2^3)} < B^{(3,2,1)}, \quad B^{(2^3)} < B^{(3,2,1)}, \quad C^{(2^3)} < B^{(3,2,1)},$$

$$(14)(25)(36)E^{(2^3)} < B^{(3,2,1)}, \quad F^{(2^3)} < B^{(3,2,1)},$$

$$(465)A^{(2^3)} < C^{(3,2,1)}, \quad (465)B^{(2^3)} < C^{(3,2,1)}, \quad C^{(2^3)} < C^{(3,2,1)},$$

$$E^{(2^3)} < C^{(3,2,1)}, \quad H^{(2^3)} < C^{(3,2,1)},$$

$$B^{(2^3)} < E^{(3,2,1)}, \quad (14)(25)(36)C^{(2^3)} < E^{(3,2,1)}, \quad E^{(2^3)} < E^{(3,2,1)},$$

$$F^{(2^3)} < E^{(3,2,1)}, \quad (123)H^{(2^3)} < E^{(3,2,1)},$$

because

$$R_{2,5}R_{1,3}A^{(2^3)} = R_{1,3}B^{(2^3)} = A^{(3,2,1)},$$

$$R_{1,4}R_{2,5}A^{(2^3)} = R_{1,4}R_{2,3}B^{(2^3)} = R_{1,2}R_{2,5}C^{(2^3)} =$$

$$R_{1,2}(14)(25)(36)E^{(2^3)} = R_{1,2}R_{2,3}F^{(2^3)} = B^{(3,2,1)},$$

$$R_{1,4}(465)A^{(2^3)} = R_{1,4}R_{2,3}(465)B^{(2^3)} = R_{1,2}R_{2,6}C^{(2^3)} =$$

$$R_{1,2}R_{2,3}E^{(2^3)} = R_{1,2}H^{(2^3)} = C^{(3,2,1)},$$

$$R_{1,4}R_{2,6}B^{(2^3)} = R_{1,2}(14)(25)(36)C^{(2^3)} = R_{1,2}R_{2,5}E^{(2^3)} =$$

$$R_{1,2}R_{2,6}F^{(2^3)} = R_{1,1}(123)H^{(2^3)} = E^{(3,2,1)}.$$
Hence all substitution reactions between \((2^3)\)- and \((3, 2, 1)\)-derivatives of ethane are:

\[
a_{(2^3)} < a_{(3, 2, 1)}, \quad b_{(2^3)} < a_{(3, 2, 1)},
\]

\[
a_{(2^3)} < b_{(3, 2, 1)}, \quad b_{(2^3)} < b_{(3, 2, 1)}, \quad c_{(2^3)} < b_{(3, 2, 1)},
\]

\[
e_{(2^3)} < b_{(3, 2, 1)}, \quad f_{(2^3)} < b_{(3, 2, 1)},
\]

\[
a_{(2^3)} < c_{(3, 2, 1)}, \quad b_{(2^3)} < c_{(3, 2, 1)}, \quad c_{(2^3)} < c_{(3, 2, 1)},
\]

\[
e_{(2^3)} < c_{(3, 2, 1)}, \quad h_{(2^3)} < c_{(3, 2, 1)},
\]

\[
b_{(2^3)} < e_{(3, 2, 1)}, \quad c_{(2^3)} < e_{(3, 2, 1)}, \quad e_{(2^3)} < e_{(3, 2, 1)},
\]

\[
f_{(2^3)} < e_{(3, 2, 1)}, \quad h_{(2^3)} < e_{(3, 2, 1)}.
\]

The set of all \(G\)-orbits is

\[
T_{(2^3); G} = \{a_{(2^3)}, b_{(2^3)}, c_{(2^3)}, e_{(2^3)}, f_{(2^3)} \cup h_{(2^3)}\},
\]

Therefore the \((2^3)\)-derivatives that correspond to the formulae \(f_{(2^3)}\) and \(h_{(2^3)}\) form a chiral pair and the \((2^3)\)-derivatives that correspond to \(a_{(2^3)}\), \(b_{(2^3)}\), \(c_{(2^3)}\), and \(e_{(2^3)}\) are dimers.

The set of all \(G'\)-orbits is

\[
T_{(2^3); G'} = \{a_{(2^3)}, b_{(2^3)}, c_{(2^3)}, e_{(2^3)} \cup f_{(2^3)} \cup h_{(2^3)}\},
\]

Hence the products that correspond to members of different sets below are structural isomers, and the members of the last set are structurally identical:

\[
\{a_{(2^3)}\}, \{b_{(2^3)}\}, \{c_{(2^3)}\}, \{e_{(2^3)}, f_{(2^3)}, h_{(2^3)}\}.
\]

Case 9. \(\lambda = (2^2, 1^2)\).

In this case we have

\[
T_{(2^2, 1^2); G} =
\]

\[
\{a_{(2^2, 1^2)}, b_{(2^2, 1^2)}, c_{(2^2, 1^2)}, e_{(2^2, 1^2)}, f_{(2^2, 1^2)}, h_{(2^2, 1^2)}, k_{(2^2, 1^2)}, \ell_{(2^2, 1^2)}, m_{(2^2, 1^2)}, p_{(2^2, 1^2)}\},
\]

where:

\(a_{(2^2, 1^2)}\) is the \(G\)-orbit

\[
\{(1, 2), \{3, 4\}, \{5\}, \{6\}\}, \{(2, 3), \{1, 4\}, \{5\}, \{6\}\}, \{(1, 3), \{2, 4\}, \{5\}, \{6\}\},
\]

\[
\{(1, 2), \{3, 5\}, \{6\}, \{4\}\}, \{(1, 2), \{3, 6\}, \{4\}, \{5\}\}, \{(2, 3), \{1, 5\}, \{6\}, \{4\}\},
\]

\[
\{(2, 3), \{1, 6\}, \{4\}, \{5\}\}, \{(1, 3), \{2, 5\}, \{6\}, \{4\}\}, \{(1, 3), \{2, 6\}, \{4\}, \{5\}\},
\]

\[
\{(4, 5), \{1, 6\}, \{2\}, \{3\}\}, \{(5, 6), \{1, 4\}, \{2\}, \{3\}\}, \{(4, 6), \{1, 5\}, \{2\}, \{3\}\},
\]

\[
\{(4, 5), \{2, 6\}, \{3\}, \{1\}\}, \{(4, 5), \{3, 6\}, \{1\}, \{2\}\}, \{(5, 6), \{2, 4\}, \{3\}, \{1\}\},
\]

\[
\{(5, 6), \{3, 4\}, \{1\}, \{2\}\}, \{(4, 6), \{2, 5\}, \{3\}, \{1\}\}, \{(4, 6), \{3, 5\}, \{1\}, \{2\}\}.
\]
of the tabloid $A^{(2^2,1^2)} = \{(1,2), \{3,4\}, \{5\}, \{6\}\};$

$b_{(2^2,1^2)}$ is the $G$-orbit

\[
\{\{(1,2), \{3,4\}, \{6\}, \{5\}\}, \{(2,3), \{1,4\}, \{6\}, \{5\}\}, \{(1,3), \{2,4\}, \{6\}, \{5\}\},
\{(1,2), \{3,5\}, \{4\}, \{6\}\}, \{(1,2), \{3,6\}, \{5\}, \{4\}\}, \{(2,3), \{1,5\}, \{4\}, \{6\}\},
\{(2,3), \{1,6\}, \{5\}, \{4\}\}, \{(1,3), \{2,5\}, \{4\}, \{6\}\}, \{(1,3), \{2,6\}, \{5\}, \{4\}\},
\{(4,5), \{1,6\}, \{3\}, \{2\}\}, \{(5,6), \{1,4\}, \{3\}, \{2\}\}, \{(4,6), \{1,5\}, \{3\}, \{2\}\},
\{(4,5), \{2,6\}, \{1\}, \{3\}\}, \{(4,5), \{3,6\}, \{2\}, \{1\}\}, \{(5,6), \{2,4\}, \{1\}, \{3\}\},
\{(5,6), \{3,4\}, \{2\}, \{1\}\}, \{(4,6), \{2,5\}, \{1\}, \{3\}\}, \{(4,6), \{3,5\}, \{2\}, \{1\}\}\}
\]

of the tabloid $B^{(2^2,1^2)} = \{(1,2), \{3,4\}, \{6\}, \{5\}\};$

c_{(2^2,1^2)}$ is the $G$-orbit

\[
\{\{(1,2), \{4,5\}, \{3\}, \{6\}\}, \{(2,3), \{4,5\}, \{1\}, \{6\}\}, \{(1,3), \{4,5\}, \{2\}, \{6\}\},
\{(1,2), \{5,6\}, \{3\}, \{4\}\}, \{(1,2), \{4,6\}, \{3\}, \{5\}\}, \{(2,3), \{5,6\}, \{1\}, \{4\}\},
\{(2,3), \{4,6\}, \{1\}, \{5\}\}, \{(1,3), \{5,6\}, \{2\}, \{4\}\}, \{(1,3), \{4,6\}, \{2\}, \{5\}\},
\{(4,5), \{1,2\}, \{6\}, \{3\}\}, \{(5,6), \{1,2\}, \{4\}, \{3\}\}, \{(4,6), \{1,2\}, \{5\}, \{3\}\},
\{(4,5), \{2,3\}, \{6\}, \{1\}\}, \{(4,5), \{1,3\}, \{6\}, \{2\}\}, \{(5,6), \{2,3\}, \{4\}, \{1\}\},
\{(5,6), \{1,3\}, \{4\}, \{2\}\}, \{(4,6), \{2,3\}, \{5\}, \{1\}\}, \{(4,6), \{1,3\}, \{5\}, \{2\}\}\}
\]

of the tabloid $C^{(2^2,1^2)} = \{(1,2), \{4,5\}, \{3\}, \{6\}\};$

e_{(2^2,1^2)}$ is the $G$-orbit

\[
\{\{(1,2), \{4,5\}, \{6\}, \{3\}\}, \{(2,3), \{4,5\}, \{6\}, \{1\}\}, \{(1,3), \{4,5\}, \{6\}, \{2\}\},
\{(1,2), \{5,6\}, \{4\}, \{3\}\}, \{(1,2), \{4,6\}, \{5\}, \{3\}\}, \{(2,3), \{5,6\}, \{4\}, \{1\}\},
\{(2,3), \{4,6\}, \{5\}, \{1\}\}, \{(1,3), \{5,6\}, \{4\}, \{2\}\}, \{(1,3), \{4,6\}, \{5\}, \{2\}\},
\{(4,5), \{1,2\}, \{3\}, \{6\}\}, \{(5,6), \{1,2\}, \{3\}, \{4\}\}, \{(4,6), \{1,2\}, \{3\}, \{5\}\},
\{(4,5), \{2,3\}, \{1\}, \{6\}\}, \{(4,5), \{1,3\}, \{2\}, \{6\}\}, \{(5,6), \{2,3\}, \{1\}, \{4\}\},
\{(5,6), \{1,3\}, \{2\}, \{4\}\}, \{(4,6), \{2,3\}, \{1\}, \{5\}\}, \{(4,6), \{1,3\}, \{2\}, \{5\}\}\}
\]

of the tabloid $D^{(2^2,1^2)} = \{(1,2), \{4,5\}, \{6\}, \{3\}\};$

$f_{(2^2,1^2)}$ is the $G$-orbit

\[
\{\{(1,4), \{2,3\}, \{5\}, \{6\}\}, \{(2,4), \{1,3\}, \{5\}, \{6\}\}, \{(3,4), \{1,2\}, \{5\}, \{6\}\},
\{(1,5), \{2,3\}, \{6\}, \{4\}\}, \{(1,6), \{2,3\}, \{4\}, \{5\}\}, \{(2,5), \{1,3\}, \{6\}, \{4\}\},
\{(2,6), \{1,3\}, \{4\}, \{5\}\}, \{(3,5), \{1,2\}, \{6\}, \{4\}\}, \{(3,6), \{1,2\}, \{4\}, \{5\}\},
\{(1,4), \{5,6\}, \{2\}, \{3\}\}, \{(1,5), \{4,6\}, \{2\}, \{3\}\}, \{(1,6), \{4,5\}, \{2\}, \{3\}\}\}
\]
of the tabloid $F^{(2^2, 1^2)} = (\{1,4\}, \{2,3\}, \{5\}, \{6\})$

$h_{(2^2, 1^2)}$ is the $G$-orbit

$\{(\{1,4\}, \{2,3\}, \{6\}, \{5\}), (\{2,4\}, \{1,3\}, \{6\}, \{5\}), (\{3,4\}, \{1,2\}, \{6\}, \{5\}),

(\{1,5\}, \{2,3\}, \{4\}, \{6\}), (\{1,6\}, \{2,3\}, \{5\}, \{4\}), (\{2,5\}, \{1,3\}, \{4\}, \{6\}),

(\{2,6\}, \{1,3\}, \{5\}, \{4\}), (\{3,5\}, \{1,2\}, \{4\}, \{6\}), (\{3,6\}, \{1,2\}, \{5\}, \{4\}),

(\{1,4\}, \{5,6\}, \{3\}, \{2\}), (\{1,5\}, \{4,6\}, \{3\}, \{2\}), (\{1,6\}, \{4,5\}, \{3\}, \{2\}),

(\{2,4\}, \{5,6\}, \{1\}, \{3\}), (\{3,4\}, \{5,6\}, \{2\}, \{1\}), (\{2,5\}, \{4,6\}, \{1\}, \{3\}),

(\{3,5\}, \{4,6\}, \{2\}, \{1\}), (\{2,6\}, \{4,5\}, \{1\}, \{3\}), (\{3,6\}, \{4,5\}, \{2\}, \{1\})\}$

of the tabloid $H^{(2^2, 1^2)} = (\{1,4\}, \{2,3\}, \{5\}, \{6\})$

$k_{(2^2, 1^2)}$ is the $G$-orbit

$\{(\{1,4\}, \{2,5\}, \{3\}, \{6\}), (\{2,4\}, \{3,5\}, \{1\}, \{6\}), (\{3,4\}, \{1,5\}, \{2\}, \{6\}),

(\{1,5\}, \{2,6\}, \{3\}, \{4\}), (\{1,6\}, \{2,4\}, \{3\}, \{5\}), (\{2,5\}, \{3,6\}, \{1\}, \{4\}),

(\{2,6\}, \{3,4\}, \{1\}, \{5\}), (\{3,5\}, \{1,6\}, \{2\}, \{4\}), (\{3,6\}, \{1,4\}, \{2\}, \{5\}),

(\{1,4\}, \{2,5\}, \{6\}, \{3\}), (\{1,5\}, \{2,6\}, \{4\}, \{3\}), (\{1,6\}, \{2,4\}, \{5\}, \{3\}),

(\{2,4\}, \{3,5\}, \{6\}, \{1\}), (\{3,4\}, \{1,5\}, \{6\}, \{2\}), (\{2,5\}, \{3,6\}, \{4\}, \{1\}),

(\{3,5\}, \{1,6\}, \{4\}, \{2\}), (\{2,6\}, \{3,4\}, \{5\}, \{1\}), (\{3,6\}, \{1,4\}, \{5\}, \{2\})\}$

of the tabloid $K^{(2^2, 1^2)} = (\{1,4\}, \{2,5\}, \{3\}, \{6\})$

$\ell_{(2^2, 1^2)}$ is the $G$-orbit

$\{(\{1,4\}, \{2,6\}, \{3\}, \{5\}), (\{2,4\}, \{3,6\}, \{1\}, \{5\}), (\{3,4\}, \{1,6\}, \{2\}, \{5\}),

(\{1,5\}, \{2,4\}, \{3\}, \{6\}), (\{1,6\}, \{2,5\}, \{3\}, \{4\}), (\{2,5\}, \{3,4\}, \{1\}, \{6\}),

(\{2,6\}, \{3,5\}, \{1\}, \{4\}), (\{3,5\}, \{1,4\}, \{2\}, \{6\}), (\{3,6\}, \{1,5\}, \{2\}, \{4\}),

(\{1,4\}, \{3,5\}, \{6\}, \{2\}), (\{1,5\}, \{3,6\}, \{4\}, \{2\}), (\{1,6\}, \{3,4\}, \{5\}, \{2\}),

(\{2,4\}, \{1,5\}, \{6\}, \{3\}), (\{3,4\}, \{2,5\}, \{6\}, \{1\}), (\{2,5\}, \{1,6\}, \{4\}, \{3\}),

(\{3,5\}, \{2,6\}, \{4\}, \{1\}), (\{2,6\}, \{1,4\}, \{5\}, \{3\}), (\{3,6\}, \{2,4\}, \{5\}, \{1\})\}$

of the tabloid $L^{(2^2, 1^2)} = (\{1,4\}, \{2,6\}, \{3\}, \{5\})$

$m_{(2^2, 1^2)}$ is the $G$-orbit

$\{(\{1,4\}, \{2,6\}, \{5\}, \{3\}), (\{2,4\}, \{3,6\}, \{5\}, \{1\}), (\{3,4\}, \{1,6\}, \{5\}, \{2\}),

(\{1,5\}, \{2,4\}, \{6\}, \{3\}), (\{1,6\}, \{2,5\}, \{4\}, \{3\}), (\{2,5\}, \{3,4\}, \{6\}, \{1\}),$
\{\{2, 6\}, \{3, 5\}, \{4\}, \{1\}\}, \{(3, 5), \{1, 4\}, \{6\}, \{2\}\}, \{(3, 6), \{1, 5\}, \{4\}, \{2\}\},
\{(1, 4), \{3, 5\}, \{2\}, \{6\}\}, \{1, 5\}, \{3, 6\}, \{2\}, \{4\}\}, \{(1, 6), \{3, 4\}, \{2\}, \{5\}\},
\{(2, 4), \{1, 5\}, \{3\}, \{6\}\}, \{(3, 4), \{2, 5\}, \{1\}, \{6\}\}, \{(2, 5), \{1, 6\}, \{3\}, \{4\}\},
\{(3, 5), \{2, 6\}, \{1\}, \{4\}\}, \{(2, 6), \{1, 4\}, \{3\}, \{5\}\}, \{(3, 6), \{2, 4\}, \{1\}, \{5\}\}\}
of the tabloid \(M^{2^{2,1^2}} = \{(1, 4), \{2, 6\}, \{5\}, \{3\}\}\);
\(p_{2^{2,1^2}}\) is the \(G\)-orbit
\{\{(1, 4), \{3, 6\}, \{2\}, \{5\}\}, \{(2, 4), \{1, 6\}, \{3\}, \{5\}\}, \{(3, 4), \{2, 6\}, \{1\}, \{5\}\},
\{(1, 5), \{3, 4\}, \{2\}, \{6\}\}, \{(1, 6), \{3, 5\}, \{2\}, \{4\}\}, \{(2, 5), \{1, 4\}, \{3\}, \{6\}\},
\{(2, 6), \{1, 5\}, \{3\}, \{4\}\}, \{(3, 5), \{2, 4\}, \{1\}, \{6\}\}, \{(3, 6), \{2, 5\}, \{1\}, \{4\}\},
\{(1, 4), \{3, 6\}, \{5\}, \{2\}\}, \{(2, 4), \{1, 6\}, \{5\}, \{3\}\}, \{(3, 4), \{2, 6\}, \{5\}, \{1\}\},
\{(1, 5), \{3, 4\}, \{6\}, \{2\}\}, \{(1, 6), \{3, 5\}, \{4\}, \{2\}\}, \{(2, 5), \{1, 4\}, \{6\}, \{3\}\},
\{(2, 6), \{1, 5\}, \{4\}, \{3\}\}, \{(3, 5), \{2, 4\}, \{6\}, \{1\}\}, \{(3, 6), \{2, 5\}, \{4\}, \{1\}\}\}
of the tabloid \(P^{2^{2,1^2}} = \{(1, 4), \{3, 6\}, \{2\}, \{5\}\}\).
The inequalities between the tabloids of shape \(2^{2,1^2}\) and the tabloids of shape \(3, 1^3\)
and \(2^3\) are as follows:

\[
A^{(2^2,1^2)} < A^{(3,1^3)},
B^{(2^2,1^2)} < B^{(3,1^3)},
C^{(2^2,1^2)} < C^{(3,1^3)},
K^{(2^2,1^2)} < C^{(3,1^3)},
(142536)M^{(2^2,1^2)} < C^{(3,1^3)},
(465)C^{(2^2,1^2)} < E^{(3,1^3)},
L^{(2^2,1^2)} < E^{(3,1^3)},
(123)P^{(2^2,1^2)} < E^{(3,1^3)},
E^{(2^2,1^2)} < F^{(3,1^3)},
(14)(25)(36)K^{(2^2,1^2)} < F^{(3,1^3)},
(142536)L^{(2^2,1^2)} < F^{(3,1^3)},
(465)E^{(2^2,1^2)} < H^{(3,1^3)},
M^{(2^2,1^2)} < H^{(3,1^3)},
(142536)P^{(2^2,1^2)} < H^{(3,1^3)},
A^{(2^2,1^2)} < K^{(3,1^3)},
F^{(2^2,1^2)} < K^{(3,1^3)},
B^{(2^2,1^2)} < L^{(3,1^3)},
H^{(2^2,1^2)} < L^{(3,1^3)},
\]

and

\[
A^{(2^2,1^2)} < A^{(2^3)},
B^{(2^2,1^2)} < A^{(2^3)},
C^{(2^2,1^2)} < B^{(2^3)},
E^{(2^2,1^2)} < B^{(2^3)},
F^{(2^2,1^2)} < C^{(2^3)},
H^{(2^2,1^2)} < C^{(2^3)},
L^{(2^2,1^2)} < E^{(2^3)},
M^{(2^2,1^2)} < E^{(2^3)},
K^{(2^2,1^2)} < F^{(2^3)},
P^{(2^2,1^2)} < H^{(2^3)},
\]

because

\[
R_{1,3} A^{(2^2,1^2)} = A^{(3,1^3)},
\]
\[ R_{1,3}B^{(2^2,1^2)} = B^{(3,1^3)}, \]
\[ R_{1,4}C^{(2^2,1^2)} = R_{1,2}K^{(2^2,1^2)} = R_{1,1}(142536)M^{(2^2,1^2)} = C^{(3,1^3)}, \]
\[ R_{1,4}(465)C^{(2^2,1^2)} = R_{1,2}L^{(2^2,1^2)} = R_{1,1}(123)P^{(2^2,1^2)} = E^{(3,1^3)}, \]
\[ R_{1,4}E^{(2^2,1^2)} = R_{1,2}(14)(25)(36)K^{(2^2,1^2)} = R_{1,1}(142536)L^{(2^2,1^2)} = F^{(3,1^3)}, \]
\[ R_{1,4}(465)E^{(2^2,1^2)} = R_{1,2}M^{(2^2,1^2)} = R_{1,1}(142536)P^{(2^2,1^2)} = H^{(3,1^3)}, \]
\[ R_{1,4}A^{(2^2,1^2)} = R_{1,2}F^{(2^2,1^2)} = K^{(3,1^3)}, \]
\[ R_{1,4}B^{(2^2,1^2)} = R_{1,2}H^{(2^2,1^2)} = L^{(3,1^3)}, \]
and
\[ R_{3,6}A^{(2^2,1^2)} = R_{3,5}B^{(2^2,1^2)} = A^{(2^3)}, \]
\[ R_{3,6}C^{(2^2,1^2)} = R_{3,3}E^{(2^2,1^2)} = B^{(2^3)}, \]
\[ R_{3,6}F^{(2^2,1^2)} = R_{3,5}H^{(2^2,1^2)} = C^{(2^3)}, \]
\[ R_{3,5}L^{(2^2,1^2)} = R_{3,3}M^{(2^2,1^2)} = E^{(2^3)}, \]
\[ R_{3,6}K^{(2^2,1^2)} = F^{(2^3)}, \]
\[ R_{3,5}P^{(2^2,1^2)} = H^{(2^3)}. \]

These inequalities yield that the following simple substitution reactions between \((2^2,1^2)\)-derivatives, and \((3,1^3)\)- and \((2^3)\)-derivatives of ethane are possible:

\[ a_{(2^2,1^2)} < a_{(3,1^3)}, \]
\[ b_{(2^2,1^2)} < b_{(3,1^3)}, \]
\[ c_{(2^2,1^2)} < c_{(3,1^3)}, \]
\[ k_{(2^2,1^2)} < c_{(3,1^3)}, \]
\[ m_{(2^2,1^2)} < c_{(3,1^3)}, \]
\[ e_{(2^2,1^2)} < e_{(3,1^3)}, \]
\[ l_{(2^2,1^2)} < e_{(3,1^3)}, \]
\[ p_{(2^2,1^2)} < e_{(3,1^3)}, \]
\[ c_{(2^2,1^2)} < c_{(2^3)}, \]
\[ f_{(2^2,1^2)} < c_{(2^3)}, \]
\[ g_{(2^2,1^2)} < c_{(2^3)}, \]
\[ h_{(2^2,1^2)} < e_{(2^3)}, \]
\[ m_{(2^2,1^2)} < e_{(2^3)}, \]
\[ k_{(2^2,1^2)} < f_{(2^3)}, \]
\[ p_{(2^2,1^2)} < h_{(2^3)}. \]
The set of all $G'$-orbits is

$$T_{(2^2,1^2);G'} = \{ a_{(2^2,1^2)} \cup b_{(2^2,1^2)}, c_{(2^2,1^2)}, e_{(2^2,1^2)}, f_{(2^2,1^2)} \cup h_{(2^2,1^2)}, \ell_{(2^2,1^2)} \cup m_{(2^2,1^2)}, k_{(2^2,1^2)} \cup p_{(2^2,1^2)} \}. \quad (2.18)$$

Therefore the $(2^2,1^2)$-products that correspond to the members of the two-element sets

$$\{ a_{(2^2,1^2)}, b_{(2^2,1^2)} \}, \{ f_{(2^2,1^2)}, h_{(2^2,1^2)} \},$$

$$\{ \ell_{(2^2,1^2)}, m_{(2^2,1^2)} \}, \{ k_{(2^2,1^2)}, p_{(2^2,1^2)} \},$$

form chiral pairs and the products that correspond to formulae $c_{(2^2,1^2)}$, $e_{(2^2,1^2)}$ are dimers.

The set of all $G''$-orbits is

$$T_{(2^2,1^2);G''} = \{ a_{(2^2,1^2)} \cup b_{(2^2,1^2)}, c_{(2^2,1^2)}, e_{(2^2,1^2)}, f_{(2^2,1^2)} \cup h_{(2^2,1^2)}, \ell_{(2^2,1^2)} \cup m_{(2^2,1^2)} \cup k_{(2^2,1^2)} \cup p_{(2^2,1^2)} \}. \quad (2.19)$$

Hence the derivatives that correspond to members of different sets below are structural isomers and those that correspond to elements of one set are structurally identical:

$$\{ a_{(2^2,1^2)}, b_{(2^2,1^2)} \}, \{ c_{(2^2,1^2)} \}, \{ e_{(2^2,1^2)} \},$$

$$\{ f_{(2^2,1^2)}, h_{(2^2,1^2)} \}, \{ \ell_{(2^2,1^2)}, m_{(2^2,1^2)}, k_{(2^2,1^2)}, p_{(2^2,1^2)} \}.$$ 

Case 10. $\lambda = (2,1^4)$.

We have

$$T_{(2,1^4);G} = \{ a_{(2,1^4)}, \bar{a}_{(2,1^4)}, b_{(2,1^4)}, \bar{b}_{(2,1^4)}, c_{(2,1^4)}, \bar{c}_{(2,1^4)}, e_{(2,1^4)}, \bar{e}_{(2,1^4)}, f_{(2,1^4)}, \bar{f}_{(2,1^4)},$$

$$h_{(2,1^4)}, \bar{h}_{(2,1^4)}, \bar{k}_{(2,1^4)}, \bar{\ell}_{(2,1^4)}, \bar{m}_{(2,1^4)}, \bar{p}_{(2,1^4)} \}, \quad (2.20)$$

where:

$a_{(2,1^4)}$ is the $G$-orbit

$$\{(1,2), \{3\}, \{4\}, \{5\}, \{6\}\}, \{(2,3), \{1\}, \{4\}, \{5\}, \{6\}\}, \{(1,3), \{2\}, \{4\}, \{5\}, \{6\}\},$$

$$\{(1,2), \{3\}, \{5\}, \{6\}, \{4\}\}, \{(1,2), \{3\}, \{6\}, \{4\}, \{5\}\}, \{(2,3), \{1\}, \{5\}, \{6\}, \{4\}\},$$

$$\{(2,3), \{1\}, \{6\}, \{4\}, \{5\}\}, \{(1,3), \{2\}, \{5\}, \{6\}, \{4\}\}, \{(1,3), \{2\}, \{6\}, \{4\}, \{5\}\},$$

$$\{(4,5), \{6\}, \{1\}, \{2\}, \{3\}\}, \{(5,6), \{4\}, \{1\}, \{2\}, \{3\}\}, \{(4,6), \{5\}, \{1\}, \{2\}, \{3\}\},$$

$$\{(4,5), \{6\}, \{2\}, \{3\}, \{1\}\}, \{(4,5), \{6\}, \{3\}, \{1\}, \{2\}\}, \{(5,6), \{4\}, \{2\}, \{3\}, \{1\}\},$$

$$\{(5,6), \{4\}, \{3\}, \{1\}, \{2\}\}, \{(4,6), \{5\}, \{2\}, \{3\}, \{1\}\}. \quad (2.20)$$

of the tabloid $A^{(2,1^4)} = (\{1,2\}, \{3\}, \{4\}, \{5\}, \{6\})$;

$\bar{a}_{(2,1^4)}$ is the $G$-orbit

$$\{(1,2), \{4\}, \{3\}, \{5\}, \{6\}\}, \{(2,3), \{4\}, \{1\}, \{5\}, \{6\}\}, \{(1,3), \{4\}, \{2\}, \{5\}, \{6\}\}. \quad (2.20)$$
the tabloid $\tilde{A}^{(2,1^4)} = \{(1,2),\{4\},\{3\},\{5\},\{6\}\}$;
$b_{(2,1^4)}$ is the $G$-orbit

$\{(1,2),\{3\},\{4\},\{6\},\{5\}\},\{(1,2),\{3\},\{4\},\{6\},\{5\}\},\{(1,2),\{3\},\{4\},\{6\},\{5\}\}$,

$\{(2,3),\{6\},\{1\},\{4\},\{5\}\},\{(1,3),\{5\},\{2\},\{6\},\{4\}\},\{(1,3),\{6\},\{2\},\{4\},\{5\}\}$,

$\{(4,5),\{1\},\{6\},\{2\},\{3\}\},\{(5,6),\{1\},\{4\},\{2\},\{3\}\},\{(4,6),\{1\},\{5\},\{2\},\{3\}\}$,

$\{(4,5),\{2\},\{6\},\{3\},\{1\}\},\{(4,5),\{3\},\{6\},\{1\},\{2\}\},\{(5,6),\{2\},\{4\},\{3\},\{1\}\}$,

$\{(5,6),\{3\},\{4\},\{1\},\{2\}\},\{(4,6),\{2\},\{5\},\{3\},\{1\}\},\{(4,6),\{3\},\{5\},\{1\},\{2\}\}$

of the tabloid $\tilde{A}^{(2,1^4)} = \{(1,2),\{4\},\{3\},\{5\},\{6\}\}$;

$\tilde{b}_{(2,1^4)}$ is the $G$-orbit

$\{(1,2),\{3\},\{4\},\{6\},\{5\}\},\{(2,3),\{1\},\{4\},\{6\},\{5\}\},\{(1,3),\{2\},\{4\},\{6\},\{5\}\}$,

$\{(2,3),\{1\},\{6\},\{5\},\{4\}\},\{(1,3),\{2\},\{5\},\{4\},\{6\}\},\{(1,3),\{2\},\{6\},\{5\},\{4\}\}$,

$\{(4,5),\{6\},\{1\},\{3\},\{2\}\},\{(5,6),\{4\},\{1\},\{3\},\{2\}\},\{(4,6),\{5\},\{1\},\{3\},\{2\}\}$,

$\{(4,5),\{6\},\{2\},\{1\},\{3\}\},\{(4,5),\{6\},\{3\},\{2\},\{1\}\},\{(5,6),\{4\},\{2\},\{1\},\{3\}\}$,

$\{(5,6),\{4\},\{3\},\{2\},\{1\}\},\{(4,6),\{5\},\{2\},\{1\},\{3\}\},\{(4,6),\{5\},\{3\},\{2\},\{1\}\}$

of the tabloid $\tilde{B}^{(2,1^4)} = \{(1,2),\{3\},\{4\},\{6\},\{5\}\}$;

$\tilde{c}_{(2,1^4)}$ is the $G$-orbit

$\{(1,2),\{4\},\{3\},\{6\},\{5\}\},\{(2,3),\{4\},\{1\},\{6\},\{5\}\},\{(1,3),\{4\},\{2\},\{6\},\{5\}\}$,

$\{(2,3),\{6\},\{1\},\{5\},\{4\}\},\{(1,3),\{5\},\{2\},\{4\},\{6\}\},\{(1,3),\{6\},\{2\},\{5\},\{4\}\}$,

$\{(4,5),\{1\},\{6\},\{3\},\{2\}\},\{(5,6),\{1\},\{4\},\{3\},\{2\}\},\{(4,6),\{1\},\{5\},\{3\},\{2\}\}$,

$\{(4,5),\{2\},\{6\},\{1\},\{3\}\},\{(4,5),\{3\},\{6\},\{2\},\{1\}\},\{(5,6),\{2\},\{4\},\{1\},\{3\}\}$,

$\{(5,6),\{3\},\{4\},\{2\},\{1\}\},\{(4,6),\{2\},\{5\},\{1\},\{3\}\},\{(4,6),\{3\},\{5\},\{2\},\{1\}\}$

of the tabloid $\tilde{B}^{(2,1^4)} = \{(1,2),\{4\},\{3\},\{6\},\{5\}\}$;

$\tilde{c}_{(2,1^4)}$ is the $G$-orbit

$\{(1,2),\{4\},\{5\},\{3\},\{6\}\},\{(2,3),\{4\},\{5\},\{1\},\{6\}\},\{(1,3),\{4\},\{5\},\{2\},\{6\}\}$,

$\{(1,2),\{5\},\{6\},\{3\},\{4\}\},\{(1,2),\{6\},\{4\},\{3\},\{5\}\},\{(2,3),\{5\},\{6\},\{1\},\{4\}\}$,

$\{(2,3),\{6\},\{4\},\{1\},\{5\}\},\{(1,3),\{5\},\{6\},\{2\},\{4\}\},\{(1,3),\{6\},\{4\},\{2\},\{5\}\}$,

$\{(4,5),\{1\},\{2\},\{6\},\{3\}\},\{(5,6),\{1\},\{2\},\{4\},\{3\}\},\{(4,6),\{1\},\{2\},\{5\},\{3\}\}$,

$\{(4,5),\{2\},\{3\},\{6\},\{1\}\},\{(4,5),\{3\},\{1\},\{6\},\{2\}\},\{(5,6),\{2\},\{3\},\{4\},\{1\}\}$,

$\{(5,6),\{3\},\{1\},\{4\},\{2\}\},\{(4,6),\{2\},\{3\},\{5\},\{1\}\},\{(4,6),\{3\},\{5\},\{1\},\{2\}\}$

of the tabloid $\tilde{C}^{(2,1^4)} = \{(1,2),\{4\},\{5\},\{3\},\{6\}\)$;
\(c_{(2,1^4)}\) is the \(G\)-orbit

\[
\{\{1, 2\}, \{5\}, \{4\}, \{3\}, \{6\}\}, \{\{2, 3\}, \{5\}, \{4\}, \{1\}, \{6\}\}, \{\{1, 3\}, \{5\}, \{4\}, \{2\}, \{6\}\}, \\
(1, 2), \{6\}, \{5\}, \{3\}, \{4\}\}, \{\{1, 2\}, \{4\}, \{6\}, \{3\}, \{5\}\}, \{\{2, 3\}, \{6\}, \{5\}, \{1\}, \{4\}\}, \\
\{\{2, 3\}, \{4\}, \{6\}, \{1\}, \{5\}\}, \{\{1, 3\}, \{6\}, \{5\}, \{2\}, \{4\}\}, \{\{1, 3\}, \{4\}, \{6\}, \{2\}, \{5\}\}, \\
\{\{4, 5\}, \{2\}, \{1\}, \{6\}, \{3\}\}, \{\{5, 6\}, \{2\}, \{1\}, \{4\}, \{3\}\}, \{\{4, 6\}, \{2\}, \{1\}, \{5\}, \{3\}\}, \\
\{\{4, 5\}, \{3\}, \{2\}, \{6\}, \{1\}\}, \{\{4, 5\}, \{1\}, \{3\}, \{6\}, \{2\}\}, \{\{5, 6\}, \{3\}, \{2\}, \{4\}, \{1\}\}, \\
\{\{5, 6\}, \{1\}, \{3\}, \{4\}, \{2\}\}, \{\{4, 6\}, \{3\}, \{2\}, \{5\}, \{1\}\}, \{\{4, 6\}, \{1\}, \{3\}, \{5\}, \{2\}\}.
\]

of the tabloid \(C_{(2,1^4)} = (\{1, 2\}, \{5\}, \{4\}, \{3\}, \{6\}\);

\(e_{(2,1^4)}\) is the \(G\)-orbit

\[
\{\{1, 2\}, \{4\}, \{5\}, \{6\}, \{3\}\}, \{\{2, 3\}, \{4\}, \{5\}, \{6\}, \{1\}\}, \{\{1, 3\}, \{4\}, \{5\}, \{6\}, \{2\}\}, \\
(1, 2), \{5\}, \{6\}, \{4\}, \{3\}\}, \{\{1, 2\}, \{6\}, \{4\}, \{5\}, \{3\}\}, \{\{2, 3\}, \{5\}, \{6\}, \{4\}, \{1\}\}, \\
\{\{2, 3\}, \{6\}, \{4\}, \{5\}, \{1\}\}, \{\{1, 3\}, \{5\}, \{6\}, \{4\}, \{2\}\}, \{\{1, 3\}, \{6\}, \{4\}, \{5\}, \{2\}\}, \\
\{\{4, 5\}, \{1\}, \{2\}, \{3\}, \{6\}\}, \{\{5, 6\}, \{1\}, \{2\}, \{3\}, \{4\}\}, \{\{4, 6\}, \{1\}, \{2\}, \{3\}, \{5\}\}, \\
\{\{4, 5\}, \{2\}, \{3\}, \{1\}, \{6\}\}, \{\{4, 5\}, \{3\}, \{1\}, \{2\}, \{6\}\}, \{\{5, 6\}, \{2\}, \{3\}, \{1\}, \{4\}\}, \\
\{\{5, 6\}, \{3\}, \{1\}, \{2\}, \{4\}\}, \{\{4, 6\}, \{2\}, \{3\}, \{1\}, \{5\}\}, \{\{4, 6\}, \{3\}, \{1\}, \{2\}, \{5\}\}.
\]

of the tabloid \(E_{(2,1^4)} = (\{1, 2\}, \{4\}, \{5\}, \{6\}, \{3\}\);

\(\bar{e}_{(2,1^4)}\) is the \(G\)-orbit

\[
\{\{1, 2\}, \{5\}, \{4\}, \{6\}, \{3\}\}, \{\{2, 3\}, \{5\}, \{4\}, \{6\}, \{1\}\}, \{\{1, 3\}, \{5\}, \{4\}, \{6\}, \{2\}\}, \\
(1, 2), \{6\}, \{5\}, \{4\}, \{3\}\}, \{\{1, 2\}, \{4\}, \{6\}, \{5\}, \{3\}\}, \{\{2, 3\}, \{6\}, \{5\}, \{4\}, \{1\}\}, \\
\{\{2, 3\}, \{4\}, \{6\}, \{5\}, \{1\}\}, \{\{1, 3\}, \{6\}, \{5\}, \{4\}, \{2\}\}, \{\{1, 3\}, \{4\}, \{6\}, \{5\}, \{2\}\}, \\
\{\{4, 5\}, \{2\}, \{1\}, \{3\}, \{6\}\}, \{\{5, 6\}, \{2\}, \{1\}, \{3\}, \{4\}\}, \{\{4, 6\}, \{2\}, \{1\}, \{3\}, \{5\}\}, \\
\{\{4, 5\}, \{3\}, \{2\}, \{1\}, \{6\}\}, \{\{4, 5\}, \{1\}, \{3\}, \{2\}, \{6\}\}, \{\{5, 6\}, \{3\}, \{2\}, \{1\}, \{4\}\}, \\
\{\{5, 6\}, \{1\}, \{3\}, \{2\}, \{4\}\}, \{\{4, 6\}, \{3\}, \{2\}, \{1\}, \{5\}\}, \{\{4, 6\}, \{1\}, \{3\}, \{2\}, \{5\}\}.
\]

of the tabloid \(\bar{E}_{(2,1^4)} = (\{1, 2\}, \{5\}, \{4\}, \{6\}, \{3\}\);

\(f_{(2,1^4)}\) is the \(G\)-orbit

\[
\{\{1, 4\}, \{2\}, \{3\}, \{5\}, \{6\}\}, \{\{2, 4\}, \{3\}, \{1\}, \{5\}, \{6\}\}, \{\{3, 4\}, \{1\}, \{2\}, \{5\}, \{6\}\}, \\
(1, 5), \{2\}, \{3\}, \{6\}, \{4\}\}, \{\{1, 6\}, \{2\}, \{3\}, \{4\}, \{5\}\}, \{\{2, 5\}, \{3\}, \{1\}, \{6\}, \{4\}\}, \\
\{\{2, 6\}, \{3\}, \{1\}, \{4\}, \{5\}\}, \{\{3, 5\}, \{1\}, \{2\}, \{6\}, \{4\}\}, \{\{3, 6\}, \{1\}, \{2\}, \{4\}, \{5\}\}, \\
\{\{1, 4\}, \{5\}, \{6\}, \{2\}, \{3\}\}, \{\{1, 5\}, \{6\}, \{4\}, \{2\}, \{3\}\}, \{\{1, 6\}, \{4\}, \{5\}, \{2\}, \{3\}\}, \\
\{\{2, 4\}, \{5\}, \{6\}, \{3\}, \{1\}\}, \{\{3, 4\}, \{5\}, \{6\}, \{1\}, \{2\}\}, \{\{2, 5\}, \{6\}, \{4\}, \{3\}, \{1\}\}, \\
\{\{2, 6\}, \{3\}, \{1\}, \{4\}, \{5\}\}, \{\{3, 5\}, \{1\}, \{2\}, \{6\}, \{4\}\}, \{\{3, 6\}, \{1\}, \{2\}, \{4\}, \{5\}\}, \\
\{\{1, 4\}, \{5\}, \{6\}, \{2\}, \{3\}\}, \{\{1, 5\}, \{6\}, \{4\}, \{2\}, \{3\}\}, \{\{1, 6\}, \{4\}, \{5\}, \{2\}, \{3\}\}, \\
\{\{2, 4\}, \{5\}, \{6\}, \{3\}, \{1\}\}, \{\{3, 4\}, \{5\}, \{6\}, \{1\}, \{2\}\}, \{\{2, 5\}, \{6\}, \{4\}, \{3\}, \{1\}\}.
\]
of the tabloid $F^{(2,1^4)} = \{(1,4), \{2\}, \{3\}, \{5\}, \{6\}\}$; 

$f_{(2,1^4)}$ is the $G$-orbit

$\{\{(1,4), \{3\}, \{2\}, \{5\}, \{6\}\}, \{(2,4), \{1\}, \{3\}, \{5\}, \{6\}\}, \{(3,4), \{2\}, \{1\}, \{5\}, \{6\}\}\}$

$\{(1,5), \{3\}, \{2\}, \{6\}, \{4\}\}, \{(1,6), \{3\}, \{2\}, \{4\}, \{5\}\}, \{(2,5), \{1\}, \{3\}, \{6\}, \{4\}\}$

$\{(2,6), \{1\}, \{3\}, \{4\}, \{5\}\}, \{(3,5), \{2\}, \{1\}, \{6\}, \{4\}\}$

$\{(1,4), \{6\}, \{5\}, \{2\}, \{3\}\}, \{(2,4), \{6\}, \{5\}, \{3\}, \{1\}\}$

$\{(3,4), \{6\}, \{5\}, \{1\}, \{2\}\}, \{(2,5), \{4\}, \{6\}, \{3\}, \{1\}\}$

$\{(3,5), \{4\}, \{6\}, \{1\}, \{2\}\}, \{(2,6), \{5\}, \{4\}, \{3\}, \{1\}\}$

of the tabloid $\tilde{F}^{(2,1^4)} = \{(1,4), \{3\}, \{2\}, \{5\}, \{6\}\}$

$h_{(2,1^4)}$ is the $G$-orbit

$\{\{(1,4), \{2\}, \{3\}, \{6\}, \{5\}\}, \{(2,4), \{3\}, \{1\}, \{6\}, \{5\}\}, \{(3,4), \{1\}, \{2\}, \{6\}, \{5\}\}\}$

$\{(1,5), \{2\}, \{3\}, \{4\}, \{6\}\}, \{(1,6), \{2\}, \{3\}, \{5\}, \{4\}\}$

$\{(2,6), \{3\}, \{1\}, \{5\}, \{4\}\}, \{(3,5), \{1\}, \{2\}, \{4\}, \{6\}\}$

$\{(1,4), \{5\}, \{6\}, \{3\}, \{2\}\}, \{(2,4), \{6\}, \{5\}, \{1\}, \{3\}\}$

$\{(3,5), \{6\}, \{4\}, \{2\}, \{1\}\}, \{(2,6), \{4\}, \{5\}, \{1\}, \{3\}\}$

of the tabloid $H^{(2,1^4)} = \{(1,4), \{2\}, \{3\}, \{6\}, \{5\}\}$

$h_{(2,1^4)}$ is the $G$-orbit

$\{\{(1,4), \{3\}, \{2\}, \{6\}, \{5\}\}, \{(2,4), \{1\}, \{3\}, \{6\}, \{5\}\}, \{(3,4), \{2\}, \{1\}, \{6\}, \{5\}\}\}$

$\{(1,5), \{3\}, \{2\}, \{4\}, \{6\}\}, \{(1,6), \{3\}, \{2\}, \{5\}, \{4\}\}$

$\{(2,6), \{1\}, \{3\}, \{5\}, \{4\}\}, \{(3,5), \{2\}, \{1\}, \{4\}, \{6\}\}$

$\{(1,4), \{6\}, \{5\}, \{3\}, \{2\}\}, \{(2,4), \{6\}, \{5\}, \{1\}, \{3\}\}$

$\{(3,5), \{4\}, \{6\}, \{2\}, \{1\}\}, \{(2,6), \{5\}, \{4\}, \{1\}, \{3\}\}$

of the tabloid $\tilde{H}^{(2,1^4)} = \{(1,4), \{3\}, \{2\}, \{6\}, \{5\}\}$

$k_{(2,1^4)}$ is the $G$-orbit

$\{\{(1,4), \{2\}, \{5\}, \{3\}, \{6\}\}, \{(2,4), \{3\}, \{5\}, \{1\}, \{6\}\}, \{(3,4), \{1\}, \{5\}, \{2\}, \{6\}\}\}$

$\{(1,5), \{2\}, \{6\}, \{3\}, \{4\}\}, \{(1,6), \{2\}, \{4\}, \{3\}, \{5\}\}$

$\{(2,5), \{3\}, \{6\}, \{1\}, \{4\}\}$

$\{(2,6), \{3\}, \{4\}, \{1\}, \{5\}\}, \{(3,5), \{1\}, \{6\}, \{2\}, \{4\}\}$

$\{(3,6), \{1\}, \{4\}, \{2\}, \{5\}\}$
of the tabloid $K^{(2,1^4)} = \{(1,4), \{2\}, \{5\}, \{3\}, \{6\}\}$;
$
\tilde{k}_{(2,1^4)}$ is the $G$-orbit

$\{(1,4), \{5\}, \{2\}, \{3\}, \{6\}\}, \{(2,4), \{5\}, \{3\}, \{1\}, \{6\}\}, \{(3,4), \{5\}, \{1\}, \{2\}, \{6\}\},

$\{(1,5), \{6\}, \{2\}, \{4\}, \{3\}\}, \{(1,6), \{4\}, \{2\}, \{3\}, \{5\}\}, \{(2,5), \{6\}, \{3\}, \{1\}, \{4\}\},

$\{(2,6), \{4\}, \{3\}, \{5\}, \{1\}\}, \{(3,5), \{6\}, \{1\}, \{4\}, \{2\}\}, \{(3,6), \{4\}, \{1\}, \{5\}, \{2\}\}$

of the tabloid $\tilde{K}^{(2,1^4)} = \{(1,4), \{5\}, \{2\}, \{3\}, \{6\}\}$;
$
\ell_{(2,1^4)}$ is the $G$-orbit

$\{(1,4), \{2\}, \{6\}, \{3\}, \{5\}\}, \{(2,4), \{3\}, \{6\}, \{1\}, \{5\}\}, \{(3,4), \{1\}, \{6\}, \{2\}, \{5\}\},

$\{(1,5), \{2\}, \{4\}, \{3\}, \{6\}\}, \{(1,6), \{2\}, \{5\}, \{3\}, \{4\}\}, \{(2,5), \{3\}, \{4\}, \{1\}, \{6\}\},

$\{(2,6), \{3\}, \{5\}, \{1\}, \{4\}\}, \{(3,5), \{1\}, \{4\}, \{2\}\}, \{(3,6), \{1\}, \{5\}, \{2\}\}$

of the tabloid $\tilde{L}^{(2,1^4)} = \{(1,4), \{2\}, \{6\}, \{3\}, \{5\}\}$;
$
\ell_{(2,1^4)}$ is the $G$-orbit

$\{(1,4), \{6\}, \{2\}, \{3\}, \{5\}\}, \{(2,4), \{6\}, \{3\}, \{1\}, \{5\}\}, \{(3,4), \{6\}, \{1\}, \{2\}, \{5\}\},

$\{(1,5), \{4\}, \{2\}, \{3\}, \{6\}\}, \{(1,6), \{5\}, \{2\}, \{3\}, \{4\}\}, \{(2,5), \{4\}, \{3\}, \{1\}, \{6\}\},

$\{(2,6), \{5\}, \{3\}, \{1\}, \{4\}\}, \{(3,5), \{4\}, \{1\}, \{2\}, \{6\}\}, \{(3,6), \{5\}, \{1\}, \{2\}, \{4\}\},

$\{(1,4), \{3\}, \{5\}, \{6\}, \{2\}\}, \{(1,5), \{3\}, \{6\}, \{4\}, \{2\}\}, \{(1,6), \{3\}, \{4\}, \{5\}, \{2\}\},

$\{(2,4), \{1\}, \{5\}, \{6\}, \{3\}\}, \{(3,4), \{2\}, \{5\}, \{6\}, \{1\}\}, \{(2,5), \{1\}, \{6\}, \{4\}, \{3\}\},

$\{(3,5), \{2\}, \{6\}, \{4\}, \{1\}\}, \{(2,6), \{1\}, \{4\}, \{5\}, \{3\}\}, \{(3,6), \{2\}, \{4\}, \{5\}, \{1\}\}$

of the tabloid $\tilde{L}^{(2,1^4)} = \{(1,4), \{6\}, \{2\}, \{3\}, \{5\}\}$;
$
m_{(2,1^4)}$ is the $G$-orbit

$\{(1,4), \{2\}, \{6\}, \{5\}, \{3\}\}, \{(2,4), \{3\}, \{6\}, \{5\}, \{1\}\}, \{(3,4), \{1\}, \{6\}, \{5\}, \{2\}\}$,
of the tabloid $M^{(2,1^4)} = \{(1,4), (2), (6), (5), (3)\};$

$\tilde{m}_{(2,1^4)}$ is the $G$-orbit

$$\{((1,5), (4), (2), (6), (3)), ((1,6), (5), (2), (4), (3)), ((2,5), (4), (3), (6), (1)), ((2,6), (5), (3), (4), (1)), ((3,5), (4), (1), (6), (2)), ((3,6), (5), (1), (4), (2)), ((1,4), (3), (5), (2), (6)), ((1,5), (3), (6), (2), (4)), ((1,6), (3), (4), (2), (5)), ((2,4), (1), (5), (3), (6)), ((3,4), (2), (5), (1), (6)), ((2,5), (1), (6), (3), (4)), ((3,5), (2), (6), (1), (4)), ((2,6), (1), (4), (3), (5)), ((3,6), (2), (4), (1), (5))\}$$

of the tabloid $\tilde{M}^{(2,1^4)} = \{(1,4), (6), (2), (5), (3)\};$

$p_{(2,1^4)}$ is the $G$-orbit

$$\{((1,5), (3), (6), (2), (5)), ((2,4), (1), (6), (3), (5)), ((3,4), (2), (6), (1), (5)), ((2,6), (1), (5), (3), (4)), ((3,5), (2), (4), (1), (6)), ((3,6), (2), (5), (1), (4)), ((1,4), (6), (3), (5), (2)), ((2,4), (6), (1), (5), (3)), ((3,4), (6), (2), (5), (1)), ((1,5), (4), (3), (6), (2)), ((1,6), (5), (3), (4), (2)), ((2,5), (4), (1), (6), (3)), ((2,6), (5), (1), (4), (3)), ((3,5), (4), (2), (6), (1)), ((3,6), (5), (2), (4), (1))\}$$

of the tabloid $P^{(2,1^4)} = \{(1,4), (3), (6), (2), (5)\};$

$\bar{p}_{(2,1^4)}$ is the $G$-orbit

$$\{((1,5), (6), (3), (2), (5)), ((2,4), (6), (1), (3), (5)), ((3,4), (6), (2), (1), (5)), ((1,6), (5), (3), (2), (4)), ((2,5), (4), (1), (3), (6)), ((2,6), (5), (1), (3), (4)), ((3,5), (4), (2), (1), (6)), ((3,6), (5), (2), (1), (4)), ((1,4), (3), (6), (5), (2)), ((2,4), (1), (6), (5), (3)), ((3,4), (2), (6), (5), (1)), ((1,5), (3), (4), (6), (2)), ((1,6), (3), (5), (4), (2)), ((2,5), (1), (4), (6), (3)), ((2,6), (1), (5), (4), (3)), ((3,5), (2), (4), (6), (1)), ((3,6), (2), (5), (4), (1))\}$$

of the tabloid $\bar{P}^{(2,1^4)} = \{(1,4), (6), (3), (2), (5)\}.$
The inequalities between the tabloids of shape $(2, 1^4)$ and the tabloids of shape $(2^2, 1^2)$ are as follows:
\[
\begin{align*}
A^{(2,1^4)} &< A^{(2^2,1^2)}, A^{(2,1^4)} < A^{(2^2,1^2)}, B^{(2,1^4)} < A^{(2^2,1^2)}, B^{(2,1^4)} < A^{(2^2,1^2)}, \\
C^{(2,1^4)} &< A^{(2^2,1^2)}, E^{(2,1^4)} < A^{(2^2,1^2)}, \\
A^{(2,1^4)} &< B^{(2^2,1^2)}, \tilde{A}^{(2,1^4)} < B^{(2^2,1^2)}, B^{(2,1^4)} < B^{(2^2,1^2)}, B^{(2,1^4)} < B^{(2^2,1^2)}, \\
(465)\tilde{C}^{(2,1^4)} &< B^{(2^2,1^2)}, (465)\tilde{E}^{(2,1^4)} < B^{(2^2,1^2)}, \\
\tilde{A}^{(2,1^4)} &< C^{(2^2,1^2)}, \tilde{B}^{(2,1^4)} < C^{(2^2,1^2)}, C^{(2,1^4)} < C^{(2^2,1^2)}, \tilde{C}^{(2,1^4)} < C^{(2^2,1^2)}, \\
E^{(2,1^4)} &< C^{(2^2,1^2)}, \tilde{E}^{(2,1^4)} < C^{(2^2,1^2)}, \\
C^{(2,1^4)} &< E^{(2^2,1^2)}, \tilde{C}^{(2,1^4)} < E^{(2^2,1^2)}, E^{(2,1^4)} < E^{(2^2,1^2)}, \tilde{E}^{(2,1^4)} < E^{(2^2,1^2)}, \\
F^{(2,1^4)} &< F^{(2^2,1^2)}, \tilde{F}^{(2,1^4)} < F^{(2^2,1^2)}, H^{(2,1^4)} < F^{(2^2,1^2)}, H^{(2,1^4)} < F^{(2^2,1^2)}, \\
K^{(2,1^4)} &< F^{(2^2,1^2)}, (14)(25)(36)\tilde{K}^{(2,1^4)} < F^{(2^2,1^2)}, \\
(14)(25)(36)\tilde{L}^{(2,1^4)} &< F^{(2^2,1^2)}, (14)(25)(36)M^{(2,1^4)} < F^{(2^2,1^2)}, \\
F^{(2,1^4)} &< H^{(2^2,1^2)}, \tilde{F}^{(2,1^4)} < H^{(2^2,1^2)}, H^{(2,1^4)} < H^{(2^2,1^2)}, \tilde{H}^{(2,1^4)} < H^{(2^2,1^2)}, \\
L^{(2,1^4)} &< H^{(2^2,1^2)}, M^{(2,1^4)} < H^{(2^2,1^2)}, \\
P^{(2,1^4)} &< H^{(2^2,1^2)}, (14)(25)(36)\tilde{P}^{(2,1^4)} < H^{(2^2,1^2)}, \\
F^{(2,1^4)} &< K^{(2^2,1^2)}, H^{(2,1^4)} < K^{(2^2,1^2)}, K^{(2,1^4)} < K^{(2^2,1^2)}, \\
\tilde{K}^{(2,1^4)} &< K^{(2^2,1^2)}, (14)(25)(36)\tilde{L}^{(2,1^4)} < K^{(2^2,1^2)}, (14)(25)(36)M^{(2,1^4)} < K^{(2^2,1^2)}, \\
F^{(2,1^4)} &< L^{(2^2,1^2)}, H^{(2,1^4)} < L^{(2^2,1^2)}, L^{(2,1^4)} < L^{(2^2,1^2)}, \tilde{L}^{(2,1^4)} < L^{(2^2,1^2)}, \\
M^{(2,1^4)} &< L^{(2^2,1^2)}, \tilde{M}^{(2,1^4)} < L^{(2^2,1^2)}, \\
(14)(25)(36)\tilde{P}^{(2,1^4)} &< L^{(2^2,1^2)}, \tilde{P}^{(2,1^4)} < L^{(2^2,1^2)}, \\
(14)(25)(36)\tilde{F}^{(2,1^4)} &< M^{(2^2,1^2)}, (14)(25)(36)\tilde{H}^{(2,1^4)} < M^{(2^2,1^2)}, \\
K^{(2,1^4)} &< M^{(2^2,1^2)}, (14)(25)(36)\tilde{K}^{(2,1^4)} < M^{(2^2,1^2)}, \\
L^{(2,1^4)} &< M^{(2^2,1^2)}, \tilde{L}^{(2,1^4)} < M^{(2^2,1^2)}, M^{(2,1^4)} < M^{(2^2,1^2)}, \tilde{M}^{(2,1^4)} < M^{(2^2,1^2)}, \\
\tilde{F}^{(2,1^4)} &< P^{(2^2,1^2)}, \tilde{H}^{(2,1^4)} < P^{(2^2,1^2)}, \tilde{L}^{(2,1^4)} < P^{(2^2,1^2)}, \\
\tilde{M}^{(2,1^4)} &< P^{(2^2,1^2)}, P^{(2,1^4)} < P^{(2^2,1^2)}, \tilde{P}^{(2,1^4)} < P^{(2^2,1^2)}, \\
\text{because} & \\
R_{2,4}R_{3,5}R_{4,6}A^{(2,1^4)} &= R_{2,3}R_{3,5}R_{4,6}\tilde{A}^{(2,1^4)} = R_{2,4}R_{3,5}B^{(2,1^4)} = R_{2,3}R_{3,5}\tilde{B}^{(2,1^4)} = R_{2,3}R_{4,6}C^{(2,1^4)} = R_{2,3}\tilde{C}^{(2,1^4)} = A^{(2^2,1^2)}, \\
R_{2,3}R_{3,5}B^{(2,1^4)} &= R_{2,3}R_{4,6}C^{(2,1^4)} = R_{2,3}\tilde{E}^{(2,1^4)} = A^{(2^2,1^2)},
\end{align*}
\]
These inequalities imply the following possible substitution reactions between $(2^1)^4$-derivatives and $(2^2,1^2)$-derivatives of ethane:

$$a_{(2^1)^4} < a_{(2^2,1^2)}, \bar{a}_{(2^1)^4} < a_{(2^2,1^2)}, b_{(2^1)^4} < a_{(2^2,1^2)}, \bar{b}_{(2^1)^4} < a_{(2^2,1^2)},$$

$$c_{(2^1)^4} < a_{(2^2,1^2)}, \bar{c}_{(2^1)^4} < a_{(2^2,1^2)},$$

$$a_{(2^1)^4} < b_{(2^2,1^2)}, \bar{a}_{(2^1)^4} < b_{(2^2,1^2)}, b_{(2^1)^4} < b_{(2^2,1^2)}, \bar{b}_{(2^1)^4} < b_{(2^2,1^2)}.$$
\[ \tilde{c}(2,1^4) < b(2^2,1^2), \tilde{e}(2,1^4) < b(2^2,1^2), \]
\[ \tilde{a}(2,1^4) < c(2^2,1^2), \tilde{b}(2,1^4) < c(2^2,1^2), c(2,1^4) < c(2^2,1^2), \tilde{c}(2,1^4) < c(2^2,1^2), \]
\[ \tilde{e}(2,1^4) < c(2^2,1^2), \tilde{e}(2,1^4) < c(2^2,1^2), \]
\[ c(2,1^4) < e(2^2,1^2), \tilde{c}(2,1^4) < e(2^2,1^2), e(2,1^4) < e(2^2,1^2), \tilde{e}(2,1^4) < e(2^2,1^2), \]
\[ f(2,1^4) < f(2^2,1^2), \tilde{f}(2,1^4) < f(2^2,1^2), h(2,1^4) < f(2^2,1^2), h(2,1^4) < f(2^2,1^2), \]
\[ k(2,1^4) < f(2^2,1^2), \tilde{k}(2,1^4) < f(2^2,1^2), \tilde{e}(2,1^4) < f(2^2,1^2), m(2,1^4) < f(2^2,1^2), \]
\[ f(2,1^4) < h(2^2,1^2), \tilde{f}(2,1^4) < h(2^2,1^2), h(2,1^4) < h(2^2,1^2), h(2,1^4) < h(2^2,1^2), \]
\[ \ell(2,1^4) < h(2^2,1^2), m(2,1^4) < h(2^2,1^2), p(2,1^4) < h(2^2,1^2), \tilde{p}(2,1^4) < h(2^2,1^2), \]
\[ f(2,1^4) < k(2^2,1^2), h(2,1^4) < k(2^2,1^2), k(2,1^4) < k(2^2,1^2), \]
\[ \tilde{k}(2,1^4) < k(2^2,1^2), \ell(2,1^4) < k(2^2,1^2), m(2,1^4) < k(2^2,1^2), \]
\[ f(2,1^4) < \ell(2^2,1^2), h(2,1^4) < \ell(2^2,1^2), \ell(2,1^4) < \ell(2^2,1^2), \]
\[ m(2,1^4) < \ell(2^2,1^2), \tilde{m}(2,1^4) < \ell(2^2,1^2), p(2,1^4) < \ell(2^2,1^2), \tilde{p}(2,1^4) < \ell(2^2,1^2), \]
\[ f(2,1^4) < m(2^2,1^2), h(2,1^4) = m(2^2,1^2), k(2,1^4) = m(2^2,1^2), \tilde{k}(2,1^4) = m(2^2,1^2), \]
\[ \ell(2,1^4) < m(2^2,1^2), \tilde{\ell}(2,1^4) < m(2^2,1^2), m(2,1^4) = m(2^2,1^2), \tilde{m}(2,1^4) = m(2^2,1^2), \]
\[ f(2,1^4) < p(2^2,1^2), h(2,1^4) = p(2^2,1^2), \tilde{\ell}(2,1^4) < p(2^2,1^2), \]
\[ \tilde{m}(2,1^4) = p(2^2,1^2), p(2,1^4) = p(2^2,1^2), \tilde{p}(2,1^4) < p(2^2,1^2). \]

The set of all $G'$-orbits is

\[
T_{(2,1^4);G'} = \{ a(2,1^4) \cup b(2,1^4), \tilde{a}(2,1^4) \cup \tilde{b}(2,1^4), c(2,1^4) \cup \tilde{c}(2,1^4), e(2,1^4) \cup \tilde{e}(2,1^4), f(2,1^4) \cup \tilde{f}(2,1^4), \}
\]
\[ \tilde{f}(2,1^4) \cup h(2,1^4), k(2,1^4) \cup p(2,1^4), \tilde{k}(2,1^4) \cup \tilde{p}(2,1^4), \ell(2,1^4) \cup \tilde{\ell}(2,1^4) \cup m(2,1^4) \}. \tag{2.21} \]

Hence the products of ethane that correspond to the members of the two-element sets

\[
\{ a(2,1^4), b(2,1^4) \}, \{ \tilde{a}(2,1^4), \tilde{b}(2,1^4) \}, \{ c(2,1^4), \tilde{c}(2,1^4) \}, \{ e(2,1^4), \tilde{e}(2,1^4) \}, \{ f(2,1^4), \tilde{h}(2,1^4) \}, \]
\[ \{ \tilde{f}(2,1^4), h(2,1^4) \}, \{ k(2,1^4), p(2,1^4) \}, \{ \tilde{k}(2,1^4), \tilde{p}(2,1^4) \}, \{ \ell(2,1^4), \tilde{\ell}(2,1^4) \}, \{ m(2,1^4) \} \]
form chiral pairs.

The set of all $G''$-orbits is

\[
T_{(2,1^4);G''} = \{ a(2,1^4) \cup b(2,1^4), \tilde{a}(2,1^4) \cup \tilde{b}(2,1^4), c(2,1^4) \cup \tilde{c}(2,1^4), e(2,1^4) \cup \tilde{e}(2,1^4), f(2,1^4) \cup \tilde{f}(2,1^4), \}
\]
\[ k(2,1^4) \cup p(2,1^4) \cup \ell(2,1^4) \cup \tilde{m}(2,1^4), \tilde{k}(2,1^4) \cup \tilde{p}(2,1^4) \cup \tilde{\ell}(2,1^4) \cup m(2,1^4) \}. \tag{2.22} \]

Therefore the derivatives of ethane that correspond to the members of different sets among

\[
\{ a(2,1^4), b(2,1^4) \}, \{ \tilde{a}(2,1^4), \tilde{b}(2,1^4) \}, \{ c(2,1^4), \tilde{c}(2,1^4) \}, \{ e(2,1^4), \tilde{e}(2,1^4) \}, \]
\[ \{ f(2,1^4), \tilde{h}(2,1^4), \tilde{f}(2,1^4), h(2,1^4) \}, \{ k(2,1^4), p(2,1^4), \ell(2,1^4), \tilde{m}(2,1^4) \}, \]
are structural isomers and those products that correspond to members of one and the same set are structurally identical.

Case II. \( \lambda = (1^6) \).

The set \( T_{(1^6)} \) of tabloids of shape \( (1^6) \) can be identified with the symmetric group \( S_6 \), and after this identification the set of all \( G \)-orbits in \( T_{(1^6)} \) coincide with the set of all right cosets of the group \( S_6 \) modulo \( G \). Thus, we identify the tabloid \( A = \{(i_1), \{i_2\}, \{i_3\}, \{i_4\}, \{i_5\}, \{i_6\}\} \) with the permutation \( \zeta = (i_1, i_2, i_3, i_4, i_5, i_6) \) of the numbers 1, 2, 3, 4, 5, 6 of unsatisfied valences, and then the \( G \)-orbit of \( A \) is identified with the right coset \( G\zeta \). We set

\[
T_{(1^6);G} = \{a_{(1^6)}, a_{(1^6)}^1, \bar{a}_{(1^6)}, \bar{a}_{(1^6)}^1, b_{(1^6)}, b_{(1^6)}^1, \tilde{b}_{(1^6)}, \tilde{b}_{(1^6)}^1, c_{(1^6)}, c_{(1^6)}^1, \\
\bar{c}_{(1^6)}, \bar{c}_{(1^6)}^1, e_{(1^6)}, e_{(1^6)}^1, \tilde{e}_{(1^6)}, \tilde{e}_{(1^6)}^1, f_{(1^6)}, f_{(1^6)}^1, \tilde{f}_{(1^6)}, \tilde{f}_{(1^6)}^1, \\
h_{(1^6)}, h_{(1^6)}^1, \tilde{h}_{(1^6)}, \tilde{h}_{(1^6)}^1, k_{(1^6)}, k_{(1^6)}^1, \bar{k}_{(1^6)}, \bar{k}_{(1^6)}^1, \ell_{(1^6)}, \ell_{(1^6)}^1, \\
\bar{\ell}_{(1^6)}, \bar{\ell}_{(1^6)}^1, m_{(1^6)}, m_{(1^6)}^1, \bar{m}_{(1^6)}, \bar{m}_{(1^6)}^1, p_{(1^6)}, p_{(1^6)}^1, \bar{p}_{(1^6)}, \bar{p}_{(1^6)}^1\},
\]

where:

\( a_{(1^6)} \) is the \( G \)-orbit

\[
\{(1, 2, 3, 4, 5, 6), (2, 3, 1, 4, 5, 6), (3, 1, 2, 4, 5, 6), (1, 2, 3, 5, 6, 4), (1, 2, 3, 6, 4, 5), \\
(2, 3, 1, 5, 6, 4), (2, 3, 1, 6, 4, 5), (3, 1, 2, 5, 6, 4), (3, 1, 2, 6, 4, 5), (4, 5, 6, 1, 2, 3), \\
(5, 6, 4, 1, 2, 3), (6, 4, 5, 1, 2, 3), (4, 5, 6, 2, 3, 1), (4, 5, 6, 3, 1, 2), (5, 6, 4, 2, 3, 1), \\
(5, 6, 4, 3, 1, 2), (6, 4, 5, 2, 3, 1), (6, 4, 5, 3, 1, 2)\}
\]

of the tabloid \( A_{(1^6)} = (1, 2, 3, 4, 5, 6) \);

\( a_{(1^6)}^1 \) is the \( G \)-orbit

\[
\{(2, 1, 3, 4, 5, 6), (3, 2, 1, 4, 5, 6), (1, 3, 2, 4, 5, 6), (2, 1, 3, 5, 6, 4), (2, 1, 3, 6, 4, 5), \\
(3, 2, 1, 5, 6, 4), (3, 2, 1, 6, 4, 5), (1, 3, 2, 5, 6, 4), (1, 3, 2, 6, 4, 5), (5, 4, 6, 1, 2, 3), \\
(6, 5, 4, 1, 2, 3), (4, 6, 5, 1, 2, 3), (5, 4, 6, 2, 3, 1), (5, 4, 6, 3, 1, 2), (6, 5, 4, 2, 3, 1), \\
(6, 5, 4, 3, 1, 2), (4, 6, 5, 2, 3, 1), (4, 6, 5, 3, 1, 2)\}
\]

of the tabloid \( A_{(1^6)}^1 = (2, 1, 3, 4, 5, 6) \);

\( \bar{a}_{(1^6)} \) is \( G \)-orbit

\[
\{(1, 2, 4, 3, 5, 6), (2, 3, 4, 1, 5, 6), (3, 1, 4, 2, 5, 6), (1, 2, 5, 3, 6, 4), (1, 2, 6, 3, 4, 5), \\
(2, 3, 5, 1, 6, 4), (2, 3, 6, 1, 4, 5), (3, 1, 5, 2, 6, 4), (3, 1, 6, 2, 4, 5), (4, 5, 1, 6, 2, 3), \\
(5, 6, 1, 4, 2, 3), (6, 4, 1, 5, 2, 3), (4, 5, 2, 6, 3, 1), (4, 5, 3, 6, 1, 2), (5, 6, 2, 4, 3, 1), \\
(5, 6, 3, 4, 1, 2), (6, 4, 2, 5, 3, 1), (6, 4, 3, 5, 1, 2)\}
\]
of the tabloid $\bar{A}^{(1^6)} = (1, 2, 4, 3, 5, 6)$; 
\(\bar{a}_{(1^6)}\) is the $G$-orbit 
\[
\{(2, 1, 4, 3, 5, 6), (3, 2, 4, 1, 5, 6), (1, 3, 4, 2, 5, 6), (2, 1, 5, 3, 6, 4), (2, 1, 6, 3, 4, 5), \\
(3, 2, 5, 1, 6, 4), (3, 2, 6, 1, 4, 5), (1, 3, 5, 2, 6, 4), (1, 3, 6, 2, 4, 5), (5, 4, 1, 6, 2, 3), \\
(6, 5, 1, 4, 2, 3), (4, 6, 1, 5, 2, 3), (5, 4, 2, 6, 3, 1), (5, 4, 3, 6, 1, 2), (6, 5, 2, 4, 3, 1), \\
(6, 5, 3, 4, 1, 2), (4, 6, 2, 5, 3, 1), (4, 6, 3, 5, 1, 2)\}\] 
of the tabloid $\bar{A}^{(1^6):1} = (2, 1, 4, 3, 5, 6)$; 
\(b_{(1^6)}\) is the $G$-orbit 
\[
\{(1, 2, 3, 4, 6, 5), (2, 3, 1, 4, 6, 5), (3, 1, 2, 4, 6, 5), (1, 2, 3, 5, 4, 6), (1, 2, 3, 6, 5, 4), \\
(2, 3, 1, 5, 4, 6), (2, 3, 1, 6, 5, 4), (3, 1, 2, 5, 4, 6), (3, 1, 2, 6, 5, 4), (4, 5, 6, 1, 3, 2), \\
(5, 6, 4, 1, 3, 2), (6, 4, 5, 1, 3, 2), (4, 5, 6, 2, 1, 3), (4, 5, 6, 3, 2, 1), (5, 6, 4, 2, 1, 3), \\
(5, 6, 4, 3, 2, 1), (6, 4, 5, 2, 1, 3), (6, 4, 5, 3, 2, 1)\}\] 
of the tabloid $\bar{B}^{(1^6)} = (1, 2, 3, 4, 6, 5)$; 
\(b_{(1^6)}^{1}\) is the $G$-orbit 
\[
\{(2, 1, 3, 4, 6, 5), (3, 2, 1, 4, 6, 5), (1, 3, 2, 4, 6, 5), (2, 1, 3, 5, 4, 6), (2, 1, 3, 6, 5, 4), \\
(3, 2, 1, 5, 4, 6), (3, 2, 1, 6, 5, 4), (1, 3, 2, 5, 4, 6), (1, 3, 2, 6, 5, 4), (5, 4, 6, 1, 3, 2), \\
(6, 5, 4, 1, 3, 2), (4, 6, 5, 1, 3, 2), (5, 4, 6, 2, 1, 3), (5, 4, 6, 3, 2, 1), (6, 5, 4, 2, 1, 3), \\
(6, 5, 4, 3, 2, 1), (4, 6, 5, 2, 1, 3), (4, 6, 5, 3, 2, 1)\}\] 
of the tabloid $\bar{B}^{(1^6):1} = (2, 1, 3, 4, 6, 5)$; 
\(\bar{b}_{(1^6)}\) is the $G$-orbit 
\[
\{(1, 2, 3, 4, 6, 5), (2, 3, 4, 1, 6, 5), (3, 1, 4, 2, 6, 5), (1, 2, 5, 3, 4, 6), (1, 2, 6, 3, 5, 4), \\
(2, 3, 5, 1, 4, 6), (2, 3, 6, 1, 5, 4), (3, 1, 5, 2, 4, 6), (3, 1, 6, 2, 5, 4), (4, 5, 1, 6, 3, 2), \\
(5, 6, 1, 4, 3, 2), (6, 4, 1, 5, 3, 2), (4, 5, 2, 6, 1, 3), (4, 5, 3, 6, 2, 1), (5, 6, 2, 4, 1, 3), \\
(5, 6, 3, 4, 2, 1), (6, 4, 2, 5, 1, 3), (6, 4, 3, 5, 2, 1)\}\] 
of the tabloid $\bar{B}^{(1^6)} = (1, 2, 4, 3, 6, 5)$; 
\(\bar{b}_{(1^6)}^{1}\) is the $G$-orbit 
\[
\{(2, 1, 4, 3, 6, 5), (3, 2, 4, 1, 6, 5), (1, 3, 4, 2, 6, 5), (2, 1, 5, 3, 4, 6), (2, 1, 6, 3, 5, 4), \\
(3, 2, 5, 1, 4, 6), (3, 2, 6, 1, 5, 4), (1, 3, 5, 2, 4, 6), (1, 3, 6, 2, 5, 4), (5, 4, 1, 6, 3, 2), \\
(6, 5, 1, 4, 3, 2), (4, 6, 1, 5, 3, 2), (5, 4, 2, 6, 1, 3), (5, 4, 3, 6, 2, 1), (6, 5, 2, 4, 1, 3), \\
(6, 5, 3, 4, 2, 1), (4, 6, 2, 5, 1, 3), (4, 6, 3, 5, 2, 1)\}\]
of the tabloid $\bar{B}^{(16):1} = (2, 1, 4, 3, 6, 5)$;
c\(_{(16)}\) is the $G$-orbit

\[
\{(1, 2, 4, 5, 3, 6), (2, 3, 4, 5, 1, 6), (3, 1, 4, 5, 2, 6), (1, 2, 5, 6, 3, 4), (1, 2, 6, 4, 3, 5), \\
(2, 3, 5, 6, 1, 4), (2, 3, 6, 4, 1, 5), (3, 1, 5, 6, 2, 4), (3, 1, 6, 4, 2, 5), (4, 5, 1, 2, 6, 3), \\
(5, 6, 1, 2, 4, 3), (6, 4, 1, 2, 5, 3), (4, 5, 2, 3, 6, 1), (4, 5, 3, 1, 6, 2), (5, 6, 2, 3, 4, 1), \\
(5, 6, 3, 1, 4, 2), (6, 4, 2, 3, 5, 1), (6, 4, 3, 1, 5, 2)\},
\]

of the tabloid $C^{(16)} = (1, 2, 4, 5, 3, 6)$;
c\(_{1(16)}\) is the $G$-orbit

\[
\{(2, 1, 4, 5, 3, 6), (3, 2, 4, 5, 1, 6), (1, 3, 4, 5, 2, 6), (2, 1, 5, 6, 3, 4), (2, 1, 6, 4, 3, 5), \\
(3, 2, 5, 6, 1, 4), (3, 2, 6, 4, 1, 5), (1, 3, 5, 6, 2, 4), (1, 3, 6, 4, 2, 5), (5, 4, 1, 2, 6, 3), \\
(6, 5, 1, 2, 4, 3), (4, 6, 1, 2, 5, 3), (5, 4, 2, 3, 6, 1), (5, 4, 3, 1, 6, 2), (6, 5, 2, 3, 4, 1), \\
(6, 5, 3, 1, 4, 2), (4, 6, 2, 3, 5, 1), (4, 6, 3, 1, 5, 2)\},
\]

of the tabloid $\bar{C}^{(16):1} = (2, 1, 4, 5, 3, 6)$;
$\bar{c}_{(16)}$ is the $G$-orbit

\[
\{(1, 2, 5, 4, 3, 6), (2, 3, 5, 4, 1, 6), (3, 1, 5, 4, 2, 6), (1, 2, 6, 5, 3, 4), (1, 2, 4, 6, 3, 5), \\
(2, 3, 6, 5, 1, 4), (2, 3, 4, 6, 1, 5), (3, 1, 6, 5, 2, 4), (3, 1, 4, 6, 2, 5), (4, 5, 2, 1, 6, 3), \\
(5, 6, 2, 1, 4, 3), (6, 4, 2, 1, 5, 3), (4, 5, 3, 2, 6, 1), (4, 5, 1, 3, 6, 2), (5, 6, 3, 2, 4, 1), \\
(5, 6, 1, 3, 4, 2), (6, 4, 3, 2, 5, 1), (6, 4, 1, 3, 5, 2)\},
\]

of the tabloid $\bar{C}^{(16)} = (1, 2, 5, 4, 3, 6)$;
$\bar{c}_{1(16)}$ is the $G$-orbit

\[
\{(2, 1, 5, 4, 3, 6), (3, 2, 5, 4, 1, 6), (1, 3, 5, 4, 2, 6), (2, 1, 6, 5, 3, 4), (2, 1, 4, 6, 3, 5), \\
(3, 2, 6, 5, 1, 4), (3, 2, 4, 6, 1, 5), (1, 3, 6, 5, 2, 4), (1, 3, 4, 6, 2, 5), (5, 4, 2, 1, 6, 3), \\
(6, 5, 2, 1, 4, 3), (4, 6, 2, 1, 5, 3), (5, 4, 3, 2, 6, 1), (5, 4, 1, 3, 6, 2), (6, 5, 3, 2, 4, 1), \\
(6, 5, 1, 3, 4, 2), (4, 6, 3, 2, 5, 1), (4, 6, 1, 3, 5, 2)\},
\]

of the tabloid $\bar{C}^{(16):1} = (2, 1, 5, 4, 3, 6)$;
e\(_{(16)}\) is the $G$-orbit

\[
\{(1, 2, 4, 5, 6, 3), (2, 3, 4, 5, 6, 1), (3, 1, 4, 5, 6, 2), (1, 2, 5, 6, 4, 3), (1, 2, 6, 4, 5, 3), \\
(2, 3, 5, 6, 4, 1), (2, 3, 6, 4, 5, 1), (3, 1, 5, 6, 4, 2), (3, 1, 6, 4, 5, 2), (4, 5, 1, 2, 3, 6), \\
(5, 6, 1, 2, 3, 4), (6, 4, 1, 2, 3, 5), (4, 5, 2, 3, 1, 6), (4, 5, 3, 1, 2, 6), (5, 6, 2, 3, 1, 4), \\
(5, 6, 3, 1, 2, 4), (6, 4, 2, 3, 1, 5), (6, 4, 3, 1, 2, 5)\},
\]
of the tabloid $E^{(1^6)} = (1, 2, 4, 5, 6, 3)$;
$e_{(1^6)}$ is the $G$-orbit
\[
\{(2, 1, 4, 5, 6, 3), (3, 2, 4, 5, 6, 1), (1, 3, 4, 5, 6, 2), (2, 1, 5, 6, 4, 3), (2, 1, 6, 4, 5, 3),
(3, 2, 5, 6, 4, 1), (3, 2, 6, 4, 5, 1), (1, 3, 5, 6, 4, 2), (1, 3, 6, 4, 5, 2), (5, 4, 1, 2, 3, 6),
(6, 5, 1, 2, 3, 4), (4, 6, 1, 2, 3, 5), (5, 4, 2, 3, 1, 6), (5, 4, 3, 1, 2, 6), (6, 5, 2, 3, 1, 4),
(6, 5, 3, 1, 2, 4), (4, 6, 2, 3, 1, 5), (4, 6, 3, 1, 2, 5)\},
\]
of the tabloid $\tilde{E}^{(1^6):1} = (2, 1, 4, 5, 6, 3)$;
$\tilde{e}_{(1^6)}$ is the $G$-orbit
\[
\{(1, 2, 5, 4, 6, 3), (2, 3, 5, 4, 6, 1), (3, 1, 5, 4, 6, 2), (1, 2, 6, 5, 4, 3), (1, 2, 4, 6, 5, 3),
(2, 3, 6, 5, 4, 1), (2, 3, 4, 6, 5, 1), (3, 1, 6, 5, 4, 2), (3, 1, 4, 6, 5, 2), (4, 5, 2, 1, 3, 6),
(5, 6, 2, 1, 3, 4), (6, 4, 2, 1, 3, 5), (4, 5, 3, 2, 1, 6), (4, 5, 1, 3, 2, 6), (5, 6, 3, 2, 1, 4),
(5, 6, 1, 3, 2, 4), (6, 4, 3, 2, 1, 5), (6, 4, 1, 3, 2, 5)\},
\]
of the tabloid $\bar{E}^{(1^6)} = (1, 2, 5, 4, 6, 3)$;
$\bar{e}_{(1^6)}$ is the $G$-orbit
\[
\{(2, 1, 5, 4, 6, 3), (3, 2, 5, 4, 6, 1), (1, 3, 5, 4, 6, 2), (2, 1, 6, 5, 4, 3), (2, 1, 4, 6, 5, 3),
(3, 2, 6, 5, 4, 1), (3, 2, 4, 6, 5, 1), (1, 3, 6, 5, 4, 2), (1, 3, 4, 6, 5, 2), (5, 4, 2, 1, 3, 6),
(6, 5, 2, 1, 3, 4), (4, 6, 2, 1, 3, 5), (5, 4, 3, 2, 1, 6), (5, 4, 1, 3, 2, 6), (6, 5, 3, 2, 1, 4),
(6, 5, 1, 3, 2, 4), (4, 6, 3, 2, 1, 5), (4, 6, 1, 3, 2, 5)\},
\]
of the tabloid $\bar{E}^{(1^6):1} = (2, 1, 5, 4, 6, 3)$;
$f_{(1^6)}$ is the $G$-orbit
\[
\{(1, 4, 2, 3, 5, 6), (2, 4, 3, 1, 5, 6), (3, 4, 1, 2, 5, 6), (1, 5, 2, 3, 6, 4), (1, 6, 2, 3, 4, 5),
(2, 5, 3, 1, 6, 4), (2, 6, 3, 1, 4, 5), (3, 5, 1, 2, 6, 4), (3, 6, 1, 2, 4, 5), (4, 1, 5, 6, 2, 3),
(5, 1, 6, 4, 2, 3), (6, 1, 4, 5, 2, 3), (4, 2, 5, 6, 3, 1), (4, 3, 5, 6, 1, 2), (5, 2, 6, 4, 3, 1),
(5, 3, 6, 4, 1, 2), (6, 2, 4, 5, 3, 1), (6, 3, 4, 5, 1, 2)\},
\]
of the tabloid $\bar{E}^{(1^6)} = (1, 4, 2, 3, 5, 6)$;
$f_{(1^6)}$ is the $G$-orbit
\[
\{(4, 1, 2, 3, 5, 6), (4, 2, 3, 1, 5, 6), (4, 3, 1, 2, 5, 6), (5, 1, 2, 3, 6, 4), (6, 1, 2, 3, 4, 5),
(5, 2, 3, 1, 6, 4), (6, 2, 3, 1, 4, 5), (5, 3, 1, 2, 6, 4), (6, 3, 1, 2, 4, 5), (1, 4, 5, 6, 2, 3),
(1, 5, 6, 4, 2, 3), (1, 6, 4, 5, 2, 3), (2, 4, 5, 6, 3, 1), (3, 4, 5, 6, 1, 2), (2, 5, 6, 4, 3, 1),
(3, 5, 6, 4, 1, 2), (2, 6, 4, 5, 3, 1), (3, 6, 4, 5, 1, 2)\},
\]
of the tabloid $F^{(1^6):1} = (4, 1, 2, 3, 5, 6)$; 
$\tilde{f}_{(1^6)}$ is the $G$-orbit 
\[
\{(1, 4, 3, 2, 5, 6), (2, 4, 1, 3, 5, 6), (3, 4, 2, 1, 5, 6), (1, 5, 3, 2, 6, 4), (1, 6, 3, 2, 4, 5), \\
(2, 5, 1, 3, 6, 4), (2, 6, 1, 3, 4, 5), (3, 5, 2, 1, 6, 4), (3, 6, 2, 1, 4, 5), (4, 1, 6, 5, 2, 3), \\
(5, 1, 4, 6, 2, 3), (6, 1, 5, 4, 2, 3), (4, 2, 6, 5, 3, 1), (4, 3, 6, 5, 1, 2), (5, 2, 4, 6, 3, 1), \\
(5, 3, 4, 6, 1, 2), (6, 2, 5, 4, 3, 1), (6, 3, 5, 4, 1, 2)\},
\]

of the tabloid $\tilde{F}^{(1^6)} = (1, 4, 3, 2, 5, 6)$; 
$\tilde{f}^{1}_{(1^6)}$ is the $G$-orbit 
\[
\{(4, 1, 3, 2, 5, 6), (4, 2, 1, 3, 5, 6), (4, 3, 2, 1, 5, 6), (5, 1, 3, 2, 6, 4), (6, 1, 3, 2, 4, 5), \\
(5, 2, 1, 3, 6, 4), (6, 2, 1, 3, 4, 5), (5, 3, 2, 1, 6, 4), (6, 3, 2, 1, 4, 5), (1, 4, 6, 5, 2, 3), \\
(1, 5, 4, 6, 2, 3), (1, 6, 5, 4, 2, 3), (2, 4, 6, 5, 3, 1), (3, 4, 6, 5, 1, 2), (2, 5, 4, 6, 3, 1), \\
(3, 5, 4, 6, 1, 2), (2, 6, 5, 4, 3, 1), (3, 6, 5, 4, 1, 2)\},
\]

of the tabloid $\bar{F}^{(1^6):1} = (4, 1, 3, 2, 5, 6)$; 
$h_{(1^6)}$ is the $G$-orbit 
\[
\{(1, 4, 2, 3, 6, 5), (2, 4, 3, 1, 6, 5), (3, 4, 1, 2, 6, 5), (1, 5, 2, 3, 4, 6), (1, 6, 2, 3, 5, 4), \\
(2, 5, 3, 1, 4, 6), (2, 6, 3, 1, 5, 4), (3, 5, 1, 2, 4, 6), (3, 6, 1, 2, 5, 4), (4, 1, 5, 6, 3, 2), \\
(5, 1, 6, 4, 3, 2), (6, 1, 4, 5, 3, 2), (4, 2, 5, 6, 1, 3), (4, 3, 5, 6, 2, 1), (5, 2, 6, 4, 1, 3), \\
(5, 3, 6, 4, 2, 1), (6, 2, 4, 5, 1, 3), (6, 3, 4, 5, 2, 1)\},
\]

of the tabloid $H^{(1^6):1} = (4, 1, 2, 3, 6, 5)$; 
$h^{1}_{(1^6)}$ is the $G$-orbit 
\[
\{(4, 1, 2, 3, 6, 5), (4, 2, 3, 1, 6, 5), (4, 3, 1, 2, 6, 5), (5, 1, 2, 3, 4, 6), (6, 1, 2, 3, 5, 4), \\
(5, 2, 3, 1, 4, 6), (6, 2, 3, 1, 5, 4), (5, 3, 1, 2, 4, 6), (6, 3, 1, 2, 5, 4), (1, 4, 5, 6, 3, 2), \\
(1, 5, 6, 4, 3, 2), (1, 6, 4, 5, 3, 2), (2, 4, 5, 6, 1, 3), (3, 4, 5, 6, 2, 1), (2, 5, 6, 4, 1, 3), \\
(3, 5, 6, 4, 2, 1), (2, 6, 4, 5, 1, 3), (3, 6, 4, 5, 2, 1)\},
\]

of the tabloid $H^{(1^6):1} = (4, 1, 2, 3, 6, 5)$; 
$\bar{h}_{(1^6)}$ is the $G$-orbit 
\[
\{(1, 4, 3, 2, 6, 5), (2, 4, 1, 3, 6, 5), (3, 4, 2, 1, 6, 5), (1, 5, 3, 2, 4, 6), (1, 6, 3, 2, 5, 4), \\
(2, 5, 1, 3, 4, 6), (2, 6, 1, 3, 5, 4), (3, 5, 2, 1, 4, 6), (3, 6, 2, 1, 5, 4), (4, 1, 6, 5, 3, 2), \\
(5, 1, 4, 6, 3, 2), (6, 1, 5, 4, 3, 2), (4, 2, 6, 5, 1, 3), (4, 3, 6, 5, 2, 1), (5, 2, 4, 6, 1, 3), \\
(5, 3, 4, 6, 2, 1), (6, 2, 5, 4, 1, 3), (6, 3, 5, 4, 2, 1)\},
\]
of the tabloid $\tilde{H}^{(16)} = (1, 4, 3, 2, 6, 5)$;
\(\tilde{h}_d^{(16)}\) is the \(G\)-orbit
\[
\{(4, 1, 3, 2, 6, 5), (4, 2, 1, 3, 6, 5), (4, 3, 2, 1, 6, 5), (5, 1, 3, 2, 4, 6), (6, 1, 3, 2, 5, 4),
(5, 2, 1, 3, 4, 6), (6, 2, 1, 3, 5, 4), (5, 3, 2, 1, 4, 6), (6, 3, 2, 1, 5, 4), (1, 4, 6, 5, 3, 2),
(1, 5, 4, 6, 3, 2), (1, 6, 5, 4, 3, 2), (2, 4, 6, 5, 1, 3), (3, 4, 6, 5, 2, 1), (2, 5, 4, 6, 1, 3),
(3, 5, 4, 6, 2, 1), (2, 6, 5, 4, 1, 3), (3, 6, 5, 4, 2, 1)\},
\]
of the tabloid $\tilde{H}^{(16)}:1 = (4, 1, 3, 2, 6, 5)$;
\(k_{(16)}\) is the \(G\)-orbit
\[
\{(1, 4, 2, 5, 3, 6), (2, 4, 3, 5, 1, 6), (3, 4, 1, 5, 2, 6), (1, 5, 2, 6, 3, 4), (1, 6, 2, 4, 3, 5),
(2, 5, 3, 6, 1, 4), (2, 6, 3, 4, 1, 5), (3, 5, 1, 6, 2, 4), (3, 6, 1, 4, 2, 5), (4, 1, 5, 2, 6, 3),
(5, 1, 6, 2, 4, 3), (6, 1, 4, 2, 5, 3), (4, 2, 5, 3, 6, 1), (4, 3, 5, 1, 6, 2), (5, 2, 6, 3, 4, 1),
(5, 3, 6, 1, 4, 2), (6, 2, 4, 3, 5, 1), (6, 3, 4, 1, 5, 2)\}
\]
of the tabloid $K^{(16)} = (1, 4, 2, 5, 3, 6)$;
\(k^{1}_{(16)}\) is the \(G\)-orbit
\[
\{(4, 1, 2, 5, 3, 6), (4, 2, 3, 5, 1, 6), (4, 3, 1, 5, 2, 6), (5, 1, 2, 6, 3, 4), (6, 1, 2, 4, 3, 5),
(5, 2, 3, 6, 1, 4), (6, 2, 3, 4, 1, 5), (5, 3, 1, 6, 2, 4), (6, 3, 1, 4, 2, 5), (1, 4, 5, 2, 6, 3),
(1, 5, 6, 2, 4, 3), (1, 6, 4, 2, 5, 3), (2, 4, 5, 3, 6, 1), (3, 4, 5, 1, 6, 2), (2, 5, 6, 3, 4, 1),
(3, 5, 6, 1, 4, 2), (2, 6, 4, 3, 5, 1), (3, 6, 4, 1, 5, 2)\}
\]
of the tabloid $K^{(16)}:1 = (4, 1, 2, 5, 3, 6)$;
\(\tilde{k}_{(16)}\) is the \(G\)-orbit
\[
\{(1, 4, 5, 2, 3, 6), (2, 4, 5, 3, 1, 6), (3, 4, 5, 1, 2, 6), (1, 5, 6, 2, 3, 4), (1, 6, 4, 2, 3, 5),
(2, 5, 6, 3, 1, 4), (2, 6, 4, 3, 1, 5), (3, 5, 6, 1, 2, 4), (3, 6, 4, 1, 2, 5), (4, 1, 2, 5, 6, 3),
(5, 1, 2, 6, 4, 3), (6, 1, 2, 4, 5, 3), (4, 2, 3, 5, 6, 1), (4, 3, 1, 5, 6, 2), (5, 2, 3, 6, 4, 1),
(5, 3, 1, 6, 4, 2), (6, 2, 3, 4, 5, 1), (6, 3, 1, 4, 5, 2)\}
\]
of the tabloid $\tilde{K}^{(16)} = (1, 4, 5, 2, 3, 6)$;
\(\tilde{k}^{1}_{(16)}\) is the \(G\)-orbit
\[
\{(4, 1, 5, 2, 3, 6), (4, 2, 5, 3, 1, 6), (4, 3, 5, 1, 2, 6), (5, 1, 6, 2, 3, 4), (6, 1, 4, 2, 3, 5),
(5, 2, 6, 3, 1, 4), (6, 2, 4, 3, 1, 5), (5, 3, 6, 1, 2, 4), (6, 3, 4, 1, 2, 5), (1, 4, 2, 5, 6, 3),
(1, 5, 2, 6, 4, 3), (1, 6, 2, 4, 5, 3), (2, 4, 3, 5, 6, 1), (3, 4, 1, 5, 6, 2), (2, 5, 3, 6, 4, 1),
(3, 5, 1, 6, 4, 2), (2, 6, 3, 4, 5, 1), (3, 6, 1, 4, 5, 2)\}
\]
of the tabloid $\bar{K}^{(16):1} = (4, 1, 5, 2, 3, 6)$;
$\ell_{(16)}$ is the $G$-orbit

$\{(1, 4, 2, 6, 3, 5), (2, 4, 3, 6, 1, 5), (3, 4, 1, 6, 2, 5), (1, 5, 2, 4, 3, 6), (1, 6, 2, 5, 3, 4),$
$(2, 5, 3, 4, 1, 6), (2, 6, 3, 5, 1, 4), (3, 5, 1, 4, 2, 6), (3, 6, 1, 5, 2, 4), (4, 1, 5, 3, 6, 2),$
$(5, 1, 6, 3, 4, 2), (6, 1, 4, 3, 5, 2), (4, 2, 5, 1, 6, 3), (4, 3, 5, 2, 6, 1), (5, 2, 6, 1, 4, 3),$
(5, 3, 6, 2, 4, 1), (6, 2, 4, 1, 5, 3), (6, 3, 4, 2, 5, 1)\}$

of the tabloid $L^{(16)} = (1, 4, 2, 6, 3, 5)$;
$\ell_{(16)}^1$ is the $G$-orbit

$\{(4, 1, 2, 6, 3, 5), (4, 2, 3, 6, 1, 5), (4, 3, 1, 6, 2, 5), (5, 1, 2, 4, 3, 6), (6, 1, 2, 5, 3, 4),$
$(5, 2, 3, 4, 1, 6), (6, 2, 3, 5, 1, 4), (5, 3, 1, 4, 2, 6), (6, 3, 1, 5, 2, 4), (1, 4, 5, 3, 6, 2),$
(1, 5, 6, 3, 4, 2), (1, 6, 4, 3, 5, 2), (2, 4, 5, 1, 6, 3), (3, 4, 5, 2, 6, 1), (2, 5, 6, 1, 4, 3),$
(3, 5, 6, 2, 4, 1), (2, 6, 4, 1, 5, 3), (3, 6, 4, 2, 5, 1)\}$

of the tabloid $\bar{L}^{(16):1} = (4, 1, 2, 6, 3, 5)$;
$\ell_{(16)}^1$ is the $G$-orbit

$\{(1, 4, 6, 2, 3, 5), (2, 4, 6, 3, 1, 5), (3, 4, 6, 1, 2, 5), (1, 5, 4, 2, 3, 6), (1, 6, 5, 2, 3, 4),$
$(2, 5, 4, 3, 1, 6), (2, 6, 5, 3, 1, 4), (3, 5, 4, 1, 2, 6), (3, 6, 5, 1, 2, 4), (4, 1, 3, 5, 6, 2),$
(5, 1, 3, 6, 4, 2), (6, 1, 3, 4, 5, 2), (4, 2, 1, 5, 6, 3), (4, 3, 2, 5, 6, 1), (5, 2, 1, 6, 4, 3),$
(5, 3, 2, 6, 4, 1), (6, 2, 1, 4, 5, 3), (6, 3, 2, 4, 5, 1)\}$

of the tabloid $L^{(16)} = (1, 4, 6, 2, 3, 5)$;
$\ell_{(16)}^1$ is the $G$-orbit

$\{(4, 1, 6, 2, 3, 5), (4, 2, 6, 3, 1, 5), (4, 3, 6, 1, 2, 5), (5, 1, 4, 2, 3, 6), (6, 1, 5, 2, 3, 4),$
$(5, 2, 4, 3, 1, 6), (6, 2, 5, 3, 1, 4), (5, 3, 4, 1, 2, 6), (6, 3, 5, 1, 2, 4), (1, 4, 3, 5, 6, 2),$
(1, 5, 3, 6, 4, 2), (1, 6, 3, 4, 5, 2), (2, 4, 1, 5, 6, 3), (3, 4, 2, 5, 6, 1), (2, 5, 1, 6, 4, 3),$
(3, 5, 2, 6, 4, 1), (2, 6, 1, 4, 5, 3), (3, 6, 2, 4, 5, 1)\}$

of the tabloid $\bar{L}^{(16):1} = (4, 1, 6, 2, 3, 5)$;
$m_{(16)}$ is the $G$-orbit

$\{(1, 4, 2, 6, 5, 3), (2, 4, 3, 6, 5, 1), (3, 4, 1, 6, 5, 2), (1, 5, 2, 4, 6, 3), (1, 6, 2, 5, 4, 3),$
$(2, 5, 3, 4, 6, 1), (2, 6, 3, 5, 4, 1), (3, 5, 1, 4, 6, 2), (3, 6, 1, 5, 4, 2), (4, 1, 5, 3, 2, 6),$
(5, 1, 6, 3, 2, 4), (6, 1, 4, 3, 2, 5), (4, 2, 5, 1, 3, 6), (4, 3, 5, 2, 1, 6), (5, 2, 6, 1, 3, 4),$
(5, 3, 6, 2, 1, 4), (6, 2, 4, 1, 3, 5), (6, 3, 4, 2, 1, 5)\}$
of the tabloid $M^{(16)} = (1, 4, 2, 6, 5, 3)$;
$m_{(16)}^1$ is the $G$-orbit
\[
\{(4, 1, 2, 6, 5, 3), (4, 2, 3, 6, 5, 1), (4, 3, 1, 6, 5, 2), (5, 1, 2, 4, 6, 3), (6, 1, 2, 5, 4, 3), \\
(5, 2, 3, 4, 6, 1), (6, 2, 3, 5, 4, 1), (5, 3, 1, 4, 6, 2), (6, 3, 1, 5, 4, 2), (1, 4, 5, 3, 2, 6), \\
(1, 5, 6, 3, 2, 4), (1, 6, 4, 3, 2, 5), (2, 4, 5, 1, 3, 6), (3, 4, 5, 2, 1, 6), (2, 5, 6, 1, 3, 4), \\
(3, 5, 6, 2, 1, 4), (2, 6, 4, 1, 3, 5), (3, 6, 4, 2, 1, 5)\}
\]
of the tabloid $\tilde{M}^{(16)} = (1, 4, 2, 6, 5, 3)$;
$\tilde{m}_{(16)}^1$ is the $G$-orbit
\[
\{(1, 4, 6, 2, 5, 3), (2, 4, 6, 3, 5, 1), (3, 4, 6, 1, 5, 2), (1, 5, 4, 2, 6, 3), (1, 6, 5, 2, 4, 3), \\
(2, 5, 4, 3, 6, 1), (2, 6, 5, 3, 4, 1), (3, 5, 4, 1, 6, 2), (3, 6, 5, 1, 4, 2), (4, 1, 3, 5, 2, 6), \\
(5, 1, 3, 6, 2, 4), (6, 1, 3, 4, 2, 5), (4, 2, 1, 5, 3, 6), (4, 3, 2, 5, 1, 6), (5, 2, 1, 6, 3, 4), \\
(5, 3, 2, 6, 1, 4), (6, 2, 1, 4, 3, 5), (6, 3, 2, 4, 1, 5)\}
\]
of the tabloid $M^{(16)^{-1}} = (1, 4, 6, 2, 5, 3)$;
P_{(16)} is the $G$-orbit
\[
\{(4, 1, 6, 2, 5, 3), (4, 2, 6, 3, 5, 1), (4, 3, 6, 1, 5, 2), (5, 1, 4, 2, 6, 3), (6, 1, 5, 2, 4, 3), \\
(5, 2, 4, 3, 6, 1), (6, 2, 5, 3, 4, 1), (5, 3, 4, 1, 6, 2), (6, 3, 5, 1, 4, 2), (1, 4, 3, 5, 2, 6), \\
(1, 5, 3, 6, 2, 4), (1, 6, 3, 4, 2, 5), (2, 4, 1, 5, 3, 6), (3, 4, 2, 5, 1, 6), (2, 5, 1, 6, 3, 4), \\
(3, 5, 2, 6, 1, 4), (2, 6, 1, 4, 3, 5), (3, 6, 2, 4, 1, 5)\}
\]
of the tabloid $\tilde{M}^{(16)^{-1}} = (1, 4, 6, 2, 5, 3)$;
P_{(16)}^1 is the $G$-orbit
\[
\{(1, 4, 3, 6, 2, 5), (2, 4, 1, 6, 3, 5), (3, 4, 2, 6, 1, 5), (1, 5, 3, 4, 2, 6), (1, 6, 3, 5, 2, 4), \\
(2, 5, 1, 4, 3, 6), (2, 6, 1, 5, 3, 4), (3, 5, 2, 4, 1, 6), (3, 6, 2, 5, 1, 4), (4, 1, 6, 3, 5, 2), \\
(5, 1, 4, 3, 6, 2), (6, 1, 5, 3, 4, 2), (4, 2, 6, 1, 5, 3), (4, 3, 6, 2, 5, 1), (5, 2, 4, 1, 6, 3), \\
(5, 3, 4, 2, 6, 1), (6, 2, 5, 1, 4, 3), (6, 3, 5, 2, 4, 1)\}
\]
of the tabloid $P^{(16)} = (1, 4, 3, 6, 2, 5)$;
P_{(16)}^1 is the $G$-orbit
\[
\{(4, 1, 3, 6, 2, 5), (4, 2, 1, 6, 3, 5), (4, 3, 2, 6, 1, 5), (5, 1, 3, 4, 2, 6), (6, 1, 3, 5, 2, 4), \\
(5, 2, 1, 4, 3, 6), (6, 2, 1, 5, 3, 4), (5, 3, 2, 4, 1, 6), (6, 3, 2, 5, 1, 4), (1, 4, 6, 3, 5, 2), \\
(1, 5, 4, 3, 6, 2), (1, 6, 5, 3, 4, 2), (2, 4, 6, 1, 5, 3), (3, 4, 6, 2, 5, 1), (2, 5, 4, 1, 6, 3), \\
(3, 5, 4, 2, 6, 1), (2, 6, 5, 1, 4, 3), (3, 6, 5, 2, 4, 1)\}
\]
of the tabloid $P^{(1^6):1} = (4, 1, 3, 6, 2, 5)$;
\(\bar{p}_{(1^6)}\) is the $G$-orbit
\[
\{(1, 4, 6, 3, 2, 5), (2, 4, 6, 1, 3, 5), (3, 4, 6, 2, 1, 5), (1, 5, 4, 3, 2, 6), (1, 6, 5, 3, 2, 4),
(2, 5, 4, 1, 3, 6), (2, 6, 5, 1, 3, 4), (3, 5, 4, 2, 1, 6), (3, 6, 5, 2, 1, 4), (4, 1, 3, 6, 5, 2),
(5, 1, 3, 4, 6, 2), (6, 1, 3, 5, 4, 2), (4, 2, 1, 6, 5, 3), (4, 3, 2, 6, 5, 1), (5, 2, 1, 4, 6, 3),
(5, 3, 2, 4, 6, 1), (6, 2, 1, 5, 4, 3), (6, 3, 2, 5, 4, 1)\},
\]
of the tabloid $\bar{P}^{(2,1^4)} = (1, 4, 6, 3, 2, 5)$;
\(\bar{p}_{(1^6)}\) is the $G$-orbit
\[
\{(4, 1, 6, 3, 2, 5), (4, 2, 6, 1, 3, 5), (4, 3, 6, 2, 1, 5), (5, 1, 4, 3, 2, 6), (6, 1, 5, 3, 2, 4),
(5, 2, 4, 1, 3, 6), (6, 2, 5, 1, 3, 4), (5, 3, 4, 2, 1, 6), (6, 3, 5, 2, 1, 4), (1, 4, 3, 6, 5, 2),
(1, 5, 3, 4, 6, 2), (1, 6, 3, 5, 4, 2), (2, 4, 1, 6, 5, 3), (3, 4, 2, 6, 5, 1), (2, 5, 1, 4, 6, 3),
(3, 5, 2, 4, 6, 1), (2, 6, 1, 5, 4, 3), (3, 6, 2, 5, 4, 1)\},
\]
of the tabloid $\bar{P}^{(2,1^4):1} = (4, 1, 6, 3, 2, 5)$.
Here are the inequalities between the tabloids of shape $(1^6)$ and the tabloids of shape $(2,1^4)$:

\[
\begin{align*}
A^{(1^6)} &< A^{(2,1^4)}, A^{(1^6):1} < A^{(2,1^4)}, \bar{A}^{(1^6)} < A^{(2,1^4)}, \bar{A}^{(1^6):1} < A^{(2,1^4)}, \\
B^{(1^6)} &< A^{(2,1^4)}, B^{(1^6):1} < A^{(2,1^4)}, \bar{B}^{(1^6)} < A^{(2,1^4)}, \bar{B}^{(1^6):1} < A^{(2,1^4)}, \\
C^{(1^6)} &< A^{(2,1^4)}, C^{(1^6):1} < A^{(2,1^4)}, E^{(1^6)} < A^{(2,1^4)}, E^{(1^6):1} < A^{(2,1^4)}, \\
A^{(1^6)} &< \bar{A}^{(2,1^4)}, A^{(1^6):1} < \bar{A}^{(2,1^4)}, \bar{A}^{(1^6)} < \bar{A}^{(2,1^4)}, \bar{A}^{(1^6):1} < \bar{A}^{(2,1^4)}, \\
B^{(1^6)} &< \bar{A}^{(2,1^4)}, B^{(1^6):1} < \bar{A}^{(2,1^4)}, \bar{B}^{(1^6)} < \bar{A}^{(2,1^4)}, \bar{B}^{(1^6):1} < \bar{A}^{(2,1^4)}, \\
C^{(1^6)} &< \bar{A}^{(2,1^4)}, C^{(1^6):1} < \bar{A}^{(2,1^4)}, E^{(1^6)} < \bar{A}^{(2,1^4)}, E^{(1^6):1} < \bar{A}^{(2,1^4)}, \\
F^{(1^6)} &< \bar{A}^{(2,1^4)}, \bar{F}^{(1^6)} < \bar{A}^{(2,1^4)}, H^{(1^6)} < \bar{A}^{(2,1^4)}, \bar{H}^{(1^6)} < \bar{A}^{(2,1^4)}, \\
K^{(1^6)} &< \bar{A}^{(2,1^4)}, (14)(25)(36)K^{(1^6):1} < \bar{A}^{(2,1^4)}, \\
(14)(25)(36)\bar{L}^{(1^6):1} &< \bar{A}^{(2,1^4)}, (14)(25)(36)\bar{M}^{(1^6):1} < \bar{A}^{(2,1^4)}, \\
A^{(1^6)} &< B^{(2,1^4)}, A^{(1^6):1} < B^{(2,1^4)}, \bar{A}^{(1^6)} < B^{(2,1^4)}, \bar{A}^{(1^6):1} < B^{(2,1^4)}, \\
B^{(1^6)} &< B^{(2,1^4)}, B^{(1^6):1} < B^{(2,1^4)}, \bar{B}^{(1^6)} < B^{(2,1^4)}, \bar{B}^{(1^6):1} < B^{(2,1^4)}, \\
(465)\bar{C}^{(1^6)} &< B^{(2,1^4)}, (465)\bar{C}^{(1^6):1} < B^{(2,1^4)}, \\
(465)\bar{E}^{(1^6)} &< B^{(2,1^4)}, (465)\bar{E}^{(1^6):1} < B^{(2,1^4)}, \\
A^{(1^6)} &< \bar{B}^{(2,1^4)}, A^{(1^6):1} < \bar{B}^{(2,1^4)}, \bar{A}^{(1^6)} < \bar{B}^{(2,1^4)}, \bar{A}^{(1^6):1} < \bar{B}^{(2,1^4)},
\end{align*}
\]
\[ B^{(16)} < \bar{B}^{(2,14)}, B^{(16):1} < \bar{B}^{(2,14)}, \bar{B}^{(16)} < \bar{B}^{(2,14)}, \bar{B}^{(16):1} < \bar{B}^{(2,14)}, \]
\[ (465) \bar{C}^{(16)} < \bar{B}^{(2,14)}, (465) \bar{C}^{(16):1} < \bar{B}^{(2,14)}, \]
\[ (465) \bar{E}^{(16)} < \bar{B}^{(2,14)}, (465) \bar{E}^{(16):1} < \bar{B}^{(2,14)}, \]
\[ F^{(16)} < \bar{B}^{(2,14)}, F^{(16)} < \bar{B}^{(2,14)}, H^{(16)} < \bar{B}^{(2,14)}, \bar{H}^{(16)} < \bar{B}^{(2,14)}, \]
\[ L^{(16)} < \bar{B}^{(2,14)}, M^{(16)} < \bar{B}^{(2,14)}, P^{(16)} < \bar{B}^{(2,14)}, (14)(25)(36) L^{(16):1} < \bar{B}^{(2,14)}, \]
\[ \bar{A}^{(16)} < C^{(2,14)}, \bar{A}^{(16):1} < C^{(2,14)}, \bar{B}^{(16)} < C^{(2,14)}, \bar{B}^{(16):1} < C^{(2,14)}, \]
\[ C^{(16)} < C^{(2,14)}, C^{(16):1} < C^{(2,14)}, \bar{C}^{(16)} < C^{(2,14)}, \bar{C}^{(16):1} < C^{(2,14)}, \]
\[ E^{(16)} < C^{(2,14)}, E^{(16):1} < C^{(2,14)}, \bar{E}^{(16)} < C^{(2,14)}, \bar{E}^{(16):1} < C^{(2,14)}, \]
\[ F^{(16)} < C^{(2,14)}, (123) \bar{F}^{(16)} < C^{(2,14)}, H^{(16)} < C^{(2,14)}, (123) \bar{H}^{(16)} < C^{(2,14)}, \]
\[ K^{(16)} < C^{(2,14)}, (14)(25)(36) K^{(16):1} < C^{(2,14)}, \]
\[ \bar{K}^{(16)} < C^{(2,14)}, (14)(25)(36) \bar{K}^{(16):1} < C^{(2,14)}, \]
\[ (14)(25)(36) L^{(16):1} < C^{(2,14)}, (142536) \bar{L}^{(16):1} < C^{(2,14)}, \]
\[ (14)(25)(36) M^{(16):1} < C^{(2,14)}, (142536) M^{(16):1} < C^{(2,14)}, \]
\[ \bar{A}^{(16)} < \bar{C}^{(2,14)}, \bar{A}^{(16):1} < \bar{C}^{(2,14)}, \bar{B}^{(16)} < \bar{C}^{(2,14)}, \bar{B}^{(16):1} < \bar{C}^{(2,14)}, \]
\[ \bar{C}^{(16)} < \bar{C}^{(2,14)}, \bar{C}^{(16):1} < \bar{C}^{(2,14)}, \bar{C}^{(16)} < \bar{C}^{(2,14)}, \bar{C}^{(16):1} < \bar{C}^{(2,14)}, \]
\[ \bar{E}^{(16)} < \bar{C}^{(2,14)}, \bar{E}^{(16):1} < \bar{C}^{(2,14)}, \bar{E}^{(16)} < \bar{C}^{(2,14)}, \bar{E}^{(16):1} < \bar{C}^{(2,14)}, \]
\[ (456) F^{(16)} < \bar{C}^{(2,14)}, (123)(456) F^{(16)} < C^{(2,14)}, \]
\[ (456) H^{(16)} < \bar{C}^{(2,14)}, (123)(456) \bar{H}^{(16)} < C^{(2,14)}, \]
\[ (456) L^{(16)} < \bar{C}^{(2,14)}, (456) \bar{L}^{(16)} < \bar{C}^{(2,14)}, \]
\[ (456) M^{(16)} < \bar{C}^{(2,14)}, (456) M^{(16)} < C^{(2,14)}, \]
\[ (123)(456) P^{(16)} < \bar{C}^{(2,14)}, (152634) P^{(16):1} < C^{(2,14)}, \]
\[ (456) \bar{B}^{(16)} < \bar{C}^{(2,14)}, (153426) \bar{P}^{(16):1} < C^{(2,14)}, \]
\[ C^{(16)} < E^{(2,14)}, C^{(16):1} < E^{(2,14)}, \bar{C}^{(16)} < E^{(2,14)}, \bar{C}^{(16):1} < E^{(2,14)}, \]
\[ E^{(16)} < E^{(2,14)}, E^{(16):1} < E^{(2,14)}, \bar{E}^{(16)} < E^{(2,14)}, \bar{E}^{(16):1} < E^{(2,14)}, \]
\[ K^{(16)} < E^{(2,14)}, (14)(25)(36) K^{(16):1} < E^{(2,14)}, \]
\[ L^{(16)} < E^{(2,14)}, (142536) L^{(16):1} < E^{(2,14)}, \]
\[ (142536) \bar{L}^{(16):1} < E^{(2,14)}, M^{(16)} < E^{(2,14)}, \]
\[(142536)M^{(16):1} < E^{(2,1^4)}, (142536)\bar{M}^{(16):1} < E^{(2,1^4)},
\]
\[(123)P^{(16)} < E^{(2,1^4)}, (142536)\bar{P}^{(16):1} < E^{(2,1^4)},
\]
\[C^{(16)} < E^{(2,1^4)}, C^{(16):1} < E^{(2,1^4)}, \bar{C}^{(16)} < E^{(2,1^4)}, \bar{C}^{(16):1} < E^{(2,1^4)},
\]
\[E^{(16)} < E^{(2,1^4)}, E^{(16):1} < E^{(2,1^4)}, \bar{E}^{(16)} < E^{(2,1^4)}, \bar{E}^{(16):1} < E^{(2,1^4)},
\]
\[(152634)\bar{F}^{(16):1} < E^{(2,1^4)}, (152634)\bar{H}^{(16):1} < E^{(2,1^4)},
\]
\[(456)K^{(16)} < E^{(2,1^4)}, (152634)\bar{K}^{(16):1} < E^{(2,1^4)},
\]
\[(456)L^{(16)} < E^{(2,1^4)}, (456)\bar{L}^{(16)} < E^{(2,1^4)},
\]
\[(153426)\bar{L}^{(16):1} < E^{(2,1^4)}, (456)M^{(16)} < E^{(2,1^4)},
\]
\[(456)\bar{M}^{(16)} < E^{(2,1^4)}, (153426)\bar{M}^{(16):1} < E^{(2,1^4)},
\]
\[(123)(456)P^{(16)} < E^{(2,1^4)}, (153426)P^{(16):1} < E^{(2,1^4)},
\]
\[(123)(456)\bar{P}^{(16)} < E^{(2,1^4)}, (153426)\bar{P}^{(16):1} < E^{(2,1^4)},
\]
\[A^{(16)} < F^{(2,1^4)}, A^{(16):1} < F^{(2,1^4)}, B^{(16)} < F^{(2,1^4)}, \bar{B}^{(16)} < F^{(2,1^4)},
\]
\[C^{(16)} < F^{(2,1^4)}, E^{(16)} < F^{(2,1^4)}, F^{(16)} < F^{(2,1^4)}, F^{(16):1} < F^{(2,1^4)},
\]
\[\bar{F}^{(16)} < F^{(2,1^4)}, \bar{F}^{(16):1} < F^{(2,1^4)}, H^{(16)} < F^{(2,1^4)}, H^{(16):1} < F^{(2,1^4)},
\]
\[\bar{H}^{(16)} < F^{(2,1^4)}, H^{(16):1} < F^{(2,1^4)}, K^{(16)} < F^{(2,1^4)}, K^{(16):1} < F^{(2,1^4)},
\]
\[(14)(25)(36)\bar{K}^{(16)} < F^{(2,1^4)}, (14)(25)(36)\bar{K}^{(16):1} < F^{(2,1^4)},
\]
\[(14)(25)(36)\bar{L}^{(16)} < F^{(2,1^4)}, (14)(25)(36)\bar{L}^{(16):1} < F^{(2,1^4)},
\]
\[(14)(25)(36)\bar{M}^{(16)} < F^{(2,1^4)}, (14)(25)(36)\bar{M}^{(16):1} < F^{(2,1^4)},
\]
\[(132)A^{(16):1} < \bar{F}^{(2,1^4)}, (132)\bar{A}^{(16):1} < \bar{F}^{(2,1^4)},
\]
\[(132)B^{(16):1} < \bar{F}^{(2,1^4)}, (132)\bar{B}^{(16):1} < \bar{F}^{(2,1^4)},
\]
\[(132)C^{(16):1} < \bar{F}^{(2,1^4)}, (132)\bar{E}^{(16):1} < \bar{F}^{(2,1^4)},
\]
\[F^{(16)} < \bar{F}^{(2,1^4)}, F^{(16):1} < \bar{F}^{(2,1^4)}, \bar{F}^{(16)} < \bar{F}^{(2,1^4)}, \bar{F}^{(16):1} < \bar{F}^{(2,1^4)},
\]
\[H^{(16)} < \bar{F}^{(2,1^4)}, H^{(16):1} < \bar{F}^{(2,1^4)}, \bar{H}^{(16)} < \bar{F}^{(2,1^4)}, \bar{H}^{(16):1} < \bar{F}^{(2,1^4)},
\]
\[K^{(16)} < \bar{F}^{(2,1^4)}, K^{(16):1} < \bar{F}^{(2,1^4)},
\]
\[(14)(25)(36)\bar{K}^{(16)} < \bar{F}^{(2,1^4)}, (14)(25)(36)\bar{K}^{(16):1} < \bar{F}^{(2,1^4)},
\]
\[(14)(25)(36)\bar{L}^{(16)} < \bar{F}^{(2,1^4)}, (14)(25)(36)\bar{L}^{(16):1} < \bar{F}^{(2,1^4)},
\]
\[(14)(25)(36)\bar{M}^{(16)} < \bar{F}^{(2,1^4)}, (14)(25)(36)\bar{M}^{(16):1} < \bar{F}^{(2,1^4)},
\]
\[A^{(16)} < H^{(2,1^4)}, A^{(16):1} < H^{(2,1^4)}, B^{(16)} < H^{(2,1^4)}, B^{(16):1} < H^{(2,1^4)},
\]
(465)\(\overline{C}^{(16)} \prec H^{(2,14)}\), \((465)\overline{E}^{(16)} \prec H^{(2,14)}\),
\(F^{(16)} \prec H^{(2,14)}\), \(F^{(16)}:1 \prec H^{(2,14)}\), \(\overline{F}^{(16)} \prec H^{(2,14)}\), \(\overline{F}^{(16)}:1 \prec H^{(2,14)}\),
\(H^{(16)} \prec H^{(2,14)}\), \(H^{(16)}:1 \prec H^{(2,14)}\), \(\overline{H}^{(16)} \prec H^{(2,14)}\), \(\overline{H}^{(16)}:1 \prec H^{(2,14)}\),
\(L^{(16)} \prec H^{(2,14)}\), \(L^{(16)}:1 \prec H^{(2,14)}\), \(M^{(16)} \prec H^{(2,14)}\), \(M^{(16)}:1 \prec H^{(2,14)}\),
\(P^{(16)} \prec H^{(2,14)}\), \(P^{(16)}:1 \prec H^{(2,14)}\),
\((14)(25)(36)\overline{P}^{(16)} \prec H^{(2,14)}\), \((14)(25)(36)\overline{P}^{(16)}:1 \prec H^{(2,14)}\),
\((132)\overline{A}^{(16)}:1 \prec H^{(2,14)}\), \((132)\overline{A}^{(16)}:1 \prec H^{(2,14)}\),
\((132)\overline{B}^{(16)}:1 \prec H^{(2,14)}\), \((132)\overline{B}^{(16)}:1 \prec H^{(2,14)}\),
\((132)(465)\overline{C}^{(16)}:1 \prec H^{(2,14)}\), \((132)(465)\overline{E}^{(16)}:1 \prec H^{(2,14)}\),
\(F^{(16)} \prec H^{(2,14)}\), \(F^{(16)}:1 \prec H^{(2,14)}\), \(\overline{F}^{(16)} \prec H^{(2,14)}\), \(\overline{F}^{(16)}:1 \prec H^{(2,14)}\),
\(H^{(16)} \prec H^{(2,14)}\), \(H^{(16)}:1 \prec H^{(2,14)}\), \(\overline{H}^{(16)} \prec H^{(2,14)}\), \(\overline{H}^{(16)}:1 \prec H^{(2,14)}\),
\(L^{(16)} \prec H^{(2,14)}\), \(L^{(16)}:1 \prec H^{(2,14)}\), \(M^{(16)} \prec H^{(2,14)}\), \(M^{(16)}:1 \prec H^{(2,14)}\),
\(P^{(16)} \prec H^{(2,14)}\), \(P^{(16)}:1 \prec H^{(2,14)}\),
\((14)(25)(36)\overline{P}^{(16)} \prec H^{(2,14)}\), \((14)(25)(36)\overline{P}^{(16)}:1 \prec H^{(2,14)}\),
\(\overline{A}^{(16)} \prec K^{(2,14)}\), \(\overline{B}^{(16)} \prec K^{(2,14)}\), \(C^{(16)} \prec K^{(2,14)}\), \(\overline{C}^{(16)} \prec K^{(2,14)}\),
\(E^{(16)} \prec K^{(2,14)}\), \(\overline{E}^{(16)} \prec K^{(2,14)}\), \(F^{(16)} \prec K^{(2,14)}\), \(F^{(16)}:1 \prec K^{(2,14)}\),
\((123)\overline{E}^{(16)}:1 \prec K^{(2,14)}\), \(H^{(16)} \prec K^{(2,14)}\), \(H^{(16)}:1 \prec K^{(2,14)}\),
\(K^{(16)} \prec K^{(2,14)}\), \(K^{(16)}:1 \prec K^{(2,14)}\), \(\overline{K}^{(16)} \prec K^{(2,14)}\), \(\overline{K}^{(16)}:1 \prec K^{(2,14)}\),
\((14)(25)(36)\overline{L}^{(16)} \prec K^{(2,14)}\), \((14)(25)(36)\overline{L}^{(16)}:1 \prec K^{(2,14)}\),
\((142536)\overline{L}^{(16)} \prec K^{(2,14)}\), \((14)(25)(36)M^{(16)} \prec K^{(2,14)}\),
\((14)(25)(36)M^{(16)}:1 \prec K^{(2,14)}\), \((142536)\overline{M}^{(16)} \prec K^{(2,14)}\),
\((14)(25)(36)C^{(16)} \prec K^{(2,14)}\), \((14)(25)(36)\overline{C}^{(16)} \prec K^{(2,14)}\),
\((14)(25)(36)E^{(16)} \prec K^{(2,14)}\), \((14)(25)(36)\overline{E}^{(16)} \prec K^{(2,14)}\),
\(F^{(16)} \prec K^{(2,14)}\), \(F^{(16)}:1 \prec K^{(2,14)}\), \(H^{(16)} \prec K^{(2,14)}\), \(H^{(16)}:1 \prec K^{(2,14)}\),
\(K^{(16)} \prec K^{(2,14)}\), \(K^{(16)}:1 \prec K^{(2,14)}\), \(\overline{K}^{(16)} \prec K^{(2,14)}\), \(\overline{K}^{(16)}:1 \prec K^{(2,14)}\),
\((456)\overline{L}^{(16)} \prec K^{(2,14)}\), \((14)(25)(36)\overline{L}^{(16)}:1 \prec K^{(2,14)}\),
\((456)\overline{L}^{(16)} \prec K^{(2,14)}\), \((456)M^{(16)} \prec K^{(2,14)}\),
\((14)(25)(36)M^{(16)}:1 < \bar{K}^{(2,14)}, (456)\bar{M}^{(16)} < \bar{K}^{(2,14)},\)
\((152634)P^{(16)}:1 < \bar{K}^{(2,14)}, (456)\bar{P}^{(16)} < \bar{K}^{(2,14)},\)
\(A^{(16)} < L^{(2,14)}, \bar{B}^{(16)} < L^{(2,14)}, (465)C^{(16)} < L^{(2,14)}, (465)\bar{C}^{(16)} < L^{(2,14)},\)
\((465)E^{(16)} < L^{(2,14)}, (465)\bar{E}^{(16)} < L^{(2,14)}, F^{(16)} < L^{(2,14)}, F^{(16)}:1 < L^{(2,14)},\)
\((123)\bar{F}^{(16)}:1 < L^{(2,14)}, H^{(16)} < L^{(2,14)},\)
\(H^{(16)}:1 < L^{(2,14)}, (123)\bar{H}^{(16)}:1 < L^{(2,14)},\)
\(L^{(16)} < L^{(2,14)}, \bar{L}^{(16)} < L^{(2,14)}, L^{(16)}:1 < L^{(2,14)},\)
\(M^{(16)} < L^{(2,14)}, M^{(16)}:1 < L^{(2,14)}, \bar{M}^{(16)} < L^{(2,14)}, \bar{M}^{(16)}:1 < L^{(2,14)},\)
\((14)(25)(36)P^{(16)} < L^{(2,14)}, (123)P^{(16)}:1 < L^{(2,14)},\)
\(P^{(16)} < L^{(2,14)}, P^{(16)}:1 < L^{(2,14)},\)
\((163524)C^{(16)}:1 < \bar{L}^{(2,14)}, (163524)\bar{C}^{(16)}:1 < \bar{L}^{(2,14)},\)
\((163524)E^{(16)}:1 < \bar{L}^{(2,14)}, (163524)\bar{E}^{(16)}:1 < \bar{L}^{(2,14)},\)
\(F^{(16)} < \bar{L}^{(2,14)}, F^{(16)}:1 < \bar{L}^{(2,14)}, H^{(16)} < \bar{L}^{(2,14)}, H^{(16)}:1 < \bar{L}^{(2,14)},\)
\((465)K^{(16)} < \bar{L}^{(2,14)}, (163524)K^{(16)}:1 < \bar{L}^{(2,14)},\)
\((465)\bar{K}^{(16)} < \bar{L}^{(2,14)}, (163524)\bar{K}^{(16)}:1 < \bar{L}^{(2,14)},\)
\(L^{(16)} < \bar{L}^{(2,14)}, \bar{L}^{(16)}:1 < \bar{L}^{(2,14)}, L^{(16)} < \bar{L}^{(2,14)}, \bar{L}^{(16)}:1 < \bar{L}^{(2,14)},\)
\(M^{(16)} < \bar{L}^{(2,14)}, M^{(16)}:1 < \bar{L}^{(2,14)}, \bar{M}^{(16)} < \bar{L}^{(2,14)}, \bar{M}^{(16)}:1 < \bar{L}^{(2,14)},\)
\((14)(25)(36)P^{(16)} < \bar{L}^{(2,14)}, (14)(25)(36)P^{(16)}:1 < \bar{L}^{(2,14)},\)
\(P^{(16)} < \bar{L}^{(2,14)}, P^{(16)}:1 < \bar{L}^{(2,14)},\)
\(C^{(16)} < M^{(2,14)}, (456)\bar{C}^{(16)} < M^{(2,14)},\)
\(E^{(16)} < M^{(2,14)}, (456)\bar{E}^{(16)} < M^{(2,14)},\)
\((14)(25)(36)\bar{F}^{(16)} < M^{(2,14)}, (14)(25)(36)\bar{F}^{(16)}:1 < M^{(2,14)},\)
\((14)(25)(36)\bar{H}^{(16)} < M^{(2,14)}, (14)(25)(36)\bar{H}^{(16)}:1 < M^{(2,14)},\)
\(K^{(16)} < M^{(2,14)}, K^{(16)}:1 < M^{(2,14)},\)
\((14)(25)(36)\bar{K}^{(16)} < M^{(2,14)}, (14)(25)(36)\bar{K}^{(16)}:1 < M^{(2,14)},\)
\(L^{(16)} < M^{(2,14)}, L^{(16)}:1 < M^{(2,14)}, \bar{L}^{(16)} < M^{(2,14)}, \bar{L}^{(16)}:1 < M^{(2,14)},\)
\(M^{(16)} < M^{(2,14)}, M^{(16)}:1 < M^{(2,14)}, \bar{M}^{(16)} < M^{(2,14)}, \bar{M}^{(16)}:1 < M^{(2,14)},\)
\((142536)P^{(16)} < M^{(2,14)}, (123)P^{(16)}:1 < M^{(2,14)},\)
(142536) \tilde{P}^{(1^6)} < M^{(2,1^4)}, (123) \tilde{P}^{(1^6)}:1 < M^{(2,1^4)},
(163524) \tilde{A}^{(1^6)}:1 < \tilde{M}^{(2,1^4)}, (163524) \tilde{B}^{(1^6)}:1 < \tilde{M}^{(2,1^4)},
(163524) \tilde{C}^{(1^6)}:1 < \tilde{M}^{(2,1^4)}, (163524) \tilde{C}^{(1^6)}:1 < \tilde{M}^{(2,1^4)},
(163524) \tilde{E}^{(1^6)}:1 < \tilde{M}^{(2,1^4)}, (163524) \tilde{E}^{(1^6)}:1 < \tilde{M}^{(2,1^4)},
(163524) F^{(1^6)}:1 < \tilde{M}^{(2,1^4)}, (14)(25)(36) \tilde{F}^{(1^6)} < \tilde{M}^{(2,1^4)},
(14)(25)(36) \tilde{F}^{(1^6)}:1 < \tilde{M}^{(2,1^4)}, (163524) H^{(1^6)}:1 < \tilde{M}^{(2,1^4)},
(14)(25)(36) \tilde{H}^{(1^6)} < \tilde{M}^{(2,1^4)}, (14)(25)(36) \tilde{H}^{(1^6)}:1 < \tilde{M}^{(2,1^4)}, K^{(1^6)}:1 < \tilde{M}^{(2,1^4)}, (465) \tilde{K}^{(1^6)} < \tilde{M}^{(2,1^4)}, (14)(25)(36) \tilde{K}^{(1^6)}:1 < \tilde{M}^{(2,1^4)},
L^{(1^6)} < \tilde{M}^{(2,1^4)}, L^{(1^6)}:1 < \tilde{M}^{(2,1^4)}, L^{(1^6)} < \tilde{M}^{(2,1^4)}, \tilde{L}^{(1^6)}:1 < \tilde{M}^{(2,1^4)},
M^{(1^6)} < \tilde{M}^{(2,1^4)}, M^{(1^6)}:1 < \tilde{M}^{(2,1^4)}, \tilde{M}^{(1^6)} < \tilde{M}^{(2,1^4)}, \tilde{M}^{(1^6)}:1 < \tilde{M}^{(2,1^4)},
(132) \tilde{A}^{(1^6)}:1 < P^{(2,1^4)}, (132) \tilde{B}^{(1^6)}:1 < P^{(2,1^4)},
(132)(465) \tilde{C}^{(1^6)}:1 < P^{(2,1^4)}, (132)(465) \tilde{C}^{(1^6)}:1 < P^{(2,1^4)},
(132)(465) \tilde{E}^{(1^6)}:1 < P^{(2,1^4)}, (132)(465) \tilde{E}^{(1^6)}:1 < P^{(2,1^4)},
(132) F^{(1^6)}:1 < P^{(2,1^4)}, F^{(1^6)} < P^{(2,1^4)},
\tilde{F}^{(1^6)}:1 < P^{(2,1^4)}, (132) H^{(1^6)}:1 < P^{(2,1^4)},
\tilde{H}^{(1^6)} < P^{(2,1^4)}, \tilde{H}^{(1^6)}:1 < P^{(2,1^4)},
(132) L^{(1^6)}:1 < P^{(2,1^4)}, \tilde{L}^{(1^6)} < P^{(2,1^4)}, \tilde{L}^{(1^6)}:1 < P^{(2,1^4)},
(132) M^{(1^6)}:1 < P^{(2,1^4)}, M^{(1^6)} < P^{(2,1^4)}, \tilde{M}^{(1^6)}:1 < P^{(2,1^4)},
P^{(1^6)} < P^{(2,1^4)}, P^{(1^6)}:1 < P^{(2,1^4)}, \tilde{P}^{(1^6)} < P^{(2,1^4)}, \tilde{P}^{(1^6)}:1 < P^{(2,1^4)},
(163524) C^{(1^6)}:1 < \tilde{P}^{(2,1^4)}, (16)(24)(35) \tilde{C}^{(1^6)}:1 < \tilde{P}^{(2,1^4)},
(163524) E^{(1^6)}:1 < \tilde{P}^{(2,1^4)}, (16)(24)(35) \tilde{E}^{(1^6)}:1 < \tilde{P}^{(2,1^4)},
\tilde{F}^{(1^6)} < \tilde{P}^{(2,1^4)}, \tilde{F}^{(1^6)}:1 < \tilde{P}^{(2,1^4)}, \tilde{H}^{(1^6)} < \tilde{P}^{(2,1^4)}, \tilde{H}^{(1^6)}:1 < \tilde{P}^{(2,1^4)},
(163524) K^{(1^6)}:1 < \tilde{P}^{(2,1^4)}, (465) \tilde{K}^{(1^6)} < \tilde{P}^{(2,1^4)},
(163524) L^{(1^6)}:1 < \tilde{P}^{(2,1^4)}, \tilde{L}^{(1^6)} < \tilde{P}^{(2,1^4)}, \tilde{L}^{(1^6)}:1 < \tilde{P}^{(2,1^4)},
\[
R_{1,2}R_{2,3}R_{4,5}R_{5,6}\bar{A}^{(16)} = R_{1,1}R_{2,3}R_{4,5}R_{5,6}\bar{A}^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,3}R_{3,4}R_{4,5}B^{(16)} = R_{1,1}R_{2,3}R_{3,4}R_{4,5}B^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,3}R_{4,5}\bar{B}^{(16)} = R_{1,1}R_{2,3}R_{4,5}\bar{B}^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,3}R_{5,6}C^{(16)} = R_{1,1}R_{2,3}R_{5,6}C^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,3}E^{(16)} = R_{1,1}R_{2,3}E^{(16)}; 1 = A^{(2,1^4)},
\]
\[
R_{1,2}R_{2,4}R_{4,5}R_{5,6}A^{(16)} = R_{1,1}R_{2,4}R_{4,5}R_{5,6}A^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,4}R_{3,4}R_{4,5}R_{5,6}\bar{A}^{(16)} = R_{1,1}R_{2,4}R_{3,4}R_{4,5}R_{5,6}\bar{A}^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,4}R_{4,5}B^{(16)} = R_{1,1}R_{2,4}R_{4,5}B^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,4}R_{3,4}R_{4,5}\bar{B}^{(16)} = R_{1,1}R_{2,4}R_{3,4}R_{4,5}\bar{B}^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,4}R_{3,4}R_{5,6}C^{(16)} = R_{1,1}R_{2,4}R_{3,4}R_{5,6}C^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,4}R_{3,4}R_{5,6}E^{(16)} = R_{1,1}R_{2,4}R_{3,4}R_{5,6}E^{(16)}; 1 =
\]
\[
R_{1,2}R_{3,4}R_{4,5}R_{5,6}F^{(16)} = R_{1,2}R_{4,5}R_{5,6}F^{(16)} =
\]
\[
R_{1,2}R_{3,4}R_{5,6}H^{(16)} = R_{1,2}R_{4,5}\bar{H}^{(16)} =
\]
\[
R_{1,2}R_{3,3}R_{5,6}K^{(16)} = R_{1,2}R_{3,3}(14)(25)(36)\bar{K}^{(16)}; 1 =
\]
\[
R_{1,2}(14)(25)(36)\bar{L}^{(16)}; 1 = R_{1,2}R_{5,6}(14)(25)(36)\bar{M}^{(16)}; 1 = A^{(2,1^4)},
\]
\[
R_{1,2}R_{2,3}R_{3,4}R_{4,6}A^{(16)} = R_{1,1}R_{2,3}R_{3,4}R_{4,6}A^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,3}R_{4,6}\bar{A}^{(16)} = R_{1,1}R_{2,3}R_{4,6}\bar{A}^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,3}R_{4,6}R_{5,5}B^{(16)} = R_{1,1}R_{2,3}R_{4,6}R_{5,5}B^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,3}R_{4,6}R_{5,5}\bar{B}^{(16)} = R_{1,1}R_{2,3}R_{4,6}R_{5,5}\bar{B}^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,3}R_{5,5}(465)\bar{C}^{(16)} = R_{1,1}R_{2,3}R_{5,5}(465)\bar{C}^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,3}(465)\bar{E}^{(16)} = R_{1,1}R_{2,3}(465)\bar{E}^{(16)}; 1 = B^{(2,1^4)},
\]
\[
R_{1,2}R_{2,4}R_{4,6}A^{(16)} = R_{1,1}R_{2,4}R_{4,6}A^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,4}R_{3,3}R_{4,6}\bar{A}^{(16)} = R_{1,1}R_{2,4}R_{3,3}R_{4,6}\bar{A}^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,4}R_{4,6}R_{5,5}B^{(16)} = R_{1,1}R_{2,4}R_{4,6}R_{5,5}B^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,4}R_{3,3}R_{4,6}R_{5,5}\bar{B}^{(16)} = R_{1,1}R_{2,4}R_{3,3}R_{4,6}R_{5,5}\bar{B}^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,4}R_{3,3}R_{5,5}(465)\bar{C}^{(16)} = R_{1,1}R_{2,4}R_{3,3}R_{5,5}(465)\bar{C}^{(16)}; 1 =
\]
\[
R_{1,2}R_{2,4}R_{3,3}(465)\bar{E}^{(16)} = R_{1,1}R_{2,4}R_{3,3}(465)\bar{E}^{(16)}; 1 =
\]
\[ R_{1,2}R_{3,3}R_{4,6}F^{(16)} = R_{1,2}R_{4,6}F^{(16)} = \]
\[ R_{1,2}R_{3,3}R_{4,6}R_{5,5}H^{(16)} = R_{1,2}R_{4,6}R_{5,5}H^{(16)} = \]
\[ R_{1,2}R_{3,3}R_{5,5}L^{(16)} = R_{1,2}R_{3,3}M^{(16)} = \]
\[ R_{1,2}R_{5,5}P^{(16)} = R_{1,2}(4)(14)(25)(36)P^{(16)}:1 = \overline{B}(2,1^4), \]
\[ R_{1,2}R_{2,4}R_{3,5}R_{5,6}A^{(16)} = R_{1,1}R_{2,4}R_{3,5}R_{5,6}A^{(16)}:1 = \]
\[ R_{1,2}R_{2,4}R_{3,5}B^{(16)} = R_{1,1}R_{2,4}R_{3,5}B^{(16)}:1 = \]
\[ R_{1,2}R_{2,4}R_{3,5}R_{4,3}R_{5,6}C^{(16)} = R_{1,1}R_{2,4}R_{3,5}R_{4,3}R_{5,6}C^{(16)}:1 = \]
\[ R_{1,2}R_{2,4}R_{4,3}R_{5,6}C^{(16)} = R_{1,1}R_{2,4}R_{4,3}R_{5,6}C^{(16)}:1 = \]
\[ R_{1,2}R_{2,4}R_{3,5}R_{4,3}E^{(16)} = R_{1,1}R_{2,4}R_{3,5}R_{4,3}E^{(16)}:1 = \]
\[ R_{1,2}R_{2,4}R_{4,3}E^{(16)} = R_{1,1}R_{2,4}R_{4,3}E^{(16)}:1 = \]
\[ R_{1,2}R_{3,5}R_{5,6}F^{(16)} = R_{1,1}R_{3,5}R_{5,6}(123)F^{(16)} = \]
\[ R_{1,2}R_{3,5}H^{(16)} = R_{1,1}R_{3,5}(123)H^{(16)} = \]
\[ R_{1,2}R_{3,5}R_{4,3}R_{5,6}K^{(16)} = R_{1,2}R_{4,3}(14)(25)(36)K^{(16)}:1 = \]
\[ R_{1,2}R_{3,5}R_{5,6}K^{(16)} = R_{1,2}R_{3,5}R_{4,3}(14)(25)(36)K^{(16)}:1 = \]
\[ R_{1,2}(14)(25)(36)L^{(16)}:1 = R_{1,1}R_{3,5}R_{4,3}(142536)L^{(16)}:1 = \]
\[ R_{1,2}R_{5,6}(14)(25)(36)M^{(16)}:1 = R_{1,1}R_{3,5}R_{4,3}R_{5,6}(142536)M^{(16)}:1 = C(2,1^4), \]
\[ R_{1,2}R_{2,5}R_{5,6}A^{(16)} = R_{1,1}R_{2,5}R_{5,6}A^{(16)}:1 = \]
\[ R_{1,2}R_{2,5}B^{(16)} = R_{1,1}R_{2,5}B^{(16)}:1 = \]
\[ R_{1,2}R_{2,5}R_{4,3}R_{5,6}C^{(16)} = R_{1,1}R_{2,5}R_{4,3}R_{5,6}C^{(16)}:1 = \]
\[ R_{1,2}R_{2,5}R_{3,4}R_{4,3}R_{5,6}C^{(16)} = R_{1,1}R_{2,5}R_{3,4}R_{4,3}R_{5,6}C^{(16)}:1 = \]
\[ R_{1,2}R_{2,5}R_{4,3}E^{(16)} = R_{1,1}R_{2,5}R_{4,3}E^{(16)}:1 = \]
\[ R_{1,2}R_{2,5}R_{3,4}R_{4,3}E^{(16)} = R_{1,1}R_{2,5}R_{3,4}R_{4,3}E^{(16)}:1 = \]
\[ R_{1,2}R_{3,4}(456)F^{(16)} = R_{1,1}R_{3,4}(123)(456)F^{(16)} = \]
\[ R_{1,2}R_{3,4}R_{5,6}(456)H^{(16)} = R_{1,1}R_{3,4}R_{5,6}(123)(456)H^{(16)} = \]
\[ R_{1,2}R_{3,4}R_{4,3}R_{5,6}(456)L^{(16)} = R_{1,2}R_{4,3}R_{5,6}(456)L^{(16)} = \]
\[ R_{1,2}R_{3,4}R_{4,3}(456)M^{(16)} = R_{1,2}R_{4,3}(456)M^{(16)} = \]
\[ R_{1,1}R_{3,4}R_{4,3}R_{5,6}(123)(456)P^{(16)} = R_{1,2}(152634)P^{(16)}:1 = \]
\[ R_{1,2}R_{5,0}(456)\tilde{P}^{(16)} = R_{1,1}R_{3,4}R_{4,3}(153426)\tilde{P}^{(16):1} = \tilde{C}^{(2,1^4)}, \]
\[ R_{1,2}R_{2,4}R_{3,5}R_{4,6}C^{(16)} = R_{1,1}R_{2,4}R_{3,5}R_{4,6}C^{(16):1} = \]
\[ R_{1,2}R_{2,4}R_{4,6}\tilde{C}^{(16)} = R_{1,1}R_{2,4}R_{4,6}\tilde{C}^{(16):1} = \]
\[ R_{1,2}R_{2,4}R_{3,5}R_{4,6}R_{5,3}E^{(16)} = R_{1,1}R_{2,4}R_{3,5}R_{4,6}R_{5,3}E^{(16):1} = \]
\[ R_{1,2}R_{2,4}R_{4,6}R_{5,3}\tilde{E}^{(16)} = R_{1,1}R_{2,4}R_{4,6}R_{5,3}\tilde{E}^{(16):1} = \]
\[ R_{1,2}R_{5,3}(14)(25)(36)\tilde{E}^{(16):1} = R_{1,2}(14)(25)(36)\tilde{E}^{(16):1} = \]
\[ R_{1,2}R_{5,3}(14)(25)(36)K^{(16)} = R_{1,2}R_{5,3}(14)(25)(36)K^{(16):1} = \]
\[ R_{1,2}R_{4,6}R_{5,3}(14)(25)(36)K^{(16):1} = \]
\[ R_{1,2}R_{3,5}R_{4,6}R_{5,3}(14)(25)(36)K^{(16):1} = \]
\[ R_{1,2}R_{3,5}L^{(16)} = R_{1,1}R_{4,6}R_{5,3}(142536)L^{(16):1} = \]
\[ R_{1,1}R_{3,5}R_{4,6}R_{5,3}(142536)\tilde{L}^{(16):1} = R_{1,2}R_{3,5}R_{5,3}M^{(16)} = \]
\[ R_{1,1}R_{4,6}(142536)M^{(16):1} = R_{1,1}R_{3,5}R_{4,6}(142536)\tilde{M}^{(16):1} = \]
\[ R_{1,1}R_{3,5}(123)P^{(16)} = R_{1,1}R_{3,5}R_{5,3}(142536)\tilde{P}^{(16):1} = E^{(2,1^4)}, \]
\[ R_{1,2}R_{2,5}R_{4,6}C^{(16)} = R_{1,1}R_{2,5}R_{4,6}C^{(16):1} = \]
\[ R_{1,2}R_{2,5}R_{3,4}R_{4,6}\tilde{C}^{(16)} = R_{1,1}R_{2,5}R_{3,4}R_{4,6}\tilde{C}^{(16):1} = \]
\[ R_{1,2}R_{2,5}R_{4,6}R_{5,3}E^{(16)} = R_{1,1}R_{2,5}R_{4,6}R_{5,3}E^{(16):1} = \]
\[ R_{1,2}R_{2,5}R_{3,4}R_{4,6}R_{5,3}\tilde{E}^{(16):1} = R_{1,2}R_{5,3}(152634)\tilde{E}^{(16):1} = R_{1,2}(152634)\tilde{E}^{(16):1} = \]
\[ R_{1,2}R_{5,3}(152634)\tilde{E}^{(16):1} = R_{1,2}(152634)\tilde{E}^{(16):1} = \]
\[ R_{1,2}R_{3,4}R_{4,6}(456)K^{(16)} = R_{1,2}R_{3,4}R_{5,3}(152634)\tilde{K}^{(16):1} = \]
\[ R_{1,2}R_{3,4}R_{4,6}(456)L^{(16)} = R_{1,2}R_{4,6}(456)\tilde{L}^{(16)} = \]
\[ R_{1,1}R_{3,4}R_{5,3}(153426)\tilde{L}^{(16):1} = R_{1,2}R_{3,4}R_{4,6}R_{5,3}(456)M^{(16)} = \]
\[ R_{1,2}R_{4,6}R_{5,3}(456)\tilde{M}^{(16)} = R_{1,1}R_{3,4}(153426)\tilde{M}^{(16):1} = \]
\[ R_{1,2}R_{4,6}R_{5,3}(123)(456)P^{(16)} = R_{1,1}R_{4,6}R_{5,3}(153426)P^{(16):1} = \]
\[ R_{1,1}R_{4,6}(123)(456)\tilde{P}^{(16)} = R_{1,1}R_{3,4}R_{4,6}R_{5,3}(153426)\tilde{P}^{(16):1} = \tilde{E}^{(2,1^4)}, \]
\[ R_{1,4}R_{4,5}R_{5,6}A^{(16)} = R_{1,4}R_{3,3}R_{4,5}R_{5,6}A^{(16)} = \]
\[ R_{1,4}R_{4,5}B^{(16)} = R_{1,4}R_{3,3}R_{4,5}B^{(16)} = \]
\[ R_{1,4}R_{3,3}R_{5,6}C^{(16)} = R_{1,4}R_{3,3}C^{(16)} = \]
\[ R_{1,4}R_{3,3}R_{5,6}F^{(16)} = R_{1,1}R_{2,2}R_{3,3}R_{4,5}R_{5,6}F^{(16):1} = \]
\[ R_{1,4} R_{2,2} R_{4,5} R_{5,6} \tilde{F}^{(16)} = R_{1,1} R_{2,2} R_{4,5} R_{5,6} \tilde{F}^{(16)}:1 = \]
\[ R_{1,4} R_{2,2} R_{3,3} R_{4,5} \tilde{H}^{(16)} = R_{1,1} R_{2,2} R_{3,3} R_{4,5} \tilde{H}^{(16)}:1 = \]
\[ R_{1,4} R_{2,2} R_{4,5} \tilde{H}^{(16)} = R_{1,1} R_{2,2} R_{4,5} \tilde{H}^{(16)}:1 = \]
\[ R_{1,4} R_{2,2} R_{3,3} R_{5,6} K^{(16)} = R_{1,1} R_{2,2} R_{3,3} R_{5,6} K^{(16)}:1 = \]
\[ R_{1,1} R_{2,2} R_{3,3}(14)(25)(36) \tilde{K}^{(16)} = R_{1,4} R_{2,2} R_{3,3}(14)(25)(36) \tilde{K}^{(16)}:1 = \]
\[ R_{1,1} R_{2,2}(14)(25)(36) \tilde{L}^{(16)} = R_{1,4} R_{2,2}(14)(25)(36) \tilde{L}^{(16)}:1 = \]
\[ R_{1,1} R_{2,2} R_{5,6}(14)(25)(36) \tilde{M}^{(16)} = R_{1,4} R_{2,2} R_{5,6}(14)(25)(36) \tilde{M}^{(16)}:1 = F^{(2,1^4)}, \]
\[ R_{1,4} R_{4,5} R_{5,6}(132) A^{(16)}:1 = R_{1,4} R_{3,2} R_{4,5} R_{5,6}(132) \tilde{A}^{(16)}:1 = \]
\[ R_{1,4} R_{4,5}(132) B^{(16)}:1 = R_{1,4} R_{3,2} R_{4,5}(132) \tilde{B}^{(16)}:1 = \]
\[ R_{1,4} R_{3,2} R_{5,6}(132) C^{(16)}:1 = R_{1,4} R_{3,2}(132) E^{(16)}:1 = \]
\[ R_{1,4} R_{2,3} R_{4,5} R_{5,6} F^{(16)} = R_{1,1} R_{2,3} R_{4,5} R_{5,6} F^{(16)}:1 = \]
\[ R_{1,4} R_{2,3} R_{3,2} R_{4,5} R_{5,6} \tilde{F}^{(16)} = R_{1,1} R_{2,3} R_{3,2} R_{4,5} R_{5,6} \tilde{F}^{(16)}:1 = \]
\[ R_{1,4} R_{2,3} R_{4,5} H^{(16)} = R_{1,1} R_{2,3} R_{4,5} H^{(16)}:1 = \]
\[ R_{1,4} R_{2,3} R_{3,2} R_{4,5} \tilde{H}^{(16)} = R_{1,1} R_{2,3} R_{3,2} R_{4,5} \tilde{H}^{(16)}:1 = \]
\[ R_{1,4} R_{2,3} R_{5,6} K^{(16)} = R_{1,1} R_{2,3} R_{5,6} K^{(16)}:1 = \]
\[ R_{1,1} R_{2,3}(14)(25)(36) \tilde{K}^{(16)} = R_{1,4} R_{2,3}(14)(25)(36) \tilde{K}^{(16)}:1 = \]
\[ R_{1,1} R_{2,3} R_{3,2}(14)(25)(36) \tilde{L}^{(16)} = R_{1,4} R_{2,3} R_{3,2}(14)(25)(36) \tilde{L}^{(16)}:1 = \]
\[ R_{1,1} R_{2,3} R_{3,2} R_{5,6}(14)(25)(36) \tilde{M}^{(16)} = R_{1,4} R_{2,3} R_{3,2} R_{5,6}(14)(25)(36) \tilde{M}^{(16)}:1 = F^{(2,1^4)}, \]
\[ R_{1,4} R_{4,6} A^{(16)} = R_{1,4} R_{3,3} R_{4,6} \tilde{A}^{(16)} = \]
\[ R_{1,4} R_{4,6} R_{5,5} B^{(16)} = R_{1,4} R_{3,3} R_{4,6} R_{5,5} \tilde{B}^{(16)} = \]
\[ R_{1,4} R_{3,3} R_{5,5}(465) \tilde{C}^{(16)} = R_{1,4} R_{3,3}(465) \tilde{E}^{(16)} = \]
\[ R_{1,4} R_{2,2} R_{3,3} R_{4,6} F^{(16)} = R_{1,1} R_{2,2} R_{3,3} R_{4,6} F^{(16)}:1 = \]
\[ R_{1,4} R_{2,2} R_{4,6} \tilde{F}^{(16)} = R_{1,1} R_{2,2} R_{4,6} \tilde{F}^{(16)}:1 = \]
\[ R_{1,4} R_{2,2} R_{3,3} R_{4,6} R_{5,5} H^{(16)} = R_{1,1} R_{2,2} R_{3,3} R_{4,6} R_{5,5} H^{(16)}:1 = \]
\[ R_{1,4} R_{2,2} R_{4,6} R_{5,5} \tilde{H}^{(16)} = R_{1,1} R_{2,2} R_{4,6} R_{5,5} \tilde{H}^{(16)}:1 = \]
\[ R_{1,4} R_{2,2} R_{3,3} R_{5,5} L^{(16)} = R_{1,1} R_{2,2} R_{3,3} R_{5,5} L^{(16)}:1 = \]
\[ R_{1,4} R_{2,2} R_{3,3} M^{(16)} = R_{1,1} R_{2,2} R_{3,3} M^{(16)}:1 = \]
\[ R_{1,4}R_{2,2}R_{5,5}P^{(1)} = R_{1,1}R_{2,2}R_{5,5}P^{(1)}:1 = \]
\[ R_{1,1}R_{2,2}(14)(25)(36)\tilde{P}^{(1)} = R_{1,4}R_{2,2}(14)(25)(36)\tilde{P}^{(1)}:1 = H^{(2,1^4)}, \]
\[ R_{1,4}R_{4,6}(132)A^{(1)}:1 = R_{1,4}R_{3,2}R_{4,6}(132)\tilde{A}^{(1)}:1 = \]
\[ R_{1,4}R_{4,6}R_{5,5}(132)B^{(1)}:1 = R_{1,4}R_{3,2}R_{4,6}R_{5,5}(132)\tilde{B}^{(1)}:1 = \]
\[ R_{1,4}R_{3,2}R_{5,5}(132)(465)\tilde{C}^{(1)}:1 = R_{1,4}R_{3,2}(132)(465)\tilde{E}^{(1)}:1 = \]
\[ R_{1,4}R_{2,3}R_{4,6}F^{(1)} = R_{1,1}R_{2,3}R_{4,6}F^{(1)}:1 = \]
\[ R_{1,4}R_{2,3}R_{3,2}R_{4,6}F^{(1)} = R_{1,1}R_{2,3}R_{3,2}R_{4,6}F^{(1)}:1 = \]
\[ R_{1,4}R_{2,3}R_{4,6}R_{5,5}H^{(1)} = R_{1,1}R_{2,3}R_{4,6}R_{5,5}H^{(1)}:1 = \]
\[ R_{1,4}R_{2,3}R_{3,2}R_{4,6}R_{5,5}\tilde{H}^{(1)} = R_{1,1}R_{2,3}R_{3,2}R_{4,6}R_{5,5}\tilde{H}^{(1)}:1 = \]
\[ R_{1,4}R_{2,3}R_{5,5}L^{(1)} = R_{1,1}R_{2,3}R_{5,5}L^{(1)}:1 = \]
\[ R_{1,4}R_{2,3}M^{(1)} = R_{1,1}R_{2,3}M^{(1)}:1 = \]
\[ R_{1,4}R_{2,3}R_{3,2}R_{5,5}P^{(1)} = R_{1,1}R_{2,3}R_{3,2}R_{5,5}P^{(1)}:1 = \]
\[ R_{1,1}R_{2,3}R_{3,2}(14)(25)(36)\tilde{P}^{(1)} = R_{1,4}R_{2,3}R_{3,2}(14)(25)(36)\tilde{P}^{(1)}:1 = \tilde{H}^{(2,1^4)}, \]
\[ R_{1,4}R_{3,5}R_{5,6}\tilde{A}^{(1)} = R_{1,4}R_{3,5}\tilde{B}^{(1)} = \]
\[ R_{1,4}R_{3,5}R_{4,3}R_{5,6}\tilde{C}^{(1)} = R_{1,4}R_{4,3}R_{5,6}\tilde{C}^{(1)} = \]
\[ R_{1,4}R_{3,5}R_{4,3}E^{(1)} = R_{1,4}R_{4,3}\tilde{E}^{(1)} = \]
\[ R_{1,4}R_{2,3}R_{3,5}R_{5,6}F^{(1)} = R_{1,1}R_{2,2}R_{3,5}R_{5,6}F^{(1)}:1 = \]
\[ R_{1,1}R_{3,5}R_{5,6}(123)\tilde{F}^{(1)}:1 = R_{1,4}R_{2,2}R_{3,5}H^{(1)} = \]
\[ R_{1,1}R_{2,2}R_{3,5}H^{(1)}:1 = R_{1,1}R_{3,5}(123)\tilde{H}^{(1)}:1 = \]
\[ R_{1,4}R_{2,3}R_{3,5}R_{4,3}R_{5,6}K^{(1)} = R_{1,1}R_{2,2}R_{3,5}R_{4,3}R_{5,6}K^{(1)}:1 = \]
\[ R_{1,4}R_{2,2}R_{4,3}R_{5,6}\tilde{K}^{(1)} = R_{1,1}R_{2,2}R_{4,3}R_{5,6}\tilde{K}^{(1)}:1 = \]
\[ R_{1,1}R_{2,2}(14)(25)(36)E^{(1)} = R_{1,4}R_{2,2}(14)(25)(36)L^{(1)}:1 = \]
\[ R_{1,1}R_{3,5}R_{4,3}(142536)\tilde{L}^{(1)} = R_{1,1}R_{2,2}R_{5,6}(14)(25)(36)M^{(1)} = \]
\[ R_{1,4}R_{2,2}R_{5,6}(14)(25)(36)M^{(1)}:1 = R_{1,1}R_{3,5}R_{4,3}R_{5,6}(142536)\tilde{M}^{(1)} = K^{(2,1^4)}, \]
\[ R_{1,1}R_{3,2}R_{4,3}(14)(25)(36)C^{(1)} = R_{1,1}R_{4,3}(14)(25)(36)\tilde{C}^{(1)} = \]
\[ R_{1,1}R_{3,2}R_{4,3}R_{5,6}(14)(25)(36)E^{(1)} = R_{1,1}R_{4,3}R_{5,6}(14)(25)(36)\tilde{E}^{(1)} = \]
\[ R_{1,4}R_{2,5}R_{5,6}F^{(1)} = R_{1,1}R_{2,5}R_{5,6}F^{(1)}:1 = \]
\[ R_{1,4}R_{2,5}H^{(16)} = R_{1,1}R_{2,5}H^{(16):1} = \]
\[ R_{1,4}R_{2,5}R_{4,3}R_{5,6}K^{(16)} = R_{1,1}R_{2,5}R_{4,3}R_{5,6}K^{(16):1} = \]
\[ R_{1,4}R_{2,5}R_{3,2}R_{4,3}R_{5,6}\bar{K}^{(16)} = R_{1,1}R_{2,5}R_{3,2}R_{4,3}R_{5,6}\bar{K}^{(16):1} = \]
\[ R_{1,4}R_{4,3}R_{5,6}(465)L^{(16)} = R_{1,4}R_{2,5}R_{3,2}(14)(25)(36)L^{(16):1} = \]
\[ R_{1,4}R_{3,2}R_{4,3}R_{5,6}(465)\bar{L}^{(16)} = R_{1,4}R_{4,3}(465)M^{(16)} = \]
\[ R_{1,4}R_{3,2}(152634)P^{(16):1} = R_{1,4}R_{3,2}R_{5,6}(465)\bar{P}^{(16)} = \bar{K}^{(2,1^4)}, \]
\[ R_{1,4}R_{3,6}A^{(16)} = R_{1,4}R_{3,6}R_{5,5}\bar{A}^{(16)} = \]
\[ R_{1,4}R_{4,3}R_{5,5}(465)C^{(16)} = R_{1,4}R_{3,6}R_{4,3}R_{5,5}(465)\bar{C}^{(16)} = \]
\[ R_{1,4}R_{4,3}(465)E^{(16)} = R_{1,4}R_{3,6}R_{4,3}(465)\bar{E}^{(16)} = \]
\[ R_{1,4}R_{2,2}R_{3,6}F^{(16)} = R_{1,1}R_{2,2}R_{3,6}F^{(16):1} = \]
\[ R_{1,1}R_{3,6}(123)\bar{F}^{(16):1} = R_{1,4}R_{2,2}R_{3,6}R_{5,5}H^{(16)} = \]
\[ R_{1,1}R_{2,2}R_{3,6}R_{5,5}H^{(16):1} = R_{1,1}R_{3,6}R_{5,5}(123)\bar{H}^{(16):1} = \]
\[ R_{1,4}R_{2,2}R_{3,6}R_{4,3}R_{5,5}L^{(16)} = R_{1,1}R_{2,2}R_{3,6}R_{4,3}R_{5,5}L^{(16):1} = \]
\[ R_{1,4}R_{2,2}R_{4,3}R_{5,5}\bar{L}^{(16)} = R_{1,1}R_{2,2}R_{4,3}R_{5,5}\bar{L}^{(16):1} = \]
\[ R_{1,4}R_{2,2}R_{3,6}R_{4,3}M^{(16)} = R_{1,1}R_{2,2}R_{3,6}R_{4,3}M^{(16):1} = \]
\[ R_{1,4}R_{2,2}R_{4,3}M^{(16)} = R_{1,1}R_{2,2}R_{4,3}\bar{M}^{(16):1} = \]
\[ R_{1,1}R_{2,2}(14)(25)(36)P^{(16):1} = R_{1,1}R_{3,6}R_{4,3}R_{5,5}(123)P^{(16):1} = \]
\[ R_{1,4}R_{2,2}R_{5,5}\bar{P}^{(16)} = R_{1,1}R_{2,2}R_{5,5}\bar{P}^{(16):1} = L^{(2,1^4)}, \]
\[ R_{1,1}R_{3,2}R_{4,3}(163524)C^{(16):1} = R_{1,1}R_{4,3}(163524)\bar{C}^{(16):1} = \]
\[ R_{1,1}R_{3,2}R_{4,3}R_{5,5}(163524)E^{(16):1} = R_{1,1}R_{4,3}R_{5,5}(163524)\bar{E}^{(16):1} = \]
\[ R_{1,4}R_{2,6}F^{(16):1} = R_{1,1}R_{2,6}F^{(16):1} = \]
\[ R_{1,1}R_{2,6}R_{5,5}H^{(16):1} = R_{1,1}R_{2,6}R_{5,5}H^{(16):1} = \]
\[ R_{1,4}R_{3,6}R_{5,5}(465)K^{(16)} = R_{1,4}R_{3,2}R_{4,3}(163524)K^{(16):1} = \]
\[ R_{1,4}R_{3,2}R_{4,3}R_{5,5}(465)\bar{K}^{(16)} = R_{1,4}R_{4,3}(163524)\bar{K}^{(16):1} = \]
\[ R_{1,4}R_{2,6}R_{4,3}R_{5,5}L^{(16)} = R_{1,1}R_{2,6}R_{4,3}R_{5,5}L^{(16):1} = \]
\[ R_{1,4}R_{2,6}R_{3,2}R_{4,3}R_{5,5}\bar{L}^{(16)} = R_{1,1}R_{2,6}R_{3,2}R_{4,3}R_{5,5}\bar{L}^{(16):1} = \]
\[ R_{1,4}R_{2,6}R_{4,3}M^{(16)} = R_{1,1}R_{2,6}R_{4,3}M^{(16)}:1 = \]
\[ R_{1,4}R_{2,6}R_{5,2}R_{4,3}M^{(16)} = R_{1,1}R_{2,6}R_{3,2}R_{4,3}M^{(16)}:1 = \]
\[ R_{1,1}R_{2,6}R_{3,2}(14)(25)(36)P^{(16)} = R_{1,4}R_{2,6}R_{3,2}(14)(25)(36)P^{(16)}:1 = \]
\[ R_{1,4}R_{2,6}R_{3,2}R_{5,5}\tilde{P}^{(16)} = R_{1,1}R_{2,6}R_{3,2}R_{5,5}\tilde{P}^{(16)}:1 = \bar{L}^{(2,1^s)} = \]
\[ R_{1,4}R_{3,6}C^{(16)} = R_{1,4}(456)C^{(16)} = \]
\[ R_{1,4}R_{3,6}R_{5,3}E^{(16)} = R_{1,4}R_{5,3}(456)E^{(16)} = \]
\[ R_{1,1}R_{2,2}R_{5,3}(14)(25)(36)F^{(16)} = R_{1,4}R_{2,2}R_{5,3}(14)(25)(36)F^{(16)}:1 = \]
\[ R_{1,1}R_{2,2}(14)(25)(36)\bar{H}^{(16)} = R_{1,4}R_{2,2}(14)(25)(36)\bar{H}^{(16)}:1 = \]
\[ R_{1,4}R_{2,2}R_{3,6}K^{(16)} = R_{1,1}R_{2,2}R_{3,6}K^{(16)}:1 = \]
\[ R_{1,1}R_{2,2}R_{3,6}R_{5,3}(14)(25)(36)\bar{K}^{(16)} = R_{1,4}R_{2,2}R_{3,6}R_{5,3}(14)(25)(36)\bar{K}^{(16)}:1 = \]
\[ R_{1,4}R_{2,2}R_{3,6}R_{4,5}L^{(16)} = R_{1,1}R_{2,2}R_{3,6}R_{4,5}L^{(16)}:1 = \]
\[ R_{1,4}R_{2,2}R_{4,5}\bar{L}^{(16)} = R_{1,1}R_{2,2}R_{4,5}\bar{L}^{(16)}:1 = \]
\[ R_{1,4}R_{2,2}R_{3,6}R_{4,5}R_{5,3}M^{(16)} = R_{1,1}R_{2,2}R_{3,6}R_{4,5}R_{5,3}M^{(16)}:1 = \]
\[ R_{1,4}R_{2,2}R_{4,5}R_{5,3}\tilde{M}^{(16)} = R_{1,1}R_{2,2}R_{4,5}R_{5,3}\tilde{M}^{(16)}:1 = \]
\[ R_{1,1}R_{4,5}R_{5,3}(142536)P^{(16)} = R_{1,1}R_{3,6}R_{4,5}(123)P^{(16)}:1 = \]
\[ R_{1,1}R_{3,6}R_{4,5}R_{5,3}(142536)\tilde{P}^{(16)} = R_{1,1}R_{4,5}(123)\tilde{P}^{(16)}:1 = M^{(2,1^s)}, \]
\[ R_{1,1}R_{3,2}R_{5,3}(163524)\bar{A}^{(16)}:1 = R_{1,1}R_{3,2}(163524)\bar{B}^{(16)}:1 = \]
\[ R_{1,1}R_{3,2}R_{4,5}R_{5,3}(163524)C^{(16)}:1 = R_{1,1}R_{4,5}R_{5,3}(163524)C^{(16)}:1 = \]
\[ R_{1,1}R_{3,2}R_{4,5}(163524)E^{(16)}:1 = R_{1,1}R_{4,5}(163524)E^{(16)}:1 = \]
\[ R_{1,4}R_{3,2}R_{5,3}(163524)F^{(16)}:1 = R_{1,1}R_{2,6}R_{3,2}R_{5,3}(14)(25)(36)F^{(16)} = \]
\[ R_{1,4}R_{2,6}R_{3,2}R_{5,3}(14)(25)(36)F^{(16)}:1 = R_{1,4}R_{3,2}(163524)\bar{H}^{(16)}:1 = \]
\[ R_{1,1}R_{2,6}R_{3,2}(14)(25)(36)\bar{H}^{(16)} = R_{1,4}R_{2,6}R_{3,2}(14)(25)(36)\bar{H}^{(16)}:1 = \]
\[ R_{1,4}R_{2,6}R_{3,2}(14)(25)(36)\bar{K}^{(16)} = R_{1,4}R_{2,6}R_{3,2}(14)(25)(36)\bar{K}^{(16)}:1 = \]
\[ R_{1,4}R_{2,6}R_{4,5}L^{(16)} = R_{1,1}R_{2,6}R_{4,5}L^{(16)}:1 = \]
\[ R_{1,4}R_{2,6}R_{3,2}R_{4,5}\bar{L}^{(16)} = R_{1,1}R_{2,6}R_{3,2}R_{4,5}\bar{L}^{(16)}:1 = \]
\[ R_{1,4}R_{2,6}R_{4,5}R_{5,3}M^{(16)} = R_{1,1}R_{2,6}R_{4,5}R_{5,3}M^{(16)}:1 = \]
$R_{1,4}R_{2,6}R_{3,2}R_{4,5}R_{5,3} \tilde{M}^{(1^6)} = R_{1,1}R_{2,6}R_{3,2}R_{4,5}R_{5,3} \tilde{M}^{(1)} = \tilde{M}^{(2,1^4)},$

$R_{1,4}R_{3,6}(132)\tilde{A}^{(1^6)}:1 = R_{1,4}R_{3,6}R_{5,5}(132)\tilde{B}^{(1^6)}:1 = \cdots$

$R_{1,4}R_{4,2}R_{5,5}(132)(465)C^{(1^6)}:1 = R_{1,4}R_{3,6}R_{4,2}R_{5,5}(132)(465)C^{(1)}:1 = \cdots$

$R_{1,4}R_{4,2}(132)(465)E^{(1^6)}:1 = R_{1,4}R_{3,6}R_{4,2}(132)(465)E^{(1^6)}:1 = \cdots$

$R_{1,1}R_{3,6}(132)F^{(1^6)}:1 = R_{1,4}R_{2,3}R_{3,6}\tilde{F}^{(1^6)} = \cdots$

$R_{1,4}R_{2,3}R_{3,6}R_{4,2}R_{5,5}P^{(1^6)} = R_{1,1}R_{2,3}R_{3,6}R_{4,2}R_{5,5}P^{(1^6)}:1 = \cdots$

$R_{1,4}R_{2,3}R_{4,2}R_{5,5}\tilde{P}^{(1^6)} = R_{1,1}R_{2,3}R_{4,2}R_{5,5}\tilde{P}^{(1^6)}:1 = \tilde{P}^{(2,1^4)},$

$R_{1,1}R_{3,3}(163524)C^{(1^6)}:1 = R_{1,1}(16)(24)(35)C^{(1^6)}:1 = \cdots$

$R_{1,1}R_{3,3}R_{5,5}(163524)E^{(1^6)}:1 = R_{1,1}R_{5,5}(16)(24)(35)E^{(1^6)}:1 = \cdots$

$R_{1,4}R_{2,6}\tilde{F}^{(1^6)} = R_{1,1}R_{2,6}\tilde{F}^{(1^6)}:1 = \cdots$

$R_{1,4}R_{2,3}R_{5,5}H^{(1^6)} = R_{1,1}R_{2,6}R_{5,5}H^{(1^6)}:1 = \cdots$

$R_{1,4}R_{3,3}(163524)K^{(1^6)}:1 = R_{1,4}R_{3,3}R_{5,5}(465)K^{(1^6)} = \cdots$

$R_{1,4}R_{3,3}R_{4,2}(163524)L^{(1^6)}:1 = R_{1,4}R_{2,6}R_{3,3}R_{5,5}L^{(1^6)} = \cdots$

$R_{1,1}R_{2,6}R_{3,3}R_{5,5}L^{(1^6)}:1 = R_{1,4}R_{3,3}R_{4,2}R_{5,5}(163524)M^{(1^6)}:1 = \cdots$

$R_{1,4}R_{2,6}R_{3,3}\tilde{M}^{(1^6)} = R_{1,1}R_{2,6}R_{3,3}\tilde{M}^{(1^6)}:1 = \cdots$

$R_{1,4}R_{2,6}R_{4,2}R_{5,5}P^{(1^6)} = R_{1,1}R_{2,6}R_{4,2}R_{5,5}P^{(1^6)}:1 = \tilde{P}^{(2,1^4)},$

$R_{1,4}R_{2,6}R_{3,3}R_{4,2}R_{5,5}\tilde{P}^{(1^6)} = R_{1,1}R_{2,6}R_{3,3}R_{4,2}R_{5,5}\tilde{P}^{(1^6)}:1 = \tilde{P}^{(2,1^4)},$

These relations yield the following substitution reactions between $(1^6)$-derivatives and $(2,1^4)$-derivatives of ethane:

$a_{(1^6)} < a_{(2,1^4)}, a^{1}_{(1^6)} < a_{(2,1^4)}, a^{1}_{(1^6)} < a_{(2,1^4)}, a^{1}_{(1^6)} < a_{(2,1^4)},$

$b_{(1^6)} < a_{(2,1^4)}, b^{1}_{(1^6)} < a_{(2,1^4)}, b^{1}_{(1^6)} < a_{(2,1^4)}, b^{1}_{(1^6)} < a_{(2,1^4)},$

$c_{(1^6)} < a_{(2,1^4)}, c^{1}_{(1^6)} < a_{(2,1^4)}, e^{1}_{(1^6)} < a_{(2,1^4)}, e^{1}_{(1^6)} < a_{(2,1^4)},$
\[
\begin{align*}
\alpha_{(16)} &< \tilde{a}_{(2,14)}, \quad \alpha_{(16)}^1 < \tilde{a}_{(2,14)}, \quad \tilde{a}_{(16)} < \tilde{a}_{(2,14)}, \quad \tilde{a}_{(16)}^1 < \tilde{a}_{(2,14)}, \\
\beta_{(16)} &< \tilde{b}_{(2,14)}, \quad \beta_{(16)}^1 < \tilde{b}_{(2,14)}, \quad \tilde{b}_{(16)} < \tilde{b}_{(2,14)}, \quad \tilde{b}_{(16)}^1 < \tilde{b}_{(2,14)}, \\
\gamma_{(16)} &< \tilde{c}_{(2,14)}, \quad \gamma_{(16)}^1 < \tilde{c}_{(2,14)}, \quad \tilde{c}_{(16)} < \tilde{c}_{(2,14)}, \quad \tilde{c}_{(16)}^1 < \tilde{c}_{(2,14)}, \\
\delta_{(16)} &< \tilde{d}_{(2,14)}, \quad \delta_{(16)}^1 < \tilde{d}_{(2,14)}, \quad \tilde{d}_{(16)} < \tilde{d}_{(2,14)}, \quad \tilde{d}_{(16)}^1 < \tilde{d}_{(2,14)}, \\
&\vdots \\
\end{align*}
\]
\[ \tilde{k}(16) < e(2,14), \tilde{\ell}(16) < e(2,14), \ell(16) < e(2,14), \ell(16) < e(2,14), \]
\[ \tilde{\ell}(16) < e(2,14), m(16) < e(2,14), m(16) < e(2,14), \tilde{m}(16) < e(2,14), \]
\[ p(16) < e(2,14), p(16) < e(2,14), \]
\[ c(16) < \tilde{e}(2,14), c(16) < \tilde{e}(2,14), \tilde{c}(16) < \tilde{e}(2,14), \tilde{c}(16) < \tilde{e}(2,14), \]
\[ e(16) < \tilde{e}(2,14), e(16) < \tilde{e}(2,14), \tilde{e}(16) < \tilde{e}(2,14), \tilde{e}(16) < \tilde{e}(2,14), \]
\[ f(16) < \tilde{e}(2,14), \tilde{h}(16) < \tilde{e}(2,14), k(16) < \tilde{e}(2,14), \tilde{h}(16) < \tilde{e}(2,14), \]
\[ \ell(16) < \tilde{e}(2,14), \ell(16) < \tilde{e}(2,14), \tilde{\ell}(16) < \tilde{e}(2,14), \tilde{\ell}(16) < \tilde{e}(2,14), m(16) < \tilde{e}(2,14), \]
\[ \tilde{m}(16) < \tilde{e}(2,14), \tilde{m}(16) < \tilde{e}(2,14), p(16) < \tilde{e}(2,14), p(16) < \tilde{e}(2,14), \]
\[ a(16) < f(2,14), a(16) < f(2,14), b(16) < f(2,14), b(16) < f(2,14), \]
\[ c(16) < f(2,14), c(16) < f(2,14), f(16) < f(2,14), f(16) < f(2,14), \]
\[ f(16) < f(2,14), f(16) < f(2,14), f(16) < f(2,14), f(16) < f(2,14), \]
\[ h(16) < f(2,14), h(16) < f(2,14), h(16) < f(2,14), h(16) < f(2,14), \]
\[ \tilde{h}(16) < f(2,14), \tilde{h}(16) < f(2,14), h(16) < f(2,14), h(16) < f(2,14), \]
\[ k(16) < f(2,14), k(16) < f(2,14), k(16) < f(2,14), k(16) < f(2,14), \]
\[ \tilde{k}(16) < f(2,14), \tilde{k}(16) < f(2,14), \tilde{k}(16) < f(2,14), \tilde{k}(16) < f(2,14), \]
\[ \tilde{m}(16) < f(2,14), \tilde{m}(16) < f(2,14), \tilde{m}(16) < f(2,14), \tilde{m}(16) < f(2,14), \]
\[ a(16) < h(2,14), a(16) < h(2,14), b(16) < h(2,14), b(16) < h(2,14), \]
\[ c(16) < h(2,14), c(16) < h(2,14), f(16) < h(2,14), f(16) < h(2,14), \]
\[ f(16) < h(2,14), f(16) < h(2,14), f(16) < h(2,14), f(16) < h(2,14), \]
\[ h(16) < h(2,14), h(16) < h(2,14), h(16) < h(2,14), h(16) < h(2,14), \]
\[ \tilde{h}(16) < h(2,14), \tilde{h}(16) < h(2,14), h(16) < h(2,14), h(16) < h(2,14), \]
\[ \ell(16) < h(2,14), \ell(16) < h(2,14), \ell(16) < h(2,14), \ell(16) < h(2,14), \]
\[ m(16) < h(2,14), m(16) < h(2,14), m(16) < h(2,14), m(16) < h(2,14), \]
\[ \tilde{p}(16) < h(2,14), \tilde{p}(16) < h(2,14), \tilde{p}(16) < h(2,14), \tilde{p}(16) < h(2,14), \]
\[ a(16) < \tilde{h}(2,14), a(16) < \tilde{h}(2,14), b(16) < \tilde{h}(2,14), b(16) < \tilde{h}(2,14), \]
\[ c(16) < \tilde{h}(2,14), c(16) < \tilde{h}(2,14), f(16) < \tilde{h}(2,14), f(16) < \tilde{h}(2,14), \]
\[ f(16) < \tilde{h}(2,14), f(16) < \tilde{h}(2,14), f(16) < \tilde{h}(2,14), f(16) < \tilde{h}(2,14), \]
\[ h(16) < \tilde{h}(2,14), h(16) < \tilde{h}(2,14), h(16) < \tilde{h}(2,14), h(16) < \tilde{h}(2,14), \]
\[ \tilde{h}(16) < \tilde{h}(2,14), \tilde{h}(16) < \tilde{h}(2,14), \tilde{h}(16) < \tilde{h}(2,14), \tilde{h}(16) < \tilde{h}(2,14), \]
\[ m(16) < \tilde{h}(2,14), m(16) < \tilde{h}(2,14), m(16) < \tilde{h}(2,14), m(16) < \tilde{h}(2,14), \]
\[ \tilde{p}(16) < \tilde{h}(2,14), \tilde{p}(16) < \tilde{h}(2,14), \tilde{p}(16) < \tilde{h}(2,14), \tilde{p}(16) < \tilde{h}(2,14), \]
\[\tilde{c}_{(16)}^1 < \tilde{h}_{(2,1^4)}, \tilde{e}_{(16)}^1 < \tilde{h}_{(2,1^4)}, f_{(16)} < \tilde{h}_{(2,1^4)}, f_{(16)}^1 < \tilde{h}_{(2,1^4)},
\]
\[\bar{f}_{(16)} < \tilde{h}_{(2,1^4)}, \bar{f}_{(16)}^1 < \tilde{h}_{(2,1^4)}, h_{(16)} < \tilde{h}_{(2,1^4)}, h_{(16)}^1 < \tilde{h}_{(2,1^4)},
\]
\[\bar{h}_{(16)} < \tilde{h}_{(2,1^4)}, \bar{h}_{(16)}^1 < \tilde{h}_{(2,1^4)}, \ell_{(16)} < \tilde{h}_{(2,1^4)}, \ell_{(16)}^1 < \tilde{h}_{(2,1^4)},
\]
\[m_{(16)} < \tilde{h}_{(2,1^4)}, m_{(16)}^1 < \tilde{h}_{(2,1^4)}, p_{(16)} < \tilde{h}_{(2,1^4)}, p_{(16)}^1 < \tilde{h}_{(2,1^4)}, \]
\[\bar{p}_{(16)} < \tilde{h}_{(2,1^4)}, \bar{p}_{(16)}^1 < \tilde{h}_{(2,1^4)},
\]
\[\bar{a}_{(16)} < k_{(2,1^4)}, \bar{h}_{(16)} < k_{(2,1^4)}, c_{(16)} < k_{(2,1^4)}, \bar{c}_{(16)} < k_{(2,1^4)},
\]
\[e_{(16)} < k_{(2,1^4)}, \bar{e}_{(16)} < k_{(2,1^4)}, f_{(16)} < k_{(2,1^4)}, f_{(16)}^1 < k_{(2,1^4)},
\]
\[\bar{f}_{(16)} < k_{(2,1^4)}, h_{(16)} < k_{(2,1^4)}, h_{(16)}^1 < k_{(2,1^4)}, \bar{h}_{(16)} < k_{(2,1^4)},
\]
\[k_{(16)} < k_{(2,1^4)}, k_{(16)}^1 < k_{(2,1^4)}, \bar{k}_{(16)} < k_{(2,1^4)}, \bar{k}_{(16)}^1 < k_{(2,1^4)},
\]
\[\ell_{(16)} < k_{(2,1^4)}, \ell_{(16)}^1 < k_{(2,1^4)}, \bar{\ell}_{(16)} < k_{(2,1^4)}, m_{(16)} < k_{(2,1^4)},
\]
\[m_{(16)}^1 < k_{(2,1^4)}, m_{(16)}^1 < k_{(2,1^4)}, c_{(16)} < k_{(2,1^4)}, \bar{c}_{(16)} < k_{(2,1^4)},
\]
\[f_{(16)} < k_{(2,1^4)}, f_{(16)}^1 < k_{(2,1^4)}, h_{(16)} < k_{(2,1^4)}, h_{(16)}^1 < k_{(2,1^4)},
\]
\[k_{(16)} < k_{(2,1^4)}, k_{(16)}^1 < k_{(2,1^4)}, \bar{k}_{(16)} < k_{(2,1^4)}, \bar{k}_{(16)}^1 < k_{(2,1^4)},
\]
\[\ell_{(16)} < k_{(2,1^4)}, \ell_{(16)}^1 < k_{(2,1^4)}, \bar{\ell}_{(16)} < k_{(2,1^4)}, m_{(16)} < k_{(2,1^4)},
\]
\[m_{(16)}^1 < k_{(2,1^4)}, m_{(16)}^1 < k_{(2,1^4)}, \bar{m}_{(16)} < k_{(2,1^4)}, \bar{m}_{(16)}^1 < k_{(2,1^4)}, \]
\[p_{(16)} < \ell_{(2,1^4)}, p_{(16)}^1 < \ell_{(2,1^4)}, \bar{p}_{(16)}^1 < \ell_{(2,1^4)}, \bar{p}_{(16)}^1 < \ell_{(2,1^4)},
\]
\[\tilde{c}_{(16)} < \ell_{(2,1^4)}, \tilde{e}_{(16)} < \ell_{(2,1^4)}, \bar{e}_{(16)} < \ell_{(2,1^4)}, \bar{e}_{(16)} < \ell_{(2,1^4)},
\]
\[f_{(16)} < \ell_{(2,1^4)}, f_{(16)}^1 < \ell_{(2,1^4)}, h_{(16)} < \ell_{(2,1^4)}, h_{(16)}^1 < \ell_{(2,1^4)},
\]
\[k_{(16)} < \ell_{(2,1^4)}, k_{(16)}^1 < \ell_{(2,1^4)}, \bar{k}_{(16)} < \ell_{(2,1^4)}, \bar{k}_{(16)}^1 < \ell_{(2,1^4)},
\]
\[\ell_{(16)} < \ell_{(2,1^4)}, \ell_{(16)}^1 < \ell_{(2,1^4)}, \bar{\ell}_{(16)} < \ell_{(2,1^4)}, \bar{\ell}_{(16)}^1 < \ell_{(2,1^4)},
\]
\[m_{(16)} < \ell_{(2,1^4)}, m_{(16)}^1 < \ell_{(2,1^4)}, \bar{m}_{(16)} < \ell_{(2,1^4)}, \bar{m}_{(16)}^1 < \ell_{(2,1^4)}, \]
\[p_{(16)} < \ell_{(2,1^4)}, p_{(16)}^1 < \ell_{(2,1^4)}, \bar{p}_{(16)} < \ell_{(2,1^4)}, \bar{p}_{(16)}^1 < \ell_{(2,1^4)},
\]
\[
c_{(16)} < m_{(2,14)}, \tilde{c}_{(16)} < m_{(2,14)}, e_{(16)} < m_{(2,14)}, \tilde{e}_{(16)} < m_{(2,14)}, \]
\[
\tilde{f}_{(16)} < m_{(2,14)}, \tilde{f}_{(16)}^{1} < m_{(2,14)}, \tilde{h}_{(16)} < m_{(2,14)}, \tilde{h}_{(16)}^{1} < m_{(2,14)}, \]
\[
k_{(16)} < m_{(2,14)}, k_{(16)}^{1} < m_{(2,14)}, \tilde{k}_{(16)} < m_{(2,14)}, \tilde{k}_{(16)}^{1} < m_{(2,14)}, \]
\[
\ell_{(16)} < m_{(2,14)}, \ell_{(16)}^{1} < m_{(2,14)}, \tilde{\ell}_{(16)} < m_{(2,14)}, \tilde{\ell}_{(16)}^{1} < m_{(2,14)}, \]
\[
m_{(16)} < m_{(2,14)}, m_{(16)}^{1} < m_{(2,14)}, \bar{m}_{(16)} < m_{(2,14)}, \bar{m}_{(16)}^{1} < m_{(2,14)}, \]
\[
p_{(16)} < m_{(2,14)}, p_{(16)}^{1} < m_{(2,14)}, \tilde{p}_{(16)} < m_{(2,14)}, \tilde{p}_{(16)}^{1} < m_{(2,14)}, \]
\[
\tilde{a}_{(16)} < m_{(2,14)}, \tilde{b}_{(16)} < m_{(2,14)}, c_{(16)}^{1} < m_{(2,14)}, \tilde{c}_{(16)}^{1} < m_{(2,14)}, \]
\[
e_{(16)} < m_{(2,14)}, \tilde{e}_{(16)} < m_{(2,14)}, \tilde{f}_{(16)}^{1} < m_{(2,14)}, \tilde{f}_{(16)} < m_{(2,14)}, \]
\[
\tilde{f}_{(16)}^{1} < m_{(2,14)}, \tilde{h}_{(16)} < m_{(2,14)}, \tilde{h}_{(16)} < m_{(2,14)}, \tilde{h}_{(16)}^{1} < m_{(2,14)}, \]
\[
k_{(16)} < m_{(2,14)}, k_{(16)}^{1} < m_{(2,14)}, \tilde{k}_{(16)} < m_{(2,14)}, \tilde{k}_{(16)}^{1} < m_{(2,14)}, \]
\[
\ell_{(16)} < m_{(2,14)}, \ell_{(16)}^{1} < m_{(2,14)}, \tilde{\ell}_{(16)} < m_{(2,14)}, \tilde{\ell}_{(16)}^{1} < m_{(2,14)}, \]
\[
m_{(16)} < m_{(2,14)}, m_{(16)}^{1} < m_{(2,14)}, \bar{m}_{(16)} < m_{(2,14)}, \bar{m}_{(16)}^{1} < m_{(2,14)}, \]
\[
\bar{a}_{(16)} < p_{(2,14)}, \tilde{b}_{(16)}^{1} < p_{(2,14)}, c_{(16)}^{1} < p_{(2,14)}, \tilde{c}_{(16)}^{1} < p_{(2,14)}, \]
\[
e_{(16)} < p_{(2,14)}, \tilde{e}_{(16)}^{1} < p_{(2,14)}, \tilde{f}_{(16)}^{1} < p_{(2,14)}, \tilde{f}_{(16)} < p_{(2,14)}, \]
\[
\tilde{f}_{(16)}^{1} < p_{(2,14)}, \tilde{h}_{(16)} < p_{(2,14)}, \tilde{h}_{(16)} < p_{(2,14)}, \tilde{h}_{(16)}^{1} < p_{(2,14)}, \]
\[
\ell_{(16)} < p_{(2,14)}, \tilde{\ell}_{(16)} < p_{(2,14)}, \tilde{\ell}_{(16)}^{1} < p_{(2,14)}, \ell_{(16)}^{1} < p_{(2,14)}, \]
\[
m_{(16)} < p_{(2,14)}, m_{(16)}^{1} < p_{(2,14)}, \bar{m}_{(16)} < p_{(2,14)}, \bar{m}_{(16)}^{1} < p_{(2,14)}, \]
\[
\bar{p}_{(16)} < p_{(2,14)}, \tilde{p}_{(16)}^{1} < p_{(2,14)}, \]
\[
c_{(16)} < \bar{p}_{(2,14)}, \tilde{c}_{(16)}^{1} < \bar{p}_{(2,14)}, \tilde{e}_{(16)}^{1} < \bar{p}_{(2,14)}, \tilde{e}_{(16)}^{1} < \bar{p}_{(2,14)}, \]
\[
\tilde{f}_{(16)} < \bar{p}_{(2,14)}, \tilde{f}_{(16)}^{1} < \bar{p}_{(2,14)}, \tilde{h}_{(16)} < \bar{p}_{(2,14)}, \tilde{h}_{(16)}^{1} < \bar{p}_{(2,14)}, \]
\[
k_{(16)} < \bar{p}_{(2,14)}, \tilde{k}_{(16)} < \bar{p}_{(2,14)}, \ell_{(16)} < \bar{p}_{(2,14)}, \tilde{\ell}_{(16)} < \bar{p}_{(2,14)}, \]
\[
m_{(16)} < \bar{p}_{(2,14)}, \bar{m}_{(16)}^{1} < \bar{p}_{(2,14)}, \tilde{m}_{(16)} < \bar{p}_{(2,14)}, \tilde{m}_{(16)}^{1} < \bar{p}_{(2,14)}, \]
\[
p_{(16)} < \bar{p}_{(2,14)}, \bar{p}_{(16)}^{1} < \bar{p}_{(2,14)}, \tilde{p}_{(16)} < \bar{p}_{(2,14)}, \tilde{p}_{(16)}^{1} < \bar{p}_{(2,14)}. \]

The set of all \(G^t\)-orbits is
\[
T_{(16):G^t} = \{ a_{(16)} \cup b_{(16)}^{1}, a_{(16)}^{1} \cup b_{(16)}, \tilde{a}_{(16)} \cup \tilde{b}_{(16)}, \bar{a}_{(16)} \cup \tilde{b}_{(16)}, \bar{a}_{(16)}^{1} \cup c_{(16)}^{1}, \bar{a}_{(16)}^{1} \cup \bar{c}_{(16)}, c_{(16)}^{1} \cup \bar{e}_{(16)}, \bar{c}_{(16)} \cup c_{(16)}^{1}, \bar{c}_{(16)} \cup \bar{e}_{(16)}, c_{(16)}^{1} \cup \bar{e}_{(16)}, \bar{c}_{(16)} \cup e_{(16)}, e_{(16)}^{1} \cup \bar{e}_{(16)}, e_{(16)}^{1} \cup e_{(16)}, f_{(16)} \cup \bar{h}_{(16)}, f_{(16)}^{1} \cup \bar{h}_{(16)}, \tilde{f}_{(16)} \cup \bar{h}_{(16)}, \tilde{f}_{(16)}^{1} \cup h_{(16)}, k_{(16)} \cup p_{(16)}, k_{(16)}^{1} \cup p_{(16)}, \tilde{k}_{(16)} \cup \tilde{p}_{(16)}, \tilde{k}_{(16)}^{1} \cup \tilde{p}_{(16)}, \ell_{(16)} \cup \bar{m}_{(16)}, \ell_{(16)}^{1} \cup \bar{m}_{(16)}, \bar{m}_{(16)} \cup \ell_{(16)}, \ell_{(16)}^{1} \cup \bar{m}_{(16)}, \bar{m}_{(16)}^{1} \cup \ell_{(16)}, \bar{m}_{(16)}^{1} \cup \ell_{(16)}^{1} \cup m_{(16)} \}.
\]
Therefore \((1^6)\)-derivatives of ethane that correspond to the members of the two-element sets
\[
\{a(1^6), b(1^6)\}, \{a(1^6), a(1^6)\}, \{\bar{a}(1^6), \bar{a}(1^6)\}, \{\bar{a}(1^6), b(1^6)\}, \{c(1^6), c(1^6)\},
\]
\[
\{e(1^6), e(1^6)\}, \{e(1^6), \bar{e}(1^6)\}, \{e(1^6), c(1^6)\}, \{f(1^6), \bar{h}(1^6)\}, \{f(1^6), f(1^6)\},
\]
\[
\{f(1^6), h(1^6)\}, \{f(1^6), \bar{f}(1^6)\}, \{f(1^6), h(1^6)\}, \{f(1^6), \bar{f}(1^6)\},
\]
\[
\{k(1^6), p(1^6)\}, \{k(1^6), \bar{p}(1^6)\}, \{k(1^6), \bar{e}(1^6)\}, \{k(1^6), e(1^6)\}, \{k(1^6), \bar{e}(1^6)\}, \{\bar{e}(1^6), m(1^6)\}, \{\bar{e}(1^6), \bar{m}(1^6)\}, \{\bar{e}(1^6), m(1^6)\}, \{\bar{e}(1^6), \bar{m}(1^6)\}
\]
form chiral pairs.

The set of all \(G''\)-orbits is
\[
T_{1(6)G''} =
\{a(1^6) \cup b(1^6) \cup a(1^6) \cup b(1^6), a(1^6) \cup \bar{a}(1^6) \cup a(1^6) \cup b(1^6), c(1^6) \cup \bar{c}(1^6) \cup c(1^6) \cup \bar{c}(1^6),
\]
\[
e(1^6) \cup e(1^6) \cup e(1^6) \cup e(1^6), f(1^6) \cup \bar{h}(1^6) \cup f(1^6) \cup h(1^6), f(1^6) \cup \bar{f}(1^6) \cup f(1^6) \cup h(1^6),
\]
\[
k(1^6) \cup p(1^6), \cup \ell(1^6) \cup \bar{m}(1^6) \cup e(1^6) \cup \bar{e}(1^6) \cup m(1^6), \bar{k}(1^6) \cup \bar{p}(1^6) \cup \bar{e}(1^6) \cup \bar{m}(1^6),
\]
\[
\bar{k}(1^6) \cup \bar{p}(1^6) \cup \bar{e}(1^6) \cup m(1^6)\}\}. \quad (2.26)
\]

Thus, the products that correspond to members of different sets below are structural isomers whereas those that correspond to elements of one and the same set are structurally identical:
\[
\{a(1^6), b(1^6), a(1^6), b(1^6)\}, \{\bar{a}(1^6), \bar{b}(1^6), \bar{a}(1^6), \bar{b}(1^6)\}, \{c(1^6), \bar{c}(1^6), c(1^6), \bar{c}(1^6)\},
\]
\[
\{e(1^6), e(1^6), e(1^6), e(1^6)\}, \{f(1^6), \bar{h}(1^6), f(1^6), h(1^6)\}, \{f(1^6), \bar{f}(1^6), f(1^6), h(1^6)\},
\]
\[
\{k(1^6), p(1^6), \ell(1^6), \bar{m}(1^6)\}, \{k(1^6), p(1^6), \ell(1^6), m(1^6)\}, \{k(1^6), \bar{p}(1^6), \bar{e}(1^6), m(1^6)\}, \{k(1^6), \bar{p}(1^6), \bar{e}(1^6), m(1^6)\}, \{\bar{k}(1^6), \bar{p}(1^6), \bar{e}(1^6), m(1^6)\}, \{\bar{k}(1^6), \bar{p}(1^6), \bar{e}(1^6), m(1^6)\}.
\]
\[
\{\bar{k}(1^6), \bar{p}(1^6), \bar{e}(1^6), \bar{m}(1^6)\}, \{\bar{k}(1^6), \bar{p}(1^6), \bar{e}(1^6), \bar{m}(1^6)\}. \quad (2.27)
\]

3. Identification of the derivatives

Here our objective is to find the Lunn-Senior’s automorphism groups \(Aut''_0(T_{D,G})\) for \(D = D_k\), \(k = 1, \ldots, 8\), where
\[
D_1 = \{(6), (5, 1), (4, 2)\}, \quad D_2 = \{(6), (5, 1), (4, 2), (4, 1^2)\},
\]
\[
D_3 = \{(6), (5, 1), (4, 2), (4, 1^2), (3^2)\}, \quad D_4 = \{(6), (5, 1), (4, 2), (4, 1^2), (3^2), (3, 2, 1)\},
\]
\[
D_5 = \{(6), (5, 1), (4, 2), (4, 1^2), (3^2), (3, 2, 1), (3, 1^3)\},
\]
\[
D_6 = \{(6), (5, 1), (4, 2), (4, 1^2), (3^2), (3, 2, 1), (3, 1^3), (2^3)\},
\]
\[
D_7 = \{(6), (5, 1), (4, 2), (4, 1^2), (3^2), (3, 2, 1), (3, 1^3), (2^3), (2^2, 1^2)\},
\]
\[
D_8 = \{(6), (5, 1), (4, 2), (4, 1^2), (3^2), (3, 2, 1), (3, 1^3), (2^3), (2^2, 1^2), (2, 1^4)\},
\]
and for \(D = P_6\), as well as for \(D = \{(2^2), (2^2, 1^2)\}\), and \(D = \{(2^2, 1^2), (2, 1^4)\}\). Then the elements of the \(Aut''_0(T_{D,G})\)-orbits in the set \(T_{D,G}\) will represent the products of ethene that can not be distinguished via substitution reactions among the elements of \(T_{D,G}\).
In a partially ordered set $V$ we denote for short the set of all $x \in V$ that are strictly greater than a fixed element $a \in V$ by $C_a(V; a)$, and call it the (right) cone of $a$ in $V$. Let $M$ and $W$ be subsets of $V$ and suppose that $M$ consists of minimal elements of $V$. We say that $W$ is a barrier of $M$ in $V$ if for any $m \in M$ and for any $b \in C_a(V; m)$ there exists $c \in C_a(W; m)$ such that $b \geq c$.

Let $V, X \subseteq T_{d,G}$ be unions of $G''$-orbits, $X \subseteq V$. The set of cones $\{ C_a(V; a) \mid a \in X \}$ is invariant with respect to the natural action of the group $Aut_0''(V)$ on the set of all subsets of $V$ because $\alpha(C_a(V; a)) = C_a(V; \alpha(a))$ and $\alpha(a) \in X$ for all $a \in X$. Thus, the rule $\alpha C_a(V; a) = C_a(V; \alpha(a))$ defines an action of the group $Aut_0''(V)$ on the set $\{ C_a(V; a) \mid a \in X \}$.

We begin with a simple technical lemma.

**Lemma 3.1.** Let $U, V, \bar{V} \subseteq T_{d,G}$ be unions of $G''$-orbits, such that $U \subseteq V$, $V \setminus U \subseteq \bar{V}$, and the difference $\bar{V} \setminus U$ consists of minimal elements of the partially ordered set $V \cup \bar{V}$. Then the following statements hold:

(i) the difference $\bar{V} \setminus U$ consists of minimal elements of the partially ordered set $V$;

(ii) the cones $C_a(V; a), C_a(\bar{V}; a)$, where $a \in V \setminus U$, are subsets of $U$, and $C_a(V; a) = C_a(\bar{V}; a) \cap V$;

(iii) if $\bar{V}$ is a barrier of $V \setminus U$ in $V$, then the map

$$\{ C_a(V; a) \mid a \in V \setminus U \} \rightarrow \{ C_a(\bar{V}; a) \mid a \in V \setminus U \},$$

$$C_a(V; a) \mapsto C_a(\bar{V}; a),$$

is an $Aut_0''(V)$-invariant bijection;

(iv) let $\beta \in Aut_0''(U)$, and let $\alpha_0: V \setminus U \rightarrow V \setminus U$ be a bijection that maps any chiral pair onto a chiral pair and any $G''$-orbit onto itself. Then the resulting bijection $\alpha$ of the set $V$ is an element of the group $Aut_0''(V)$ if and only if $\alpha(C_a(\bar{V}; a)) = C_a(\bar{V}; \alpha(a))$ for all $a \in V \setminus U$.

**Proof:** (i) It is enough to note that $\bar{V} \setminus U = (V \setminus U) \cup (\bar{V} \setminus U)$.

(ii) If $b > a$, $b \in V$, (respectively, $b \in \bar{V}$), then the assumption $b \in V \setminus U$ (respectively, $b \in \bar{V} \setminus U$) leads to contradiction with the minimality of the elements of the set $V \setminus U$ (respectively, $\bar{V} \setminus U$).

(iii) We have

$$\alpha C_a(V; a) = C_a(V; \alpha(a)) \mapsto C_a(\bar{V}; \alpha(a)) = \alpha C_a(\bar{V}; a),$$

for any $a \in V \setminus U$. Suppose that $C_a(\bar{V}; a) = C_a(\bar{V}; a')$, $a, a' \in V \setminus U$, and let $b \in C_a(V; a)$. Since $\bar{V}$ is a barrier of $V \setminus U$ in $V$, there exists $c \in \bar{V}$ with $b \geq c > a$. Then $b \geq a'$ and hence $b \in C_a(V; a')$. Thus, $C_a(V; a) = C_a(\bar{V}; a')$.

(iv) Since the elements of the set $V \setminus U$ are minimal in $V$, all strict inequalities in $V$ have the form $b > a$, where $a, b \in U$, or $b > a$, where $a \in V \setminus U$, $b \in U$. It is enough to prove that for any $a \in V \setminus U$ the equality $\alpha C_a(\bar{V}; a) = C_a(\bar{V}; \alpha(a))$ yields the equality $\alpha C_a(V; a) = C_a(V; \alpha(a))$. Suppose that $b \in C_a(V; a)$. Since $\bar{V}$ is a barrier of $V \setminus U$ in $V$, there exists $c \in \bar{V}$ with $b \geq c > a$. Then $\alpha(c) > \alpha(a)$ and in accord to (i) we have $b, c \in U$, so $\alpha(b) \geq \alpha(c)$. Thus, $\alpha(b) > \alpha(a)$, that is, $\alpha(b) \in C_a(V; \alpha(a))$. Now, let $b' \in C_a(V; \alpha(a))$. Again, there exists $c' \in \bar{V}$ with $b' \geq c' > \alpha(a)$, and hence $c' = \alpha(c)$ for some $c \in C_a(\bar{V}; a)$. Since $b' \in U$, there exists $b \in U$ with $\alpha(b) = b'$, and the inequality $\alpha(b) \geq \alpha(c)$ implies $b \geq c$. Thus, $b \in C_a(V; a)$.
Corollary 3.2. Under the conditions of lemma 3.1 one has:

(i) if $H$ is the subgroup of $Aut''_0(U)$ consisting of all automorphisms $\beta$ that satisfy the equality $\beta(C_{\gamma}(\bar{V}; a)) = C_{\gamma}(\bar{V}; a)$ for each $a \in V \setminus U$, then $H \leq Aut''_0(V)$; if, in addition, $C_{\gamma}(\bar{V}; A) = C_{\gamma}(\bar{V}; B) = \cdots$, and $C_{\gamma}(\bar{V}; A^1) = C_{\gamma}(\bar{V}; B^1) = \cdots$, and if $s$ is a bijection of $V \setminus U$ onto itself, such that $s^2 = id$, and $s$ leaves the sets $\{A, B, \ldots\}$ and $\{A^1, B^1, \ldots\}$ invariant, then $Hs \subset Aut''_0(V)$;

(ii) if $V \setminus U$ consists of several chiral pairs $\{A, A^1\}$, $\{B, B^1\}$, and eventually, of dimers $\Delta$, . . . , if the automorphism $w \in Aut''_0(U)$ is such that $w^2 = id$, $w(C_{\gamma}(\bar{V}; A)) = C_{\gamma}(\bar{V}; A^1)$, $w(C_{\gamma}(\bar{V}; B)) = C_{\gamma}(\bar{V}; B^1)$, . . . , $w$ leaves the cones $C_{\gamma}(\bar{V}; \Delta)$, . . . , of the dimers invariant, and if $s = (A, A^1)(B, B^1)$, . . . , then $Hws \subset Aut''_0(V)$; if, in addition, $C_{\gamma}(\bar{V}; A) = C_{\gamma}(\bar{V}; B) = \cdots$ and $C_{\gamma}(\bar{V}; A^1) = C_{\gamma}(\bar{V}; B^1) = \cdots$, and if $s$ is a bijection of $V \setminus U$ onto itself, such that $s^2 = id$, and $s$ maps the set $\{A, B, \ldots\}$ onto the set $\{A^1, B^1, \ldots\}$, then $Hws \subset Aut''_0(V)$;

(iii) if $V \setminus U$ consists of two chiral pairs $\{A, A^1\}$, $\{B, B^1\}$, and eventually, of dimers $\Delta$, . . . , if the automorphism $w \in Aut''_0(U)$ is such that $w = id$, $w(C_{\gamma}(\bar{V}; A)) = C_{\gamma}(\bar{V}; B)$, $w(C_{\gamma}(\bar{V}; A^1)) = C_{\gamma}(\bar{V}; B^1)$, $w$ leaves the cones $C_{\gamma}(\bar{V}; \Delta)$, . . . , of the dimers invariant, and if $s = (A, A^1)(B, B^1)$, . . . , then $ws \in Aut''_0(V)$.

Proof: Straightforward application of lemma 3.1, (iv).

In lemmas 3.3 – 3.7 below, under the conditions of lemma 3.1, we suppose, in addition, that $V$ is a barrier of $\bar{V} \setminus U$ in $V$, the automorphism group $Aut''_0(U)$ is a commutative 2-group, and set $H = \{\beta \in Aut''_0(U) \mid \beta(C_{\gamma}(\bar{V}; a)) = C_{\gamma}(\bar{V}; a), a \in V \setminus U\}$. Moreover, for any pair $X, V \subset T_{dG}$ of sets that are unions of $G''$-orbits with $X \subset V$ we denote by $I_{V,X}$ the image of the restriction homomorphism

$$\varrho_{V,X}: Aut''_0(V) \rightarrow Aut''_0(X).$$

Lemma 3.3. Suppose that the difference $V \setminus U$ consists of several dimers $\Delta, \Theta$, . . . , and that the cones $C_{\gamma}(\bar{V}; \Delta), C_{\gamma}(\bar{V}; \Theta)$, . . . , are pairwise different and invariant under the action of the group $Aut''_0(U)$. Then the restriction homomorphism $\varrho_{V,U}$ is an isomorphism and its inverse is the extension as identity on $V \setminus U$.

Proof: Corollary 3.2, (i), implies $H \leq Aut''_0(V)$. Conversely, let $\alpha \in Aut''_0(V)$. We have $\alpha(C_{\gamma}(\bar{V}; \Delta)) = C_{\gamma}(\bar{V}; \Delta)$; on the other hand, lemma 3.1, (iv), shows that $\alpha(C_{\gamma}(\bar{V}; \Delta)) = C_{\gamma}(\bar{V}; \alpha(\Delta))$, so $\alpha(\Delta) = \Delta$. By analogy, $\alpha(\Theta) = \Theta$, . . . , that is, $\varrho_{V,X \setminus U}(\alpha) = id$. Thus, $Aut''_0(V) = H$, and the proof is done.

Lemma 3.4. Suppose that the difference $V \setminus U$ consists of several chiral pairs

$$\{A, A^1\}, \{B, B^1\}, \ldots,$$

such that $C_{\gamma}(\bar{V}; A) = C_{\gamma}(\bar{V}; B) = \cdots = P$, $C_{\gamma}(\bar{V}; A^1) = C_{\gamma}(\bar{V}; B^1) = \cdots = P^1$, and there exists a decomposition $I_{V,U} = H \times \langle w \rangle$, where $w(P) = P^1$. Then the following two statements hold:
(i) if the chiral pairs in $V \setminus U$ are $G''$-orbits, then there exists a decomposition
\[ Aut''_0(V) = H \times \langle ws \rangle, \]
where $s = (A, A^1)(B, B^1) \ldots$, the restriction homomorphism $\varphi_{V,U}$ is injective, and $Aut''_0(V)$ is a commutative 2-group.

(ii) if $V \setminus U$ is a $G''$-orbit that consists of two chiral pairs $\{A, A^1\}$, $\{B, B^1\}$, then there exists a decomposition
\[ Aut''_0(V) = H \times \langle z \rangle \times \langle wx \rangle, \]
where $x = (A, A^1)(B, B^1)$, $z = (A, B)(A^1, B^1)$, the restriction homomorphism $\varphi_{V,U}$ has kernel $\{id, z\}$, and $Aut''_0(V)$ is a commutative 2-group.

**Proof:** Since $w \notin H$, $w^2 = id$, and $w(P) = P_1$, we obtain $P \neq P_1$.

(i) In compliance with corollary 3.2, (i), (ii), we have $H \cup Hws \subset Aut''_0(V)$. Now, let $\alpha \in Aut''_0(V)$ with $\varphi_{V,U}(\alpha) \in H$ (respectively, $\varphi_{V,U}(\alpha) \in Hw$). Then lemma 3.1, (iv), yields $\alpha(\{A, B, \ldots\}) = \{A, B, \ldots\}$ and $\alpha(\{A^1, B^1, \ldots\}) = \{A^1, B^1, \ldots\}$ (respectively, $\alpha(\{A, B, \ldots\}) = \{A^1, B^1, \ldots\}$ and $\alpha(\{A^1, B^1, \ldots\}) = \{A^1, B^1, \ldots\}$). Since $\alpha$ maps any chiral pair in $V \setminus U$ onto itself, we get $\varphi_{V,V \setminus U}(\alpha) = id$ (respectively, $\varphi_{V,V \setminus U}(\alpha) = s$). In other words, $\alpha \in H$ (respectively, $\alpha \in Hws$), hence $Aut''_0(V) = H \cup Hws$. Moreover, $(ws)^2 = id$ and the automorphism $ws$ commutes with the elements of $H$, so part (i) is proved.

(ii) Again, by virtue of corollary 3.2, (i), (ii), we have $H \cup Hz \cup Hwx \cup Hwy \subset Aut''_0(V)$, where $y = xz = (A, B^1)(A^1, B)$. Now, suppose that $\alpha \in Aut''_0(V)$ with $\varphi_{V,U}(\alpha) \in H \cup Hz$ (respectively, $\varphi_{V,U}(\alpha) \in Hwx \cup Hwy$). Using lemma 3.1, (iv), we obtain $\alpha(\{A, B\}) = \{A, B\}$, $\alpha(\{A^1, B^1\}) = \{A^1, B^1\}$ (respectively, $\alpha(\{A, B\}) = \{A^1, B^1\}$, $\alpha(\{A^1, B^1\}) = \{A, B\}$). Since $\alpha$ maps any chiral pair in $V \setminus U$ onto a chiral pair, we obtain $\varphi_{V,V \setminus U}(\alpha) = id$ or $\varphi_{V,V \setminus U}(\alpha) = z$ (respectively, $\varphi_{V,V \setminus U}(\alpha) = x$ or $\varphi_{V,V \setminus U}(\alpha) = y$). In other words, $\alpha \in H$ or $\alpha \in Hz$ (respectively, $\alpha \in Hwx$ or $\alpha \in Hwy$), hence $Aut''_0(V) = H \cup Hz \cup Hwx \cup Hwy$. We have $(wx)^2 = id$, the elements of the subgroups $H$ and $\langle z \rangle$ commute, and each one of them commutes with the automorphism $wz$. This finishes the proof of part (ii).

**Lemma 3.5.** Suppose that the difference $V \setminus U$ consists of several chiral pairs
\[ \{A, A^1\}, \{B, B^1\}, \ldots, \]
such that $P = C_\prec (\bar{V}; A) = C_\prec (\bar{V}; A^1), Q = C_\prec (\bar{V}; B) = C_\prec (\bar{V}; B^1), \ldots$, and the cones $P, Q, \ldots$, are pairwise different and invariant under the action of the group $Aut''_0(U)$. Then there exists a decomposition
\[ Aut''_0(V) = Aut''_0(U) \times \langle s \rangle \times \langle t \rangle \cdots, \]
where $s = (A, A^1)$, $t = (B, B^1), \ldots$, the restriction homomorphism $\varphi_{V,U}$ coincides with the first projection, and $Aut''_0(V)$ is a commutative 2-group.

**Proof:** After the condition, $H = Aut''_0(U)$. Corollary 3.2, (i), (ii) yield that $H \times \langle s \rangle \times \langle t \rangle \times \cdots \leq Aut''_0(V)$. Now, take an $\alpha \in Aut''_0(V)$. Since $\varphi_{V,U}(\alpha) \in H$, and since the cones $P, Q, \ldots$, are pairwise different, lemma 3.1, (iv), implies $\alpha(\{A, A^1\}) = \{A, A^1\}$, $\alpha(\{B, B^1\}) = \{B, B^1\}, \ldots$, so $\alpha \in H \times \langle s \rangle \times \langle t \rangle \times \cdots$, and the above decomposition holds. Moreover, $s^2 = id, t^2 = id, \ldots$, and the automorphisms $s, t, \ldots$, commute with the elements of $H$ and among themselves. Thus, our lemma is proved.
Lemma 3.6. Let the difference $V \setminus U$ be a $G''$-orbit that consists of two chiral pairs \{A, A^1\}, \{B, B^1\}. Suppose that the cones

$$C_>(V; A) = P, C_>(V; A^1) = P^1, C_>(V; B) = Q, C_>(V; B^1) = Q^1,$$

are pairwise different, and let $x = (A, A^1)(B, B^1)$, $z = (A, B)(A^1, B^1)$.

(i) If there exists a decomposition

$$I_{V,U} = H \times \langle u \rangle$$

where $u(P) = P^1, u(Q) = Q^1$, then there exists a decomposition

$$Aut''_0(V) = H \times \langle ux \rangle,$$

the restriction homomorphism $\varrho_{V,U}$ is injective, and $Aut''_0(V)$ is a commutative 2-group;

(ii) if there exists a decomposition

$$I_{V,U} = H \times \langle u, w \rangle$$

where $u(P) = P^1, u(Q) = Q^1, w(P) = Q, w(P^1) = Q^1$, then there exists a decomposition

$$Aut''_0(V) = H \times \langle ux, wz \rangle,$$

the restriction homomorphism $\varrho_{V,U}$ is injective, and $Aut''_0(V)$ is a commutative 2-group.

Proof: (i) Corollary 3.2, (i), (ii), yield $H \cup Hux \subset Aut''_0(V)$. Let $\alpha \in Aut''_0(V)$. If $\varrho_{V,U}(\alpha) \in H$ (respectively, $\varrho_{V,U}(\alpha) \in H_u$), then lemma 3.1, (iv), implies $\varrho_{V,V \setminus U}(\alpha) = id$ (respectively, $\varrho_{V,V \setminus U}(\alpha) = x$), that is, $\alpha \in H \cup Hux$, so $Aut''_0(V) = H \cup Hux$. Finally, $(ux)^2 = id$ and the automorphism $ux$ commutes with the elements of $H$.

(ii) Corollary 3.2, (i), (ii), implies $H \cup Hux \cup Hvvy \cup Hwz \subset Aut''_0(V)$, where $v = uvw$ with $v(P) = Q^1, v(P^1) = Q$, and $y = xz = (A, B^1)(A^1, B)$. Now, suppose that $\alpha \in Aut''_0(V)$ and let $\varrho_{V,U}(\alpha) \in H$ (respectively, $\varrho_{V,U}(\alpha) \in H_u, \varrho_{V,U}(\alpha) \in Hv, \varrho_{V,U}(\alpha) \in Hw$). Under the terms of lemma 3.1, (iv), we have $\varrho_{V,V \setminus U}(\alpha) = id$ (respectively, $\varrho_{V,V \setminus U}(\alpha) = x, \varrho_{V,V \setminus U}(\alpha) = y, \varrho_{V,V \setminus U}(\alpha) = z$), that is, $\alpha \in H \cup Hux \cup Hvvy \cup Hwz$. Therefore $Aut''_0(V) = H \cup Hux \cup Hvvy \cup Hwz$. In order to finish the proof, we have to note that $(ux)^2 = (wz)^2 = id$, the automorphisms $ux$ and $wz$ commute, and, moreover, commute with the elements of the subgroup $H$.

Lemma 3.7. Let the difference $V \setminus U$ be a $G''$-orbit that consists of a chiral pair \{A, A^1\}, and a dimer $\Delta$. Suppose that the cone $Q = C_>(V; \Delta)$ is $Aut''_0(U)$-invariant, and there exists a decomposition

$$I_{V,U} = H \times \langle w \rangle$$

with $w(P) = P^1$, where $P = C_>(V; A), P^1 = C_>(V; A^1)$. Then there exists a decomposition

$$Aut''_0(V) = H \times \langle ws \rangle,$$

where $s = (A, A^1)$, the restriction homomorphism $\varrho_{V,U}$ is injective, and $Aut''_0(V)$ is a commutative 2-group.

Proof: Corollary 3.2, (i), (ii), yield $H \cup Hws \subset Aut''_0(V)$. Since $w \notin H$, we have $P \neq P^1$, and both cones are different from $Q$ because the last one is $Aut''_0(U)$-invariant. Let us suppose that $\alpha \in Aut''_0(V)$ with $\varrho_{V,U}(\alpha) \in H$ (respectively, $\varrho_{V,U}(\alpha) \in H_w$). Using lemma 3.1, (iv), we get $\varrho_{V,V \setminus U}(\alpha) = id$ (respectively, $\varrho_{V,V \setminus U}(\alpha) = s$), that is, $\alpha \in H \cup Hws$, and therefore $Aut''_0(V) = H \cup Hws$. Moreover, $(ws)^2 = id$ and the automorphism $ws$ commutes with the elements of $H$. 
Theorem 3.8. One has:

(i) $\text{Aut}_0''(T_{D;G}) \simeq (id)$;

(ii) $\text{Aut}_0''(T_{D_2;G}) = \langle (a_{(4,12)}, b_{(4,12)}) \rangle \simeq C_2$;

(iii) $\text{Aut}_0''(T_{D_3;G}) = \langle (a_{(4,12)}, b_{(4,12)}) \rangle \simeq C_2$;

(iv) $\text{Aut}_0''(T_{D_4;G}) = \langle (a_{(4,12)}, b_{(4,12)}), c_{(3,2,1)}, c_{(3,2,1)} \rangle \simeq C_2$;

(v) $\text{Aut}_0''(T_{D_5;G}) = \langle a_{(4,12)}, b_{(4,12)}, (c_{(3,1,3)}, e_{(3,1,3)})(f_{(3,1,3)}, h_{(3,1,3)}), (a_{(3,1,3)}, b_{(3,1,3)}), (k_{(3,1,3)}, l_{(3,1,3)}) \rangle \simeq C_2 \times C_2 \times C_2$;

(vi) $\text{Aut}_0''(T_{D_6;G}) = \langle a_{(4,12)}, b_{(4,12)}, (c_{(3,2,1)}, e_{(3,1,3)})(f_{(3,1,3)}, h_{(3,1,3)}), (a_{(3,1,3)}, b_{(3,1,3)}), (k_{(3,1,3)}, l_{(3,1,3)}) \rangle \simeq C_2 \times C_2 \times C_2$;

(vii) $\text{Aut}_0''(T_{D_7;G}) = \langle a_{(4,12)}, b_{(4,12)}, (c_{(3,2,1)}, e_{(3,1,3)})(f_{(3,1,3)}, h_{(3,1,3)}), (a_{(3,1,3)}, b_{(3,1,3)}), (k_{(3,1,3)}, l_{(3,1,3)}), (f_{(3,1,3)}, h_{(3,1,3)}), (a_{(2,1,2)}, b_{(2,1,2)})(f_{(2,1,2)}, h_{(2,1,2)}) \rangle \simeq C_2 \times C_2$;

(viii) $\text{Aut}_0''(T_{D_8;G}) = \langle a_{(2,1,4)}, b_{(2,1,4)}, (a_{(2,1,4)}, b_{(2,1,4)}), c_{(2,1,4)}, b_{(2,1,4)}(e_{(2,1,4)}, l_{(2,1,4)})(k_{(2,1,4)}, l_{(2,1,4)})(m_{(2,1,4)})(b_{(2,1,4)}) \rangle \simeq C_2 \times C_2 \times C_2 \times C_2$;

where

$$w_0 = \langle a_{(4,12)}, b_{(4,12)}(c_{(2,2,1)}, c_{(3,1,3)})(a_{(3,1,3)}, b_{(3,1,3)}), (c_{(3,1,3)}, e_{(3,1,3)})(f_{(3,1,3)}, h_{(3,1,3)}), (k_{(3,1,3)}, l_{(3,1,3)})(f_{(3,1,3)}, h_{(3,1,3)}), (a_{(2,1,2)}, b_{(2,1,2)})(f_{(2,1,2)}, h_{(2,1,2)}), (k_{(2,1,2)}, l_{(2,1,2)}), (m_{(2,1,2)})(c_{(2,1,4)}, b_{(2,1,4)}), (e_{(2,1,4)}, l_{(2,1,4)})(k_{(2,1,4)}, l_{(2,1,4)})(m_{(2,1,4)})(b_{(2,1,4)}) \rangle$$

$$x_0 = \langle (f_{(2,1,4)}, h_{(2,1,4)})(g_{(2,1,4)}, h_{(2,1,4)}) \rangle$$

and $z_0 = \langle (f_{(2,1,4)}, h_{(2,1,4)})(g_{(2,1,4)}, h_{(2,1,4)}) \rangle$;

(ix) $\text{Aut}_0''(T_{D_9;G}) = \langle z_1, z_2, z_3, z_4, z_5 \rangle \simeq C_2 \times C_2 \times C_2 \times C_2$;

where

$$z_1 = \langle a_{(1,6)}, b_{(1,6)}, a_{(1,6)}(b_{(1,6)}, b_{(1,6)}), z_2 = \langle a_{(1,6)}, b_{(1,6)}, a_{(1,6)}(b_{(1,6)}, b_{(1,6)}), z_3 = \langle f_{(1,6)}, h_{(1,6)})(f_{(1,6)}, h_{(1,6)}) \rangle, z_4 = \langle f_{(1,6)}, h_{(1,6)})(f_{(1,6)}, h_{(1,6)}) \rangle, z_5 = w_0 x_0 a_{(2,1,4)}, b_{(2,1,4)}(a_{(2,1,4)}, b_{(2,1,4)})(a_{(1,6)}, b_{(1,6)})(a_{(1,6)}, b_{(1,6)})$$

and the automorphism

$$z_5 = w_0 x_0 a_{(2,1,4)}, b_{(2,1,4)}(a_{(2,1,4)}, b_{(2,1,4)})(a_{(1,6)}, b_{(1,6)})(a_{(1,6)}, b_{(1,6)})$$
Let $\alpha \in Aut_0''(T_{D_1;G})$ be a Lunn-Senior’s automorphism. Then because of (2.1) and (2.2) we obtain $\alpha(a(6)) = a(6)$ and $\alpha(a(5,1)) = a(5,1)$. Since $a(4,2)$ and $b(4,2)$ are structural isomers, see (2.3), we have $\alpha(a(4,2)) = a(4,2)$ and $\alpha(b(4,2)) = b(4,2)$. Hence the group $Aut_0''(T_{D_1;G})$ is the trivial one.

(i) Let $\alpha \in Aut_0''(T_{D_1;G})$ be a Lunn-Senior’s automorphism. Then because of (2.1) and (2.2) we obtain $\alpha(a(6)) = a(6)$ and $\alpha(a(5,1)) = a(5,1)$. Since $a(4,2)$ and $b(4,2)$ are structural isomers, see (2.3), we have $\alpha(a(4,2)) = a(4,2)$ and $\alpha(b(4,2)) = b(4,2)$. Hence the group $Aut_0''(T_{D_1;G})$ is the trivial one.

(ii) In accord to (2.4) – (2.6), $\{a(4,12), b(4,12)\}$ is a chiral pair, $c(4,12)$ is a dimer and both are structural isomers. We set $U_1^{(1)} = T_{D_1;G} \cup \{a(4,12), b(4,12)\}$ and $V = T_{D_2;G}$, where $D_2' = \{(4,2), (4,12)\}$. We note that $V$ is a barrier of $T_{(4,12);G}$ in $T_{D_2;G}$. The above figure shows that all conditions of lemma 3.5 are satisfied, so $Aut_0''(U_1^{(1)}) = \{(a(4,12), b(4,12))\}$. Now, lemma 3.3 finishes the proof of (ii).
(iii) In compliance with (2.7), on level \((3^2)\) we have two dimers that are in the same time structural isomers. Let us set \(\bar{V} = T_{D_2^4;G}\), where \(D_2^4 = \{(4, 2), (3^2)\}\). We have that \(\bar{V}\) is a barrier of \(T_{(3^2);G}\) in \(T_{D_2^4;G}\). Taking into account the figure above we can see immediately that all conditions of lemma 3.3 are fulfilled, so we obtain (iii).

(iv) On level \((3,2,1)\), in accordance with the equalities (2.8) – (2.10), we have one chiral pair \(\{b_{(3,2,1)}, c_{(3,2,1)}\}\) and two dimers \(a_{(3,2,1)}, e_{(3,2,1)}\), and all three are pairwise structural isomers. We set \(\bar{V} = T_{D_2^4;G}\), where \(D_2^4 = \{(4, 1^2), (3^2), (3, 2, 1)\}\) and note that \(\bar{V}\) is a barrier of \(T_{(3,2,1);G}\) in \(T_{D_2^4;G}\). Let \(U_4^{(1)} = T_{D_2^4;G} \cup \{a_{(3,2,1)}, e_{(3,2,1)}\}\). The inequalities from section 2, Case 6, yield

\[
C_\succ(\bar{V}; a_{(3,2,1)},) = \{a_{(4,1^2)}, b_{(4,1^2)}, a_{(3^2)}\}, \quad C_\succ(\bar{V}; e_{(3,2,1)}) = \{c_{(4,1^2)}, b_{(3^2)}\},
\]

and these cones are stable under the action of the group

\[
\text{Aut}_0''(T_{D_2^4}) = \{(a_{(4,1^2)}, b_{(4,1^2)})\}.
\]

Therefore lemma 3.3 implies \(\text{Aut}_0''(U_4^{(1)}) = \{(a_{(4,1^2)}, b_{(4,1^2)})\}\). Again, from the inequalities from section 2, Case 6, we get

\[
P = C_\succ(\bar{V}; b_{(3,2,1)}) = \{a_{(4,1^2)}, c_{(4,1^2)}, b_{(3^2)}\},
\]

and

\[
P^1 = C_\succ(\bar{V}; c_{(3,2,1)}) = \{b_{(4,1^2)}, c_{(4,1^2)}, b_{(3^2)}\}.
\]

We have \(\text{Aut}_0''(U_4^{(1)}) = H \times \langle w \rangle\), where \(H = \{\beta \in \text{Aut}_0''(U_4^{(1)}) \mid \beta(P) = P, \beta(P^1) = P^1\}\) is the unit group, \(w = (a_{(4,1^2)}, b_{(4,1^2)})\), and \(w(P) = P^1\). By virtue of corollary 3.2, (i), (ii), the restriction homomorphism \(\vartheta_{T_{D_2^4;G}, U_4^{(1)}}\) is surjective and then lemma 3.4, (i), implies part (iv).

(v) The equalities (2.11), (2.12) and the form of the \(G''\)-orbits (2.13) yield that on \((3,1^3)\)-level we have four chiral pairs

\[
\{a_{(3,1^3)}, b_{(3,1^3)}\}, \quad \{c_{(3,1^3)}, e_{(3,1^3)}\}, \quad \{f_{(3,1^3)}, h_{(3,1^3)}\}, \quad \{k_{(3,1^3)}, \ell_{(3,1^3)}\},
\]

that are pairwise structural isomers. Set \(\bar{V} = T_{D_2^5;G}\), where \(D_2^5 = \{(3,2,1, (3,1^3)\}\). The set \(\bar{V}\) is a barrier of \(T_{(3,1^3);G}\) in \(T_{D_2^5;G}\). The inequalities from section 2, Case 7, imply

\[
C_\succ(\bar{V}; a_{(3,1^3)}) = C_\succ(\bar{V}; b_{(3,1^3)}) = \{a_{(3,2,1)}\},
\]

\[
C_\succ(\bar{V}; k_{(3,1^3)}) = C_\succ(\bar{V}; \ell_{(3,1^3)}) = \{b_{(3,2,1)}, c_{(3,2,1)}\},
\]

and

\[
P = C_\succ(\bar{V}; c_{(3,1^3)}) = C_\succ(\bar{V}; f_{(3,1^3)}) = \{b_{(3,2,1)}, e_{(3,2,1)}\},
\]

\[
P^1 = C_\succ(\bar{V}; e_{(3,1^3)}) = C_\succ(\bar{V}; h_{(3,1^3)}) = \{c_{(3,2,1)}, e_{(3,2,1)}\}.
\]

Since the common cones of the members of the chiral pairs \(\{a_{(3,1^3)}, b_{(3,1^3)}\}\) and \(\{k_{(3,1^3)}, \ell_{(3,1^3)}\}\) are \(\text{Aut}_0''(T_{D_2^5;G})\)-invariant, lemma 3.5 shows that adding these chiral pairs to the set \(T_{D_2^5;G}\) results in appearance of two new generators of the automorphism group \(\text{Aut}_0''(T_{D_2^5;G})\), so

\[
\text{Aut}_0''(U_5^{(1)}) = \{(a_{(4,1^2)}, b_{(4,1^2)}), (b_{(3,2,1)}, c_{(3,2,1)}), (a_{(3,1^3)}, b_{(3,1^3)}), (k_{(3,1^3)}, \ell_{(3,1^3)})\}.
\]
where $U_{5}^{(1)} = T_{D_{4};G} \cup \{a_{(3,1^{3})}, b_{(3,1^{3})}, k_{(3,1^{3})}, \ell_{(3,1^{3})}\}$.

Further, there exists a decomposition $Aut_{0}''(U_{5}^{(1)}) = H \times \langle w \rangle$, where

$$H = \langle (a_{(3,1^{3})}, b_{(3,1^{3})}), (k_{(3,1^{3})}, \ell_{(3,1^{3})}) \rangle, \quad w = (a_{(4,1^{2})}, b_{(4,1^{2})})(b_{(3,2,1)}, c_{(3,2,1)}).$$

We have $H = \{ \beta \in Aut_{0}''(U_{5}^{(1)}) \mid \beta(P) = P, \beta(P^{1}) = P^{1} \}$ and $w(P) = P^{1}$. Moreover, if we add to $U_{5}^{(1)}$ the chiral pairs $\{c_{(3,1^{3})}, e_{(3,1^{3})}\}$ and $\{f_{(3,1^{3})}, h_{(3,1^{3})}\}$, then in compliance with corollary 3.2, (i), (ii), the restriction homomorphism $\varrho_{T_{D_{5};G};U_{5}^{(1)}}$ is surjective. We apply lemma 3.4, (i), and obtain (v).

(vi) On $(2^{3})$-level, using the equalities (2.14) – (2.16) we get four dimers $a_{(2^{3})}$, $b_{(2^{3})}$, $c_{(2^{3})}$, $e_{(2^{3})}$, which are at the same time structural isomers, and a chiral pair $\{f_{(2^{3})}, h_{(2^{3})}\}$.

Moreover, the dimer $e_{(2^{3})}$ is structurally identical to (each member of) the chiral pair $\{f_{(2^{3})}, h_{(2^{3})}\}$. Let us consider the set $\bar{V} = T_{D_{6};G}$ where $D_{6} = \{(3,2,1), (2^{3})\}$. Note that $\bar{V}$ is a barrier of $T_{(2^{3});G}$ in $T_{D_{6};G}$. All inequalities in $\bar{V}$ are listed in section 2, Case 8.

In particular, the cones

$$C_{>}(\bar{V}; a_{(2^{3})}) = \{a_{(3,2,1)}, b_{(3,2,1)}, c_{(3,2,1)}\},$$

$$C_{>}(\bar{V}; b_{(2^{3})}) = \{a_{(3,2,1)}, b_{(3,2,1)}, c_{(3,2,1)}, e_{(3,2,1)}\},$$

$$C_{>}(\bar{V}; c_{(2^{3})}) = \{b_{(3,2,1)}, c_{(3,2,1)}, e_{(3,2,1)}\},$$

are pairwise different and $Aut_{0}''(T_{D_{6};G})$-invariant. Thus, if we add the dimers $a_{(2^{3})}$, $b_{(2^{3})}$, $c_{(2^{3})}$, to the set $T_{D_{6};G}$, after lemma 3.3 we get $Aut_{0}''(U_{6}^{(1)}) = Aut_{0}''(T_{D_{6};G})$, where $U_{6}^{(1)} = T_{D_{6};G} \cup \{a_{(2^{3})}, b_{(2^{3})}, c_{(2^{3})}\}$.

Further, we add the dimer $e_{(2^{3})}$ and the chiral pair $\{f_{(2^{3})}, h_{(2^{3})}\}$ to the set $U_{6}^{(1)}$ in order to obtain $T_{D_{6};G}$. The equalities

$$C_{>}(\bar{V}; e_{(2^{3})}) = \{b_{(3,2,1)}, c_{(3,2,1)}, e_{(3,2,1)}\},$$

$$P = C_{>}(\bar{V}; f_{(2^{3})}) = \{b_{(3,2,1)}, e_{(3,2,1)}\},$$

$$P^{1} = C_{>}(\bar{V}; h_{(2^{3})}) = \{c_{(3,2,1)}, e_{(3,2,1)}\},$$

show that the cone $C_{>}(\bar{V}; e_{(2^{3})})$ is $Aut_{0}''(U_{6}^{(1)})$-invariant. Moreover, $Aut_{0}''(U_{6}^{(1)}) = H \times \langle w \rangle$, where

$$H = \langle (a_{(3,1^{3})}, b_{(3,1^{3})}), (k_{(3,1^{3})}, \ell_{(3,1^{3})}) \rangle,$$

and

$$w = (a_{(4,1^{2})}, b_{(4,1^{2})})(b_{(3,2,1)}, c_{(3,2,1)})(c_{(3,1^{3})}, e_{(3,1^{3})})(f_{(3,1^{3})}, h_{(3,1^{3})}).$$

We have $w(P) = P^{1}$, and the subgroup $H$ consists of all automorphisms from $Aut_{0}''(U_{6}^{(1)})$ that leave the cones $P$ and $P^{1}$ invariant. In accordance to corollary 3.2, (i), (ii), the restriction homomorphism $\varrho_{T_{D_{6};G};U_{6}^{(1)}}$ is surjective. Applying lemma 3.7, we obtain (vi).

(vii) On $(2^{2}, 1^{2})$-level the equalities (2.17) – (2.19) yield that there are four chiral pairs $\{a_{(2^{2}, 1^{2})}, b_{(2^{2}, 1^{2})}\}$, $\{f_{(2^{2}, 1^{2})}, h_{(2^{2}, 1^{2})}\}$, $\{\ell_{(2^{2}, 1^{2})}, m_{(2^{2}, 1^{2})}\}$ $\{k_{(2^{2}, 1^{2})}, p_{(2^{2}, 1^{2})}\}$, the last two being structurally identical, two dimers $c_{(2^{2}, 1^{2})}$, $e_{(2^{2}, 1^{2})}$, and all of them represent five structural isomers. Let $D'_{7} = \{(3,1^{3}), (2^{3}), (2^{2}, 1^{2})\}$. We set $\bar{V} = T_{D'_{7};G}$ and note that
\( \bar{V} \) is a barrier of \( T_{(2^2, 1^2)}; G \) in \( T_{D_6; G} \). The list of inequalities from section 2, Case 9, show that

\[
C_\supset (\bar{V}; a(2^2, 1^2)) = \{a(3,1^3), k(3,1^3), a(2^3)\},
C_\supset (\bar{V}; b(2^2, 1^2)) = \{b(3,1^3), \ell(3,1^3), a(2^3)\},
C_\supset (\bar{V}; c(2^2, 1^2)) = \{c(3,1^3), e(3,1^3), b(2^3)\},
C_\supset (\bar{V}; e(2^2, 1^2)) = \{f(3,1^3), h(3,1^3), b(2^3)\},
C_\supset (\bar{V}; f(2^2, 1^2)) = \{k(3,1^3), c(2^3)\},
C_\supset (\bar{V}; h(2^2, 1^2)) = \{\ell(3,1^3), c(2^3)\},
C_\supset (\bar{V}; m(2^2, 1^2)) = \{c(3,1^3), h(3,1^3), c(2^3)\},
C_\supset (\bar{V}; p(2^2, 1^2)) = \{c(3,1^3), h(3,1^3), h(2^3)\}.
\]

Let us set \( U_7^{(1)} = T_{D_6; G} \cup \{\ell(2^2, 1^2), m(2^2, 1^2), k(2^2, 1^2), p(2^2, 1^2)\} \). We have \( Aut''_0(U_7^{(1)}) = H \times \langle u \rangle \), where the automorphism

\[
u = (a(4,1^2), b(4,1^2))(b(3,2,1), c(3,2,1))(c(3,1^3), e(3,1^3))(f(3,1^3), h(3,1^3))(f(2^3), h(2^3))
\]

transposes the cones of the members of any one of the chiral pairs

\[\{\ell(2^2, 1^2), m(2^2, 1^2)\}, \{k(2^2, 1^2), p(2^2, 1^2)\}\]

and the subgroup \( H = \langle(a(3,1^3), b(3,1^3)), (k(3,1^3), \ell(3,1^3))\rangle \) consists of all automorphisms that leave these cones invariant. After corollary 3.2, (i), (ii), the restriction homomorphism \( g_{U_7^{(1)}}; T_{D_6; G} \) is surjective, and lemma 3.6, (i), yields

\[
Aut''_0(U_7^{(1)}) =
\langle (a(4,1^2), b(4,1^2))(b(3,2,1), c(3,2,1))(c(3,1^3), e(3,1^3))(f(3,1^3), h(3,1^3))(f(2^3), h(2^3))
\rangle
\]

Now, we add the chiral pair \( \{f(2^2, 1^2), h(2^2, 1^2)\} \) to the set \( U_7^{(1)} \) and denote the corresponding union by \( U_7^{(2)} \). We have \( Aut''_0(U_7^{(1)}) = H \times \langle w \rangle \), where

\[
H = \langle (a(4,1^2), b(4,1^2))(b(3,2,1), c(3,2,1))(c(3,1^3), e(3,1^3))(f(3,1^3), h(3,1^3))
\rangle
\]

\[
(f(2^3), h(2^3))(\ell(2^2, 1^2), m(2^2, 1^2))(k(2^2, 1^2), p(2^2, 1^2)), (a(3,1^3), b(3,1^3)), (k(3,1^3), \ell(3,1^3))\rangle.
\]

and \( w = (k(3,1^3), \ell(3,1^3)) \). Here the subgroup \( H \) consists of all automorphisms that leave the cones of \( f(2^2, 1^2) \) and \( h(2^2, 1^2) \) invariant, and \( w \) transposes these cones. Corollary 3.2, (i), (ii), imply that the restriction homomorphism \( g_{U_7^{(2)}}; U_7^{(1)} \) is surjective, and then lemma 3.4, (i), yields

\[
Aut''_0(U_7^{(2)}) =
\]
\[(a_{(4,1)}, b_{(4,1)})(b_{(3,2,1)}, c_{(3,2,1)})(c_{(3,1)}, e_{(3,1)})(f_{(3,1)}), h_{(3,1)}\]
\[(f_{(2)}), h_{(2)})(\ell_{(2,12)}, m_{(2,12)})(k_{(2,12)}, p_{(2,12)})\]
\[(a_{(3,1)}, b_{(3,1)}), (k_{(3,1)}, \ell_{(3,1)})(f_{(2,12)}, h_{(2,12)})\].

Next, we add the chiral pair \(\{a_{(2,12)}, b_{(2,12)}\}\) to the set \(U^{(2)}_7\) and denote the union by \(U^{(3)}_7\). Let us set
\[H = \langle (a_{(4,1)}, b_{(4,1)})(b_{(3,2,1)}, c_{(3,2,1)})(c_{(3,1)}, e_{(3,1)})(f_{(3,1)}), h_{(3,1)}\rangle\]
\[(f_{(2)}), h_{(2)})(\ell_{(2,12)}, m_{(2,12)})(k_{(2,12)}, p_{(2,12)})\].

The elements of the cosets \(H(a_{(3,1)}, b_{(3,1)}), H(k_{(3,1)}, \ell_{(3,1)})(f_{(2,12)}, h_{(2,12)})\) of the group \(\text{Aut}_0''(U^{(2)}_7)\) do not permute the cones
\[P = C_>(V; a_{(2,12)}), P^1 = C_>(V; b_{(2,12)}).\]

Hence after lemma 3.1, (iv), the elements of these cosets do not belong to the image of the restriction homomorphism \(\vartheta_{U^{(3)}_7, U^{(2)}_7}\). On the other hand, the elements of \(H\) leave \(P\) and \(P^1\) invariant, and if \(w = (a_{(3,1)}, b_{(3,1)})(k_{(3,1)}, \ell_{(3,1)})(f_{(2,12)}, h_{(2,12)})\), then \(w(P) = P^1\). Therefore corollary 3.2, (i), (ii), yield \(I_{U^{(3)}_7, U^{(2)}_7} = H \times \langle w \rangle\). Now, by virtue of lemma 3.4, (i), we obtain
\[\text{Aut}_0''(U^{(3)}_7) = \langle (a_{(4,1)}, b_{(4,1)})(b_{(3,2,1)}, c_{(3,2,1)})(c_{(3,1)}, e_{(3,1)})(f_{(3,1)}), h_{(3,1)}\rangle\]
\[(f_{(2)}), h_{(2)})(\ell_{(2,12)}, m_{(2,12)})(k_{(2,12)}, p_{(2,12)})\]
\[(a_{(3,1)}, b_{(3,1)}), (k_{(3,1)}, \ell_{(3,1)})(f_{(2,12)}, h_{(2,12)})(a_{(2,12)}, b_{(2,12)})\].

Finally, we add the dimers \(c_{(2,12)}\) and \(e_{(2,12)}\) to the set \(U^{(3)}_7\) and get \(T_{D_7;G}\). Since the cones of \(c_{(2,12)}\) and \(e_{(2,12)}\) are \(\text{Aut}_0''(U^{(3)}_7)\)-invariant, lemma 3.3 finishes the proof of (vii).

(viii) On level \((2,1^4)\) the equalities (2.20) – (2.22) yield that there are ten chiral pairs distributed among seven \(G''\)-orbits — see (2.23). We set \(D_8 = \{(2^2, 1^2), (2, 1^4)\}\) and \(\bar{V} = T_{D_8;G}\). We have that \(\bar{V}\) is a barrier of \(T_{(2,1^4);G}\) in \(T_{D_8;G}\). In accord to the list of inequalities from section 2, Case 10, we obtain
\[C_>(\bar{V}, a_{(2,1^4)}) = C_>(\bar{V}, b_{(2,1^4)}) = \{a_{(2,12)}, b_{(2,12)}\},\]
\[C_>(\bar{V}, a_{(2,1^4)}) = C_>(\bar{V}, b_{(2,1^4)}) = \{a_{(2,12)}, b_{(2,12)}, c_{(2,12)}\},\]
\[C_>(\bar{V}, c_{(2,1^4)}) = C_>(\bar{V}, e_{(2,1^4)}) = \{a_{(2^2, 1^2)}, c_{(2,1^2)}, e_{(2^2, 1^2)}\},\]
\[C_>(\bar{V}, \tilde{c}_{(2,1^4)}) = C_>(\bar{V}, \tilde{e}_{(2,1^4)}) = \{b_{(2^2, 1^2)}, c_{(2,1^2)}, e_{(2^2, 1^2)}\},\]
\[C_>(\bar{V}, f_{(2,1^4)}) = C_>(\bar{V}, h_{(2,1^4)}) = \{f_{(2^2, 1^2)}, h_{(2^2, 1^2)}, k_{(2^2, 1^2)}, \ell_{(2^2, 1^2)}\}\]
\[C_>(\bar{V}, \tilde{f}_{(2,1^4)}) = C_>(\bar{V}, \tilde{h}_{(2,1^4)}) = \{f_{(2^2, 1^2)}, h_{(2^2, 1^2)}, m_{(2^2, 1^2)}, p_{(2^2, 1^2)}\},\]
\[C_>(\bar{V}, k_{(2,1^4)}) = C_>(\bar{V}, \tilde{k}_{(2,1^4)}) = \{f_{(2^2, 1^2)}, k_{(2^2, 1^2)}, m_{(2^2, 1^2)}\}.\]
we obtain
\[ C_{>} (\overline{V}, p_{(2,1^4)}) = C_{>} (\overline{V}, \overline{p}_{(2,1^4)}) = \{ h_{(2^2,1^2)}, \ell_{(2^2,1^2)}, p_{(2^2,1^2)} \}, \quad (3.16) \]
\[ C_{>} (\overline{V}, m_{(2,1^4)}) = C_{>} (\overline{V}, \ell_{(2,1^4)}) = \{ h_{(2^2,1^2)}, k_{(2^2,1^2)}, \ell_{(2^2,1^2)}, m_{(2^2,1^2)} \}, \quad (3.17) \]
\[ C_{>} (\overline{V}, \overline{\ell}_{(2,1^4)}) = C_{>} (\overline{V}, \overline{m}_{(2,1^4)}) = \{ f_{(2^2,1^2)}, \ell_{(2^2,1^2)}, m_{(2^2,1^2)}, p_{(2^2,1^2)} \}. \quad (3.18) \]

The cones (3.9) and (3.10) are $\text{Aut}_0''(T_{D'}, C)$-invariant, so lemma 3.5 can be applied and we obtain

\[ \text{Aut}_0''(U_8^{(1)}) = \langle (a_{(4,1^2)}, b_{(4,1^2)})(b_{(3,2,1)}, c_{(3,2,1)})(c_{(3,1^3)}, e_{(3,1^3)})(f_{(3,1^3)}, h_{(3,1^3)}) \]
\[ (f_{(2^3)}, h_{(2^3)})(\ell_{(2^2,1^2)}, m_{(2^2,1^2)})(k_{(2^2,1^2)}, p_{(2^2,1^2)}), \]
\[ (a_{(3,1^3)}, b_{(3,1^3)})(k_{(3,1^3)}, \ell_{(3,1^3)})(f_{(2^2,1^2)}, h_{(2^2,1^2)})(a_{(2^2,1^2)}, b_{(2^2,1^2)}), \]
\[ (a_{(2,1^4)}, b_{(2,1^4)}), (\overline{a}_{(2,1^4)}, \overline{b}_{(2,1^4)}), \rangle, \]

where $U_8^{(1)}$ is the union of the set $T_{D', C}$ with the two chiral pairs

\[ \{ a_{(2,1^4)}, b_{(2^2,1^2)} \}, \{ \overline{a}_{(2,1^4)}, \overline{b}_{(2,1^4)} \}. \]

Further, all generators of the group $\text{Aut}_0''(U_8^{(1)})$ except

\[ w = (a_{(3,1^3)}, b_{(3,1^3)})(k_{(3,1^3)}, \ell_{(3,1^3)})(f_{(2^2,1^2)}, h_{(2^2,1^2)})(a_{(2^2,1^2)}, b_{(2^2,1^2)}), \]

leave the cones (3.11) and (3.12) invariant, and $w$ transposes them. Thus, for the set

\[ U_8^{(2)} = U_8^{(1)} \cup \{ c_{(2,1^4)}, \overline{c}_{(2,1^4)}, e_{(2,1^4)}, \overline{e}_{(2,1^4)} \} \]

lemma 3.4, (i), yields

\[ \text{Aut}_0''(U_8^{(2)}) = \]
\[ \langle (a_{(4,1^2)}, b_{(4,1^2)})(b_{(3,2,1)}, c_{(3,2,1)})(c_{(3,1^3)}, e_{(3,1^3)})(f_{(3,1^3)}, h_{(3,1^3)}) \]
\[ (f_{(2^3)}, h_{(2^3)})(\ell_{(2^2,1^2)}, m_{(2^2,1^2)})(k_{(2^2,1^2)}, p_{(2^2,1^2)}), \]
\[ (a_{(3,1^3)}, b_{(3,1^3)})(k_{(3,1^3)}, \ell_{(3,1^3)})(f_{(2^2,1^2)}, h_{(2^2,1^2)}), \]
\[ (a_{(2^2,1^2)}, b_{(2^2,1^2)})(c_{(2,1^4)}, \overline{c}_{(2,1^4)})(e_{(2,1^4)}, \overline{e}_{(2,1^4)}), \]
\[ (a_{(2,1^4)}, b_{(2,1^4)}), (\overline{a}_{(2,1^4)}, \overline{b}_{(2,1^4)}). \]

Now, we add the chiral pairs $\{ k_{(2,1^4)}, p_{(2,1^4)} \}, \{ \ell_{(2,1^4)}, m_{(2,1^4)} \}$ to the set $U_8^{(2)}$ and denote the union by $U_8^{(3)}$. We set

\[ H = \langle (a_{(2,1^4)}, b_{(2,1^4)}), (\overline{a}_{(2,1^4)}, \overline{b}_{(2,1^4)}) \rangle, \]
\[ u = (a_{(4,1^2)}, b_{(4,1^2)})(b_{(3,2,1)}, c_{(3,2,1)})(c_{(3,1^3)}, e_{(3,1^3)})(f_{(3,1^3)}, h_{(3,1^3)}) \]
\[ (f_{(2^3)}, h_{(2^3)})(\ell_{(2^2,1^2)}, m_{(2^2,1^2)})(k_{(2^2,1^2)}, p_{(2^2,1^2)}), \]
\[ (a_{(3,1^3)}, b_{(3,1^3)})(k_{(3,1^3)}, \ell_{(3,1^3)})(f_{(2^2,1^2)}, h_{(2^2,1^2)}), \]
\[ (a_{(2^2,1^2)}, b_{(2^2,1^2)})(c_{(2,1^4)}, \overline{c}_{(2,1^4)})(e_{(2,1^4)}, \overline{e}_{(2,1^4)}). \]
The elements of $H$ leave the cones (3.15) – (3.18) of $k_{(2,1^4)}$, $p_{(2,1^4)}$, $\ell_{(2,1^4)}$, $\bar{m}_{(2,1^4)}$, respectively, invariant, and the elements of the cosets

$$H(a_{(4,1^2)}, b_{(4,1^2)})(b_{(3,2,1)}, c_{(3,2,1)})(c_{(3,1^3)}, e_{(3,1^3)})(f_{(3,1^3)}, h_{(3,1^3)})$$

$$\quad (f_{(2^3)}, h_{(2^3)})(\ell_{(2^2,1^2)}, m_{(2^2,1^2)})(k_{(2^2,1^2)}, p_{(2^2,1^2)})$$

$$H(a_{(3,1^3)}, b_{(3,1^3)})(k_{(3,1^3)}, \ell_{(3,1^3)})(f_{(2^2,1^2)}, h_{(2^2,1^2)})$$

$$\quad (a_{(2^2,1^2)}, b_{(2^2,1^2)})(c_{(2,1^4)}, \bar{c}_{(2,1^4)})(e_{(2,1^4)}, \bar{e}_{(2,1^4)})$$

of the group $\text{Aut}_0''(U_8^{(2)})$ do not permute these cones. We have

$$u(C_>(V, k_{(2,1^4)})) = C_>(V, p_{(2,1^4)}), \quad u(C_>(V, \ell_{(2,1^4)})) = C_>(V, \bar{m}_{(2,1^4)}),$$

and, in particular, $H$ consists of all automorphisms from $\text{Aut}_0''(U_8^{(2)})$ that leave these four cones invariant. Thus, lemma 3.1, (iv), and corollary 3.2, (i), (ii), imply $I_{U_8^{(2)}, U_8^{(3)}} = H \times \langle u \rangle$, and then in keeping with lemma 3.6, (i), we get

$$\text{Aut}_0''(U_8^{(3)}) =$$

$$\langle (a_{(4,1^2)}, b_{(4,1^2)})(b_{(3,2,1)}, c_{(3,2,1)})(c_{(3,1^3)}, e_{(3,1^3)})(f_{(3,1^3)}, h_{(3,1^3)})$$

$$\quad (f_{(2^3)}, h_{(2^3)})(\ell_{(2^2,1^2)}, m_{(2^2,1^2)})(k_{(2^2,1^2)}, p_{(2^2,1^2)})$$

$$\quad (a_{(3,1^3)}, b_{(3,1^3)})(k_{(3,1^3)}, \ell_{(3,1^3)})(f_{(2^2,1^2)}, h_{(2^2,1^2)})$$

$$\quad (a_{(2^2,1^2)}, b_{(2^2,1^2)})(c_{(2,1^4)}, \bar{c}_{(2,1^4)})(e_{(2,1^4)}, \bar{e}_{(2,1^4)})(k_{(2,1^4)}, p_{(2,1^4)})(\ell_{(2,1^4)}, \bar{m}_{(2,1^4)}),$$

$$\quad (a_{(2,1^4)}, b_{(2,1^4)}), (\bar{a}_{(2,1^4)}, \bar{b}_{(2,1^4)})).$$

Let us add the chiral pairs $\{\bar{k}_{(2,1^4)}, \bar{p}_{(2,1^4)}\}$, $\{\bar{\ell}_{(2,1^4)}, m_{(2,1^4)}\}$ to the set $U_8^{(3)}$ and denote the set thus obtained by $U_8^{(4)}$. Since the cones of the elements of $U_8^{(4)} \setminus U_8^{(2)}$ coincide with the cones of the elements of $U_8^{(3)} \setminus U_8^{(2)}$, we apply again lemma 3.6, (i), and have

$$\text{Aut}_0''(U_8^{(4)}) =$$

$$\langle (a_{(4,1^2)}, b_{(4,1^2)})(b_{(3,2,1)}, c_{(3,2,1)})(c_{(3,1^3)}, e_{(3,1^3)})(f_{(3,1^3)}, h_{(3,1^3)})$$

$$\quad (f_{(2^3)}, h_{(2^3)})(\ell_{(2^2,1^2)}, m_{(2^2,1^2)})(k_{(2^2,1^2)}, p_{(2^2,1^2)})$$

$$\quad (a_{(3,1^3)}, b_{(3,1^3)})(k_{(3,1^3)}, \ell_{(3,1^3)})(f_{(2^2,1^2)}, h_{(2^2,1^2)})$$

$$\quad (a_{(2^2,1^2)}, b_{(2^2,1^2)})(c_{(2,1^4)}, \bar{c}_{(2,1^4)})(e_{(2,1^4)}, \bar{e}_{(2,1^4)})(k_{(2,1^4)}, p_{(2,1^4)})(\ell_{(2,1^4)}, \bar{m}_{(2,1^4)}),$$

$$\quad (a_{(2,1^4)}, b_{(2,1^4)}), (\bar{a}_{(2,1^4)}, \bar{b}_{(2,1^4)})).$$

Finally, we add the chiral pairs $\{f_{(2,1^4)}, \bar{h}_{(2,1^4)}\}$, $\{h_{(2,1^4)}, \bar{f}_{(2,1^4)}\}$ to the set $U_8^{(4)}$ and get $T_{D_8; G}$. We set $P = C_>(V, f_{(2,1^4)}) = C_>(V, h_{(2,1^4)})$, $P^1 = C_>(V, \bar{h}_{(2,1^4)}) = C_>(V, \bar{f}_{(2,1^4)})$ (see 3.13 – 3.14), and $H = \langle (a_{(2,1^4)}, b_{(2,1^4)}), (\bar{a}_{(2,1^4)}, \bar{b}_{(2,1^4)}) \rangle$. Let $w_0$ be the first generator of $\text{Aut}_0''(U_8^{(4)})$. Then the elements of $H$ leave the cones $P$ and $P^1$
invariant, and $w_0(P) = P^1$. After corollary 3.2, (i), (ii), the corresponding restriction homomorphism is surjective and then lemma 3.4, (ii), yields part (viii).

(ix) On the last level $(1^6)$, in accord to the equalities (2.24) – (2.26) we have twenty chiral pairs distributed among ten $G''$-orbits — see (2.27). Let us set $D'_6 = \{(2,1^4), (1^6)\}$ and $\bar{V} = T_{D'_6,G}$. We have that $\bar{V}$ is a barrier of $T_{(1^6),G}$ in $T_{6,G}$. We extend the set $T_{D'_6;G}$ step by step by adding these $G''$-orbits first:

$$U_9^{(1)} = T_{D'_6;G} \cup \{a_{(1^6)}, b_{(1^6)}, a_{(1^6)}, b_{(1^6)}\}, \quad U_9^{(2)} = U_9^{(1)} \cup \{\bar{a}_{(1^6)}, \bar{b}_{(1^6)}, \bar{a}_{(1^6)}, \bar{b}_{(1^6)}\},$$

$$U_9^{(3)} = U_9^{(2)} \cup \{f_{(1^6)}, \tilde{h}_{(1^6)}, \tilde{f}_{(1^6)}, h_{(1^6)}\}, \quad U_9^{(4)} = U_9^{(3)} \cup \{f_{(1^6)}, \tilde{h}_{(1^6)}, \tilde{f}_{(1^6)}, h_{(1^6)}\}.$$ The list of inequalities from section 2, Case 11, yields that

$$C_>(\bar{V}, a_{(1^6)}) = C_>(\bar{V}, b_{(1^6)}) =$$

$$\{a_{(2,1^4)}, \bar{a}_{(2,1^4)}, b_{(2,1^4)}, \bar{b}_{(2,1^4)}, f_{(2,1^4)}, h_{(2,1^4)}\},$$

$$C_>(\bar{V}, a_{(1^6)}) = C_>(\bar{V}, b_{(1^6)}) =$$

$$\{a_{(2,1^4)}, \bar{a}_{(2,1^4)}, b_{(2,1^4)}, \bar{b}_{(2,1^4)}, f_{(2,1^4)}, \bar{h}_{(2,1^4)}\},$$

$$C_>(\bar{V}, a_{(1^6)}) = C_>(\bar{V}, b_{(1^6)}) =$$

$$\{a_{(2,1^4)}, \bar{a}_{(2,1^4)}, b_{(2,1^4)}, \bar{b}_{(2,1^4)}, c_{(2,1^4)}, \tilde{c}_{(2,1^4)}, f_{(2,1^4)}, h_{(2,1^4)}, k_{(2,1^4)}, \ell_{(2,1^4)}\},$$

$$C_>(\bar{V}, a_{(1^6)}) = C_>(\bar{V}, b_{(1^6)}) =$$

$$\{a_{(2,1^4)}, \bar{a}_{(2,1^4)}, b_{(2,1^4)}, \bar{b}_{(2,1^4)}, c_{(2,1^4)}, \tilde{c}_{(2,1^4)}, f_{(2,1^4)}, \tilde{h}_{(2,1^4)}, m_{(2,1^4)}, p_{(2,1^4)}\},$$

$$C_>(\bar{V}, f_{(1^6)}) = C_>(\bar{V}, h_{(1^6)}) =$$

$$\{a_{(2,1^4)}, \bar{a}_{(2,1^4)}, b_{(2,1^4)}, \bar{b}_{(2,1^4)}, c_{(2,1^4)}, f_{(2,1^4)}, \tilde{f}_{(2,1^4)}, h_{(2,1^4)}, k_{(2,1^4)}, \tilde{h}_{(2,1^4)}, \ell_{(2,1^4)}\},$$

$$C_>(\bar{V}, \tilde{f}_{(1^6)}) = C_>(\bar{V}, \tilde{h}_{(1^6)}) =$$

$$\{a_{(2,1^4)}, \bar{a}_{(2,1^4)}, b_{(2,1^4)}, \bar{b}_{(2,1^4)}, c_{(2,1^4)}, f_{(2,1^4)}, \tilde{f}_{(2,1^4)}, h_{(2,1^4)}, m_{(2,1^4)}, p_{(2,1^4)}\},$$

$$C_>(\bar{V}, f_{(1^6)}) = C_>(\bar{V}, h_{(1^6)}) =$$

$$\{a_{(2,1^4)}, f_{(2,1^4)}, \tilde{f}_{(2,1^4)}, h_{(2,1^4)}, k_{(2,1^4)}, \tilde{h}_{(2,1^4)}, \ell_{(2,1^4)}\},$$

$$C_>(\bar{V}, f_{(1^6)}) = C_>(\bar{V}, h_{(1^6)}) =$$

$$\{\tilde{e}_{(2,1^4)}, f_{(2,1^4)}, \tilde{f}_{(2,1^4)}, h_{(2,1^4)}, k_{(2,1^4)}, \tilde{h}_{(2,1^4)}, \ell_{(2,1^4)}\},$$

$$C_>(\bar{V}, f_{(1^6)}) = C_>(\bar{V}, h_{(1^6)}) =$$

$$\{\tilde{e}_{(2,1^4)}, f_{(2,1^4)}, \tilde{f}_{(2,1^4)}, h_{(2,1^4)}, k_{(2,1^4)}, \tilde{h}_{(2,1^4)}, \ell_{(2,1^4)}\}.$$ The elements of the subgroup $\langle (a_{(2,1^4)}, b_{(2,1^4)}), (\bar{a}_{(2,1^4)}, \bar{b}_{(2,1^4)}), z_0 \rangle$ of $Aut''(T_{D'_6;G})$ leave the cones of the chiral pairs thus added invariant. Moreover, the automorphism $w_0 z_0$ transposes the following couples of cones: (3.19) – (3.20), (3.21) – (3.22), (3.23) – (3.24), (3.25), (3.26).
and (3.25) – (3.26). After corollary 3.2, the corresponding restriction homomorphisms are surjective, so using lemma 3.4, (ii), we obtain consecutively:

$$Aut''_0(U_g^{(1)}) = \langle (a(2,1), b(2,1)), (\tilde{a}(2,1), \tilde{b}(2,1)), z_0, z_1, w_0x_0x_1, \rangle,$$

where $$x_1 = (a(1,0), b(1,0))(a(1,0), b(1,0)), z_1 = (a(1,0), b(1,0))(a(1,0), b(1,0))$$,

$$Aut''_0(U_g^{(2)}) = \langle (a(2,1), b(2,1)), (\tilde{a}(2,1), \tilde{b}(2,1)), z_0, z_1, z_2, w_0x_0x_1x_2, \rangle,$$

where $$x_2 = (a(1,0), b(1,0))(a(1,0), b(1,0)), z_2 = (a(1,0), b(1,0))(a(1,0), b(1,0)),$$

$$Aut''_0(U_g^{(3)}) = \langle (a(2,1), b(2,1)), (\tilde{a}(2,1), \tilde{b}(2,1)), z_0, z_1, z_2, z_3, w_0x_0x_1x_2x_3, \rangle,$$

where $$x_3 = (f(1,0), \tilde{h}(1,0))(f(1,0), h(1,0)), z_3 = (f(1,0), h(1,0))(f(1,0), \tilde{h}(1,0)),$$ and

$$Aut''_0(U_g^{(4)}) = \langle (a(2,1), b(2,1)), (\tilde{a}(2,1), \tilde{b}(2,1)), z_0, z_1, z_2, z_3, z_4, w_0x_0x_1x_2x_3x_4, \rangle$$

Further, we set

$$U_g^{(5)} = U_g^{(4)} \cup \{c(1,0), \tilde{c}(1,0), c(1,0), \tilde{c}(1,0)\}, \quad U_g^{(6)} = U_g^{(5)} \cup \{e(1,0), \tilde{e}(1,0), e(1,0), \tilde{e}(1,0)\}. $$

From the list of inequalities in section 2, Case 11, we get

$$P = C_>(V, c(1,0)) = C_>(V, \tilde{e}(1,0)) = \{a(2,1), \tilde{a}(2,1), c(2,1), \tilde{c}(2,1), e(2,1), \tilde{e}(2,1), f(2,1), k(2,1), \tilde{k}(2,1), \ell(2,1), m(2,1)\},$$

$$P^1 = C_>(V, \tilde{c}(1,0)) = C_>(V, e(1,0)) = \{b(2,1), \tilde{b}(2,1), c(2,1), \tilde{c}(2,1), e(2,1), \tilde{e}(2,1), h(2,1), \tilde{h}(2,1), \tilde{\ell}(2,1), \tilde{m}(2,1), \tilde{p}(2,1)\},$$

$$Q = C_>(V, c(1,0)) = C_>(V, e(1,0)) = \{a(2,1), \tilde{a}(2,1), c(2,1), \tilde{c}(2,1), e(2,1), \tilde{e}(2,1), f(2,1), \tilde{k}(2,1), \tilde{m}(2,1), \tilde{p}(2,1)\},$$

$$Q^1 = C_>(V, c(1,0)) = C_>(V, \tilde{e}(1,0)) = \{b(2,1), \tilde{b}(2,1), c(2,1), \tilde{c}(2,1), e(2,1), \tilde{e}(2,1), h(2,1), \tilde{k}(2,1), \tilde{\ell}(2,1), \tilde{m}(2,1)\}. $$

The elements of the subgroup $$H = \langle z_1, z_2, z_3, z_4 \rangle$$ of $$Aut''_0(U_g^{(4)})$$ leave the cones $$P$$, $$P^1$$, $$Q$$, and $$Q^1$$ from the equalities (3.27) – (3.30) invariant, the elements of the coset $$Hu_1$$, where $$u_1 = (a(2,1), b(2,1))(a(2,1), b(2,1))w_0x_0x_1x_2x_3x_4$$, transpose the members of the pairs of cones $$P$$, $$P^1$$ and $$Q$$, $$Q^1$$, the elements of the coset $$Hw_1$$, where $$w_1 = w_0x_0x_1x_2x_3x_4z_0$$, transpose the members of the pairs of cones $$P$$, $$Q$$, $$P^1$$, and the other automorphisms from the group $$Aut''_0(U_g^{(4)})$$ do not permute these cones. Therefore lemma 3.1, (iv), and corollary 3.2, (i), (ii), imply $$I_{U_g^{(5)}, U_g^{(4)}} = H \times \langle u_1, w_1 \rangle$$, and in keeping with lemma 3.6, (ii), we obtain

$$Aut''_0(U_g^{(5)}) = \langle z_1, z_2, z_3, z_4, u_2, w_2 \rangle,$$
where \( u_2 = u_1(c_{(16)}, \bar{c}_{(16)})(c_{(16)}, \bar{c}_{(16)}) \), \( w_2 = w_1(c_{(16)}, c_{(16)})(\bar{c}_{(16)}, \bar{c}_{(16)}) \). Now, under the terms of corollary 3.2, the restriction homomorphism \( \vartheta_{U^{(6)_1}, U^{(6)}} \) is surjective. Applying lemma 3.6, (ii), once again we get

\[
Aut^{(6)}_9(U^{(6)_1}) = \langle z_1, z_2, z_3, z_4, u_3, w_3 \rangle,
\]

where \( u_3 = u_2(e_{(16)}, \bar{e}_{(16)})(c_{(16)}, \bar{c}_{(16)}) \), \( w_3 = w_2(e_{(16)}, \bar{e}_{(16)})(c_{(16)}, \bar{c}_{(16)}) \), that is,

\[
u_3 = (a(2,1), b(2,1), \bar{a}(2,1), \bar{b}(2,1), c(16), \bar{c}(16))(c_{(16)}, \bar{c}_{(16)}),
\]

\[
\nu_3 = (e_{(16)}, \bar{e}_{(16)})(c_{(16)}, \bar{c}_{(16)}), w_3 = w_{x_0 x_1 x_2 x_3 x_4},
\]

Finally, we set

\[
U^{(7)}_9 = U^{(6)}_9 \cup \{ k(16), p(16), \ell(16), m(16), \bar{a}(16) \}, \quad U^{(8)}_9 = U^{(7)}_9 \cup \{ k_1(16), p_1(16), m(16), \bar{a}(16) \},
\]

\[
U^{(9)}_9 = U^{(8)}_9 \cup \{ k_1(16), p_1(16), \ell(16), m(16), \bar{a}(16) \}, \quad U^{(10)}_9 = U^{(9)}_9 \cup \{ k_1(16), p_1(16), m(16), \bar{a}(16) \},
\]

and we have \( U^{(10)}_9 = T_{6; G} \). The list of inequalities in section 2, Case 11, implies

\[
C_>(V, k_{(16)}) = C_>(V, \bar{k}_{(16)}) =
\]

\[
\{ \bar{a}(2,1), c(2,1), e(2,1), \bar{e}(2,1), f(2,1), \bar{f}(2,1), k(2,1), \bar{k}(2,1), \ell(2,1), \bar{\ell}(2,1), m(2,1), \bar{m}(2,1) \},
\]

\[
C_>(V, p_{(16)}) = C_>(V, \bar{p}_{(16)}) =
\]

\[
\{ \bar{b}(2,1), \bar{c}(2,1), e(2,1), \bar{e}(2,1), h(2,1), \bar{h}(2,1), \ell(2,1), \bar{\ell}(2,1), m(2,1), \bar{m}(2,1), p(2,1), \bar{p}(2,1) \},
\]

\[
C_>(V, \ell_{(16)}) = C_>(V, m_{(16)}) =
\]

\[
\{ \bar{b}(2,1), \bar{c}(2,1), e(2,1), \bar{e}(2,1), h(2,1), \bar{h}(2,1), k(2,1), \bar{k}(2,1), \ell(2,1), \bar{\ell}(2,1), m(2,1), \bar{m}(2,1) \},
\]

\[
C_>(\bar{V}, \bar{m}_{(16)}) = C_>(\bar{V}, \bar{\ell}_{(16)}) =
\]

\[
\{ \bar{a}(2,1), c(2,1), e(2,1), \bar{e}(2,1), f(2,1), \bar{f}(2,1), \ell(2,1), \bar{\ell}(2,1), m(2,1), \bar{m}(2,1), p(2,1), \bar{p}(2,1) \},
\]

\[
C_>(\bar{V}, \bar{k}_{(16)}) = C_>(\bar{V}, \bar{k}_{(16)}) =
\]

\[
\{ c(2,1), e(2,1), f(2,1), \bar{f}(2,1), k(2,1), \bar{k}(2,1), \ell(2,1), \bar{\ell}(2,1), m(2,1), \bar{m}(2,1), p(2,1), \bar{p}(2,1) \},
\]

\[
C_>(\bar{V}, \bar{p}_{(16)}) = C_>(\bar{V}, \bar{p}_{(16)}) =
\]

\[
\{ \bar{c}(2,1), \bar{e}(2,1), h(2,1), \bar{h}(2,1), k(2,1), \bar{k}(2,1), \ell(2,1), \bar{\ell}(2,1), m(2,1), \bar{m}(2,1), p(2,1), \bar{p}(2,1) \},
\]

\[
C_>(\bar{V}, \ell_{(16)}) = C_>(\bar{V}, m_{(16)}) =
\]

\[
\{ c(2,1), e(2,1), h(2,1), \bar{h}(2,1), k(2,1), \bar{k}(2,1), \ell(2,1), \bar{\ell}(2,1), m(2,1), \bar{m}(2,1), p(2,1), \bar{p}(2,1) \}.
\]
\[ C_{>}(\bar{V}, \bar{m}_{(18)}) = C_{>}(\bar{V}, \bar{\ell}_{(18)}) = \{ \bar{c}(2,1^4), \bar{e}(2,1^4), f(2,1^4), \bar{f}(2,1^4), k(2,1^4), \bar{k}(2,1^4), \ell(2,1^4), \bar{\ell}(2,1^4), m(2,1^4), \bar{m}(2,1^4), p(2,1^4), \bar{p}(2,1^4) \}. \] (3.38)

The elements of the subgroup \( H = \langle z_1, z_2, z_3, z_4 \rangle \) of \( Aut_0(U_9^{(6)}) \) leave the cones (3.31) – (3.38) invariant, the elements of the coset \( Hw_3 \) transpose the cones of the members of each chiral pair in \( U_9^{(j)} \setminus U_9^{(j-1)} \), \( j = 7, 8, 9, 10 \), and the elements of the coset \( Hw_3 \) do not permute the cones of the elements of each difference \( U_9^{(j)} \setminus U_9^{(j-1)} \). Thus, \( I_{U_9^{(j)}(U_9^{(j-1)})} = \langle z_1, z_2, z_3, z_4, u_3 \rangle \). We apply lemma 3.6, (i), once and, in particular, using corollary 3.2, (i), get that the homomorphisms \( g_{U_9^{(j)}(U_9^{(j-1)})} \), \( j = 8, 9, 10 \) are surjective. Now, we apply lemma 3.6, (i), three more times and finish the proof of (ix).

**Corollary 3.39.** The members of the chiral pairs
\[ \{ f(2,1^4), \bar{h}(2,1^4) \}, \{ f(1^6), h(1^6) \} \]
can not be distinguished via substitution reactions among the elements of the set \( T_{D_s;G} \), but can be distinguished via substitution reactions (among the elements of the whole set \( T_{0;G} \)).

**Proof:** Consider the automorphism \( z_0 \in Aut_0(T_{D_s;G}) \) that maps the members of each one of these chiral pairs onto the members of the other. On the other hand, there is no automorphism in the group \( Aut_0(T_{0;G}) \) with this property.

**Corollary 3.40.** The members of the chiral pairs in the following couples can not be distinguished via substitution reactions:
\[ \{ a(1^6), b(1^6) \}, \{ a(1^6), b(1^6) \}, \]
\[ \{ a(1^6), b(1^6) \}, \{ a(1^6), b(1^6) \}, \]
\[ \{ f(1^6), h(1^6) \}, \{ f(1^6), h(1^6) \}, \]
\[ \{ f(1^6), h(1^6) \}, \{ f(1^6), h(1^6) \}. \]

**Proof:** Consider the automorphisms \( z_1, z_2, z_3, z_4 \in Aut_0(T_{0;G}) \). We have \( z_1(a(1^6)) = b(1^6), z_2(a(1^6)) = \bar{b}(1^6), z_3(f(1^6)) = h(1^6), \) and \( z_4(f(1^6)) = h(1^6) \).

**Theorem 3.41.** For \( D = \{(2^3), (2^2, 1^2)\} \) one has
\[ Aut_0''(T_{D';G}) = \langle (a(2^2,1^2), b(2^2,1^2)), (f(2^2,1^2), h(2^2,1^2)), (\ell(2^2,1^2), m(2^2,1^2)), (f(2^3), h(2^3))(k(2^2,1^2), p(2^2,1^2)) \rangle. \]

**Proof:** We have \( Aut_0''(T_{(2^3);G}) = \langle (f(2^3), h(2^3)) \rangle \), the set \( T_{(2^2,1^2);G} \) consists of minimal elements of \( T_{D';G} \), and \( T_{D';G} \) is a barrier of \( T_{(2^2,1^2);G} \) in \( T_{D';G} \). We set
\[ U_{10}^{(1)} = T_{(2^3);G} \cup \{ c(2^2,1^2), e(2^2,1^2) \}, \]
\[ U_{10}^{(2)} = U_{10}^{(1)} \cup \{ k(2^2,1^2), p(2^2,1^2) \}, \]
so
\[ T_{D';G} = U_{10}^{(2)} \cup \{ a(2^2,1^2), b(2^2,1^2), f(2^2,1^2), h(2^2,1^2), \ell(2^2,1^2), m(2^2,1^2) \}. \]

Lemma 3.3 yields \( Aut_0''(U_{10}^{(1)}) = Aut_0''(T_{(2^3);G}) \). Further, in compliance with lemma 3.4, (i), we obtain \( Aut_0''(U_{10}^{(2)}) = \langle (f(2^3), h(2^3))(k(2^2,1^2), p(2^2,1^2)) \rangle \), and, finally, lemma 3.5 implies the theorem.
COROLLARY 3.42. The members of the chiral pairs

\[ \{\ell(2^2, 1^2), m(2^2, 1^2)\}, \{k(2^2, 1^2), p(2^2, 1^2)\} \]

can be distinguished via substitution reactions among the elements of the set \( T_{D'_8;G} \).

PROOF: There is no automorphism from the group \( Aut''(T_{D'_8;G}) \) that maps a member of the first chiral pair to a member of the second one.

REMARK 3.43. We can also say that corollary 3.42 is true since both members of the chiral pair \( \{\ell(2^2, 1^2), m(2^2, 1^2)\} \) can be obtained from one product of type \((2^3)\), whereas the members of the chiral pair \( \{k(2^2, 1^2), p(2^2, 1^2)\} \) do not possess this property (each one of them can be obtained from one product of type \((2^3)\), but these products are different).

THEOREM 3.44. For \( D'_8 = \{(2^2, 1^2), (2, 1^4)\} \) one has

\[ Aut''(T_{D'_8;G}) = \]

\[ ((a(2,1^4), b(2,1^4)), (\bar{a}(2,1^4), \bar{b}(2,1^4)), (a(2,1^4), b(2,1^4)) \]

\[ (c(2,1^4), \bar{c}(2,1^4))(e(2,1^4), \bar{e}(2,1^4)), z_0, w_0x_0), \]

where

\[ w_0 = (f(2^2, 1^2), h(2^2, 1^2))(\ell(2^2, 1^2), m(2^2, 1^2))(k(2^2, 1^2), p(2^2, 1^2))(k(2,1^4), p(2,1^4)) \]

\[ (\ell(2,1^4), \bar{m}(2,1^4))(\bar{k}(2,1^4), \bar{p}(2,1^4))((\ell(2,1^4), m(2,1^4)), \]

\[ x_0 = (f(2,1^4), \bar{h}(2,1^4))(\bar{f}(2,1^4), h(2,1^4)), \text{ and } z_0 = (f(2,1^4), h(2,1^4))(\bar{f}(2,1^4), \bar{h}(2,1^4)) \]

PROOF: We have

\[ Aut_0''(T_{2^2,1^2};G) = \]

\[ ((a(2,1^4), b(2,1^4)), (f(2,1^4), h(2,1^4)), (\ell(2,1^2), m(2,1^2))(k(2^2, 1^2), p(2^2, 1^2)), \]

\[ (\ell(2,1^2), \bar{m}(2,1^2)), (m(2,1^2), p(2,1^2))) \simeq C_2 \times C_2 \times \Delta_4, \]

where \( \Delta_4 \) is the group of order 8 consisting of all permutations of the elements of the \( G'' \)-orbit \( \{\ell(2^2, 1^2), m(2^2, 1^2), k(2^2, 1^2), p(2^2, 1^2)\} \), which transform any chiral pair onto a chiral pair. Moreover, the set \( T_{2^2,1^2};G \) consists of minimal elements of \( T_{D'_8;G} \), and \( T_{D'_8;G} \) is a barrier of \( T_{2^2,1^2};G \) in \( T_{D'_8;G} \).

The group \( Aut''(T_{2^2,1^2};G) \) leaves the cones (3.9) and (3.10) invariant, so lemma 3.5 can be applied and we obtain

\[ Aut''(U^{(1)}_{11}) = \]

\[ ((a(2,1^4), b(2,1^4)), (f(2,1^4), h(2,1^4)), (\ell(2,1^2), m(2,1^2))(k(2^2, 1^2), p(2^2, 1^2)), \]

\[ (\ell(2^2, 1^2), k(2^2, 1^2)), (m(2^2, 1^2), p(2^2, 1^2)), (a(2,1^4), b(2,1^4)), (\bar{a}(2,1^4), \bar{b}(2,1^4))) \]

where \( U^{(1)}_{11} \) is the union of the set \( T_{2^2,1^2};G \) with the two chiral pairs \( \{a(2,1^4), b(2,1^2)\} \) and \( \{\bar{a}(2,1^4), \bar{b}(2,1^4)\} \).
Further, all generators of the group $Aut_0''(U^{(1)}_{11})$ except $w = (a(2,1), b(2,1))$, leave the cones (3.11) and (3.12) invariant, and $w$ transposes them. Thus, for the set $U^{(2)}_{11} = U^{(1)}_{11} \cup \{c(2,1), \bar{c}(2,1), e(2,1), \bar{e}(2,1)\}$ lemma 3.4, (i), yields

$$Aut_0''(U^{(2)}_{11}) = \langle \langle f(2,1), h(2,1), (\ell(2,1), m(2,1), k(2,1), p(2,1)) \rangle \rangle,$$

$$\langle \langle f(2,1), h(2,1), (\ell(2,1), m(2,1), k(2,1), p(2,1)) \rangle \rangle.$$

Let us set

$$U^{(3)}_{11} = U^{(2)}_{11} \cup \{k(2,1), p(2,1), \ell(2,1), \bar{m}(2,1)\}.$$

We denote

$$H = \langle \langle a(2,1), b(2,1), (\bar{a}(2,1), \bar{b}(2,1)), (a(2,1), b(2,1)), (c(2,1), \bar{c}(2,1)), (e(2,1), \bar{e}(2,1)) \rangle \rangle,$$

and denote the set thus obtained by $U^{(3)}_{11}$ and denote the set thus obtained by $U^{(4)}_{11}$. Since the cones of the elements of $U^{(4)}_{11} \backslash U^{(3)}_{11}$ coincide with the cones of the elements of $U^{(3)}_{11} \backslash U^{(2)}_{11}$, we apply again lemma 3.6, (i), and have

$$Aut_0''(U^{(4)}_{11}) = \langle \langle a(2,1), b(2,1), (\bar{a}(2,1), \bar{b}(2,1)), (a(2,1), b(2,1)), (c(2,1), \bar{c}(2,1)), (e(2,1), \bar{e}(2,1)) \rangle \rangle.$$

Now, we add the chiral pairs $\{f(2,1), h(2,1)\}$, $\{\bar{f}(2,1), \bar{h}(2,1)\}$ to the set $U^{(4)}_{11}$ and get $T_{D_{16}G}$. We set $P = C_>(V, f(2,1)) = C_>(V, h(2,1))$, $P^1 = C_>(V, \bar{f}(2,1)) = C_>(V, \bar{h}(2,1))$ (see (3.13), (3.14)),

$$H = \langle \langle a(2,1), b(2,1), (\bar{a}(2,1), \bar{b}(2,1)), (a(2,1), b(2,1)), (c(2,1), \bar{c}(2,1)), (e(2,1), \bar{e}(2,1)) \rangle \rangle.$$

and let $w_0$ be the last generator of the group $Aut_0''(U^{(4)}_{11})$. Then the elements of $H$ leave the cones $P$, $P^1$ invariant, and $w_0(P) = P^1$. After corollary 3.2, (i), (ii), the corresponding restriction homomorphism is surjective and then lemma 3.4, (ii), yields the theorem.
COROLLARY 3.45. The members of the chiral pairs in the following couples can be distinguished via substitution reactions among the elements of the set $T_{D_8';G}$:

\[
\{k(2,1^4), p(2,1^4)\}, \{\ell(2,1^4), \bar{m}(2,1^4)\},
\]
\[
\{\bar{k}(2,1^4), \bar{p}(2,1^4)\}, \{\bar{\ell}(2,1^4), m(2,1^4)\}.
\]

PROOF: There is no automorphism from the group $\text{Aut}_0''(T_{D_8';G})$ that maps a member of one of these chiral pairs to a member of the other.

REMARK 3.46. We can also argue that corollary 3.45 is true since the members of the chiral pairs

\[
\{k(2,1^4), p(2,1^4)\}, \{\bar{k}(2,1^4), \bar{p}(2,1^4)\},
\]

can be obtained from three product of type $(2^2, 1^2)$, whereas the members of the chiral pairs

\[
\{\ell(2,1^4), \bar{m}(2,1^4)\}, \{\bar{\ell}(2,1^4), m(2,1^4)\}
\]

can be obtained from four such products.

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