

Point Groups, *RS*-Stereoisomeric Groups,
Stereoisomeric Groups, and Isoskeletal Groups for
Characterizing Allene Derivatives. Hierarchy of
Groups for Restructuring Stereochemistry (Part 1)

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Abstract

Chirality, *RS*-stereogenicity, stereogenicity, and isoskeletal isomerism for allene derivatives have been comprehensively discussed, where we have proposed a new terminology for groups (point groups, *RS*-permutation groups, *RS*-stereoisomeric groups, stereoisomeric groups, and isoskeletal groups) as well as a new terminology for isomers (enantiomers, holantimers, *RS*-diastereomers, diastereomers, and isoskeletal isomers). In the case of allene derivatives, *RS*-stereoisomeric groups have been clarified to coincide with stereoisomeric groups so that diastereomers are identical with *RS*-diastereomers. We have proposed the concept of *stereoisogram set* in order to discuss the relationship between (*RS*-)diastereomers and isoskeletal isomers. As for allene derivatives, each stereoisogram set contains three stereoisograms to represent isoskeletal isomerism. Each of the three stereoisograms has been categorized into five stereogenicity types (Types I-V) so that the combination of such stereogenicity types can be used to characterize the isomerism of allene derivatives. The term "pseudoasymmetry" for allene derivatives has been discussed in terms of stereoisograms.

1 Introduction

Stereochemistry is concerned with at least two aspects of organic molecules. The one is a geometrical aspect, i.e., chirality, and the other is a nomenclature aspect, i.e., stereogenicity.

The geometrical aspect (chirality) has been studied by means of point groups, which are a standard repertoire for treating such continuous objects as wavefunctions as found in various textbooks on chemical group theory [1, 2, 3]. However, this type of approaches is unsuitable to solve such stereochemical problems that are characterized by the discrete nature, e.g., isomer classification and combinatoric enumeration.

The other aspect (stereogenicity) has been related to the *RS* nomenclature [4, 5], where the permutability of ligands is a main matter of concern for determining *R*- and *S*-descriptors. Although such permutability has been treated by permutation groups, these groups have been proved to be insufficient in discussing chirality. This type of insufficiency has been discussed in terms of the violation of chirality fittingness [6].

To integrate point groups and permutation groups, we have proposed the USCI (unit-subduced-cycle-index) approach, which is based on coset representations of point groups and their subductions [7]. Mark tables related to the USCI approach have been featured in a special issue of *MATCH Commun. Math. Comput. Chem.* (Vol. 46) on "Applications of Tables of Marks in Chemistry". The USCI approach has been extended by the concept of sphericity so that it is capable of treating chirality and prochirality comprehensively as well as of solving the stereochemical problems described above [8, 9, 10].

To discuss stereogenicity and chirality more intimately, we have proposed the concept of stereoisomeric groups, which is a combination between coset representations of point groups and permutation groups [11]. We have recently proposed the concepts of holantimers and stereoisograms, where we have concluded that *RS*-stereogenicity is more definite than stereogenicity in discussing the *RS*-nomenclature [12]. This integrated approach has been formulated by *RS*-stereoisomeric groups by using tetrahedral molecules as examples [13]. As a result, further questions have emerged: how stereogenicity and *RS*-stereogenicity (or stereoisomeric groups and *RS*-stereoisomeric groups) are different and what happens if they are identical with each other. Since these questions have not been fully clarified by using tetrahedral molecules only, the integrated approach should be applied to other types of molecules. In this paper, we select allene derivatives to demonstrate the versatility of the integrated approach, where we compare point groups, *RS*-stereoisomeric groups, and stereoisomeric groups as subgroups of isoskeletal groups.

2 Groups for Characterizing an Allene Skeleton

To comprehend the stereochemistry of allene derivatives, we take account of groups collected in Fig. 1, which act on the four positions of an allene skeleton. We adopt a convention for drawing allene derivatives, as shown in Fig. 2.

1. To discuss the geometrical aspect (chirality) of allenes, we consider the point group D_{2d} of order 8. Precisely speaking, we take account of the coset representation $D_{2d}/(C_s)$. This coset representation is regarded as a permutation representation, where permutations corresponding to improper rotations (rotoreflections) are designated by overbars. Thereby, it is capable of mediating the point group D_{2d} and

the corresponding permutation group of degree 4 [10]. We here refer to the coset representation $D_{2d}/(C_s)$ as the group D_{2d} , if such usage causes no confusion.

2. To discuss the nomenclature aspect (stereogenicity) of allenes, we consider the permutation group $S_9^{[4]}$, which is a subgroup of the symmetric group of degree 4, i.e., $S^{[4]}$. The group $S_9^{[4]}$ and the coset representation $D_{2d}/(C_s)$ have common permutations with respect to the proper rotations of D_{2d} , i.e., the $S_7^{[4]}$ -group and the D_2 -group. Moreover, the remaining permutations of $S_9^{[4]}$ are identical with those of the coset representation $D_{2d}/(C_s)$, if the overbars for the roto reflections of D_{2d} are omitted; i.e., the coset $(1)(2\ 3)(4)S_7^{[4]}$ and $D_{2d}\sigma_{d(1)}$. Such a permutation group as $S_9^{[4]}$ is here called *RS-permutation group* [13].
3. To integrate the two aspects (chirality and stereogenicity), we define the *RS-stereoisomeric group* of the allene skeleton as a direct product represented by $S_9^{[4]} \times \{I, \sigma\}$. Note that the operation σ can be selected from roto reflections. For example, we can place $\sigma = \tilde{I}$, which represents an operation in which the chirality of each ligand is changed into the opposite one at the same position of the allene skeleton. This group contains the point group D_{2d} and the *RS-permutation group* $S_9^{[4]}$ as normal subgroups. In addition, there emerges another normal subgroup, which is called *inversion group* [13]:

$$\overline{D_{2d}} = \{I, C_{2(3)}, C_{2(1)}, C_{2(2)}; \tilde{I}, \tilde{C}_{2(3)}, \tilde{C}_{2(1)}, \tilde{C}_{2(2)}\} \quad (1)$$

$$= \{(1)(2)(3)(4), (1\ 4)(2\ 3), (1\ 2)(3\ 4), (1\ 3)(2\ 4); \\ (1)(2)(3)(4), (1\ 4)(2\ 3), (1\ 2)(3\ 4), (1\ 3)(2\ 4)\}, \quad (2)$$

where we place $\tilde{I} = \overline{(1)(2)(3)(4)}$ to convert eq. 1 into eq. 2.

4. To introduce the viewpoint of the conventional way of stereochemistry, we should take usual stereogenicity into consideration. However, as for allene derivatives, *RS-stereogenicity* coincides with stereogenicity. In other words, stereoisomers can be described within the framework of the *RS-stereoisomeric group* represented by $S_9^{[4]} \times \{I, \sigma\}$. It follows that, in case of allenes, such a stereoisomeric group defined in a previous paper [11] is identical with the *RS-stereoisomeric group* $S_9^{[4]} \times \{I, \sigma\}$, as shown in Fig. 1.
5. When we consider the action of the symmetric group $S^{[4]}$ on the four positions of allene derivatives, we can obtain constitutional isomers that has an allene skeleton as a common skeleton. Such isomers that have an allene skeleton as a common skeleton are called here *isoskeletal isomers*, which are a special kind of constitutional isomers. To describe these isoskeletal isomers, we consider a direct product represented by $S^{[4]} \times \{I, \sigma\}$, which is called *isoskeletal groups*.

It should be noted that (1) enantiomeric relationships are concerned with the point group D_{2d} or the coset representation $D_{2d}/(C_s)$, (2) *RS*-diastereomeric relationships are concerned with the *RS-permutation group* $S_9^{[4]}$, and (3) holantimeric relationships [12] are concerned with the inversion group $\overline{D_{2d}}$. Thus, the three relationships are ascribed to such subgroups as involved in the *RS-stereoisomeric group* $S_9^{[4]} \times \{I, \sigma\}$,

				$S_9^{[4]}$		$S^{[4]} \times \{I, \sigma\}$ $- S_9^{[4]} \times \{I, \sigma\}$
				$S_7^{[4]}$	$(1)(2\ 3)(4)S_7^{[4]}$	
D_2	I	\sim	$(1)(2)(3)(4)$	$(1)(2\ 3)(4)$		
	$C_{2(3)}$	\sim	$(1\ 4)(2\ 3)$	$(1\ 4)(2)(3)$		
	$C_{2(1)}$	\sim	$(1\ 2)(3\ 4)$	$(1\ 2\ 4\ 3)$		
	$C_{2(2)}$	\sim	$(1\ 3)(2\ 4)$	$(1\ 3\ 4\ 2)$		
$D_{2\sigma_d(1)}$	$\sigma_d(1)$	\sim	$\overline{(1)(2\ 3)(4)}$	$\overline{(1)(2)(3)(4)}$		
	$\sigma_d(2)$	\sim	$\overline{(1\ 4)(2)(3)}$	$\overline{(1\ 4)(2\ 3)}$		
	S_4	\sim	$\overline{(1\ 2\ 4\ 3)}$	$\overline{(1\ 2)(3\ 4)}$		
	S_4^3	\sim	$\overline{(1\ 3\ 4\ 2)}$	$\overline{(1\ 3)(2\ 4)}$		
point group: $D_{2d}/(C_s)$						
RS -stereoisomeric group: $S_9^{[4]} \times \{I, \sigma\}$						
stereoisomeric group: $S_9^{[4]} \times \{I, \sigma\}$						
				isoskeletal group: $S^{[4]} \times \{I, \sigma\}$		

Figure 1: Groups for characterizing an allene skeleton.

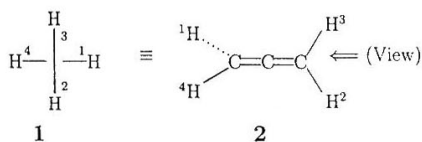


Figure 2: Convention for drawing allene derivatives

3 Isoskeletal Groups and Stereoisogram Sets

Let us next consider the conversions of an allene molecule having ligands ABCp under the action of the isoskeletal group $S_9^{[4]} \times \{I, \sigma\}$ (Figs. 3 to 5).

3.1 Action of *RS*-Stereoisomeric Groups

First, Fig. 3 depicts the action of the *RS*-stereoisomeric group $S_9^{[4]} \times \{I, \sigma\}$, which is classified into four parts according to the following cosets: $S_9^{[4]}$, $(1)(2\ 3)(4)S_9^{[4]}$, $(\overline{1})(2\ 3)(\overline{4})S_9^{[4]}$, and $(1)(2)(\overline{3})(\overline{4})S_9^{[4]}$. The numbering of the original molecule 3a is permuted so as to place ligands in accord with the function: $f(1) = A$, $f(2) = C$, $f(3) = p$, $f(4) = B$. Thereby we obtain molecules collected in Fig. 3.

Each part contains a set of homomers: the $S_9^{[4]}$ -part (the top-left part) is a homomer set of the original molecule, the $(1)(2\ 3)(4)S_9^{[4]}$ -part (the top-right part) is a homomer set of the *RS*-diastereomer, the $(\overline{1})(2\ 3)(\overline{4})S_9^{[4]}$ -part (the bottom-left part) is a homomer set of the enantiomer, and the $(1)(2)(\overline{3})(\overline{4})S_9^{[4]}$ -part (the bottom-right part) is a homomer set of the holantimer. The homomer sets can be expressed by respective representatives, i.e., the original molecule 3a, the *RS*-diastereomer 4a, the enantiomer $\overline{3a}$, and the holantimer $\overline{3a}$, which construct a stereoisogram of Type III, as described later (Fig. 6).

		$S_9^{[4]}$ (stabilizer: $S_9^{[4]})$			
D_{2d}	I	$B^4 \begin{array}{c} p \\ 3 \\ A \\ C \end{array} \begin{array}{c} 1 \\ 3 \\ a \end{array}$	$p^3 \begin{array}{c} B \\ 2 \\ C \\ A \end{array} \begin{array}{c} 1 \\ 2 \\ b \end{array}$	$C^2 \begin{array}{c} A \\ 1 \\ B \\ C \end{array} \begin{array}{c} 1 \\ 3 \\ c \end{array}$	$A^1 \begin{array}{c} C \\ 2 \\ B \\ A \end{array} \begin{array}{c} 1 \\ 3 \\ d \end{array}$
	$C_{2(1)} \sim$	$B^4 \begin{array}{c} C \\ 2 \\ A \\ p \end{array} \begin{array}{c} 1 \\ 3 \\ a \end{array}$	$p^3 \begin{array}{c} B \\ 2 \\ C \\ A \end{array} \begin{array}{c} 1 \\ 2 \\ b \end{array}$	$C^2 \begin{array}{c} A \\ 1 \\ B \\ C \end{array} \begin{array}{c} 1 \\ 3 \\ c \end{array}$	$A^1 \begin{array}{c} C \\ 2 \\ B \\ A \end{array} \begin{array}{c} 1 \\ 3 \\ d \end{array}$
	$C_{2(2)} \sim$	$B^4 \begin{array}{c} C \\ 2 \\ A \\ p \end{array} \begin{array}{c} 1 \\ 3 \\ a \end{array}$	$p^3 \begin{array}{c} B \\ 2 \\ C \\ A \end{array} \begin{array}{c} 1 \\ 2 \\ b \end{array}$	$C^2 \begin{array}{c} A \\ 1 \\ B \\ C \end{array} \begin{array}{c} 1 \\ 3 \\ c \end{array}$	$A^1 \begin{array}{c} C \\ 2 \\ B \\ A \end{array} \begin{array}{c} 1 \\ 3 \\ d \end{array}$
	$C_{2(3)} \sim$	$B^4 \begin{array}{c} C \\ 2 \\ A \\ p \end{array} \begin{array}{c} 1 \\ 3 \\ a \end{array}$	$p^3 \begin{array}{c} B \\ 2 \\ C \\ A \end{array} \begin{array}{c} 1 \\ 2 \\ b \end{array}$	$C^2 \begin{array}{c} A \\ 1 \\ B \\ C \end{array} \begin{array}{c} 1 \\ 3 \\ c \end{array}$	$A^1 \begin{array}{c} C \\ 2 \\ B \\ A \end{array} \begin{array}{c} 1 \\ 3 \\ d \end{array}$
		$(1)(2\ 3)(4)$	$(1\ 2)(3\ 4)$	$(1\ 3)(2\ 4)$	$(1\ 4)(2\ 3)$
		$(1)(2\ 3)(4)$	$(1\ 2)(3\ 4)$	$(1\ 3)(2\ 4)$	$(1\ 4)(2\ 3)$
		$(1)(2\ 3)(4)$	$(1\ 2)(3\ 4)$	$(1\ 3)(2\ 4)$	$(1\ 4)(2\ 3)$
		$(1)(2\ 3)(4)$	$(1\ 2)(3\ 4)$	$(1\ 3)(2\ 4)$	$(1\ 4)(2\ 3)$

Figure 3: Isomer equivalence for ABCp under $S_9^{[4]} \times \{I, \sigma\}$ (Part 1).

3.2 Action of Isoskeletal Groups

Second, we regards the *RS*-stereoisomeric group $S_9^{[4]} \times \{I, \sigma\}$ as a subgroup of the isoskeletal group $S_9^{[4]} \times \{I, \sigma\}$ (cf. Fig. 1). Thereby, we can consider the corresponding coset decomposition, where one of the cosets is listed in Fig. 4. Each permutation is shown in the first row of Fig. 4. The resulting isomers are not stereoisomers but isoskeletal isomers (a kind of constitutional isomers) of the allene derivatives collected in Fig. 3. If the first derivative 5a is selected as an original molecule and its numbering is adopted as an initial one, we obtain permutations shown in the second row of Fig. 4, which is regarded as a direct product of another permutation group, i.e., $S_9^{[4]} \times \{I, \sigma\}$. Note that $S_9^{[4]}$ which is

isomorphic to $S_9^{[4]}$ is a stabilizer group of the coset $(1)(2)(3\ 4)S_9^{[4]}$. These permutations ($\in S_9^{[4]} \times \{I, \sigma\}$) allow us to generate another stereoisogram by starting from Fig. 4. According to the four parts, we select the following representatives of the homomer sets: the second original molecule **5a**, the *RS*-diastereomer **6a**, the enantiomer **5a**, and the holantimer **6a**, which construct a stereoisogram of Type III, as described later (Fig. 6).

		(1)(2)(3 4)S ₉ ^[4] (stabilizer: S ₉ ^[4])							
D _{2d}									
		(1)(2)(3 4)	(1 2)(3)(4)	(1 4 2 3)	(1 3 2 4)	(1)(2 4 3)	(1 2 3)(4)	(1 4 2)(3)	(1 3 4)(2)
		I	$C_{2(1)} \sim$	$C_{2(2)} \sim$	$C_{2(3)} \sim$				
		(1)(2)(3)(4)	(1 2)(3 4)	(1 4)(2 3)	(1 3)(2 4)	(1)(2 4)(3)	(1 2 3 4)	(1 4 3 2)	(1 3)(2)(4)
D _{2d}									
		(1)(2 4 3)	(1 2 3)(4)	(1 4 2)(3)	(1 3 4)(2)	(1)(2)(3 4)	(1 2)(3)(4)	(1 4 2 3)	(1 3 2 4)
		$\sigma_{d(1)} \sim$	$S_4 \sim$	$S_3^2 \sim$	$\sigma_{d(2)} \sim$				
		(1)(2 4)(3)	(1 2 3 4)	(1 4 3 2)	(1 3)(2)(4)	(1)(2)(3 4)	(1 2)(3 4)	(1 4)(2 3)	(1 3)(2 4)

Figure 4: Isomer equivalence for ABC_p under $S^{[4]} \times \{I, \sigma\}$ (Part 2).

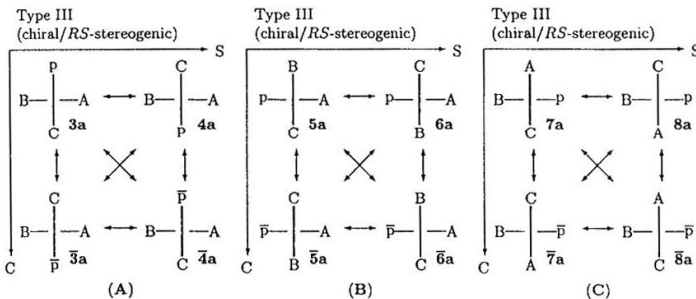
Third, the remaining coset is similarly obtained, as listed in Fig. 5, where each permutation is shown in the first row. Again, the resulting isomers are not stereoisomers but isoskeletal isomers (a kind of constitutional isomers) of the allene derivatives collected in Fig. 3. The first derivative **7a** is selected as a third original molecule and its numbering is adopted as an initial one. This means that the stabilizer group $S_9^{[4]'$ (isomorphic to $S_9^{[4]}$) for the coset $(1\ 3)(2)(4)S_9^{[4]}$ is taken into consideration. Thereby, we obtain permutations shown in the second row of Fig. 4, where these permutations ($\in S_9^{[4]'} \times \{I, \sigma\}$) allow us to generate another stereoisogram by starting from Fig. 5. According to the four parts, we select the following representatives of the homomer sets: the third original molecule **7a**, the *RS*-diastereomer **8a**, the enantiomer **7a**, and the holantimer **8a**, which construct a stereoisogram of Type III.

3.3 Definition of Stereoisogram Sets

Finally, we collect the representatives selected from Figs. 3 to 5 so as to give a set of three stereoisograms (A, B, and C) as shown in Fig. 6. We call the resulting diagram *isoskeletal set of stereoisograms* or simply *stereoisogram set*.

Each stereoisogram shown in Fig. 6 (A, B, or C) is categorized into Type III (chiral/*RS*-stereogenic) according to the scheme shown in previous papers [12, 13]. The original molecule is converted into its *RS*-diastereomer, enantiomer, or holantimer. Such molecules as contained in each stereoisogram are *RS*-stereoisomers, which coincide with stereoisomers in the present case. The three stereoisograms shown in Fig. 6 correspond to three sets (A, B, and C) of *RS*-stereoisomers (stereoisomers), where the sets are isoskeletally iso-

		(1 3)(2)(4)S ₉ ^[4] (stabilizer: S ₉ ^{[4]''})							
D _{2d}	I	B ⁴ _C ¹ ₃ p	A ¹ _p ⁴ ₂ C	C ² _B ³ ₁ A	p ³ _A ² ₄ B	B ⁴ _C ² ₃ p	A ¹ _p ³ ₂ C	C ² _B ⁴ ₁ A	p ³ _A ¹ ₄ B
		C ² _{7a}	p ³ _{7b}	B ⁴ _{7c}	A ¹ _{7d}	A ¹ _{8a}	B ⁴ _{8b}	p ³ _{8c}	C ² _{8d}
		(1 3)(2)(4)	(1 2 3 4)	(1)(2 3)(4)'	(1 4 3 2)	(1 3 2)(4)	(1 2 4)(3)	(1)(2 3 4)	(1 4 3)(2)
		C ₂₍₁₎ ~	C ₂₍₂₎ ~	C ₂₍₃₎ ~					
D _{2d}	I	B ⁴ _C ² ₃ p	A ¹ _p ³ ₂ C	C ² _B ⁴ ₁ A	p ³ _A ¹ ₄ B	B ⁴ _C ¹ ₃ p	A ¹ _p ⁴ ₂ C	C ² _B ³ ₁ A	p ³ _A ² ₄ B
		C ² _{7a}	p ³ _{7b}	B ⁴ _{7c}	A ¹ _{7d}	A ¹ _{8a}	B ⁴ _{8b}	p ³ _{8c}	C ² _{8d}
		(1 3)(2)(4)	(1 4)(2 3)	(1 3)(2 4)	(1 2)(3 4)	(1 2)(3)(4)	(1 3 2 4)	(1 4 2 3)	(1)(2)(3 4)
		C ₂₍₁₎ ~	C ₂₍₂₎ ~	C ₂₍₃₎ ~					
D _{2d}	I	B ⁴ _C ³ ₂ p	A ¹ _p ² ₄ C	C ² _B ¹ ₃ A	p ³ _A ⁴ ₁ B	B ⁴ _C ³ ₂ p	A ¹ _p ² ₄ C	C ² _B ¹ ₃ A	p ³ _A ⁴ ₁ B
		C ² _{7a}	p ³ _{7b}	B ⁴ _{7c}	A ¹ _{7d}	A ¹ _{8a}	B ⁴ _{8b}	p ³ _{8c}	C ² _{8d}
		(1 3 2)(4)	(1 2 4)(3)	(1)(2 3 4)	(1 4 3)(2)	(1 3 2)(4)	(1 2 4)(3)	(1)(2 4)(3)	(1 4 3 2)
		σ _{d(1)} ~	S ₄ ~	S ₄ ² ~	σ _{d(2)} ~				
D _{2d}	I	B ⁴ _C ³ ₂ p	A ¹ _p ² ₄ C	C ² _B ¹ ₃ A	p ³ _A ⁴ ₁ B	B ⁴ _C ³ ₂ p	A ¹ _p ² ₄ C	C ² _B ¹ ₃ A	p ³ _A ⁴ ₁ B
		C ² _{7a}	p ³ _{7b}	B ⁴ _{7c}	A ¹ _{7d}	A ¹ _{8a}	B ⁴ _{8b}	p ³ _{8c}	C ² _{8d}
		(1 2)(3)(4)	(1 4)(2 3)	(1 3)(2 4)	(1 2)(3 4)	(1 2)(3)(4)	(1 4)(2 3)	(1 3)(2 4)	(1 2)(3 4)
		σ _{d(1)} ~	S ₄ ~	S ₄ ² ~	σ _{d(2)} ~				

Figure 5: Isomer equivalence for ABCp under S^[4] × {I, σ} (Part 3).Figure 6: Stereoisogram set (III/III/III) containing three stereoisograms of Type III (A, B, and C) for isoskeletal isomerism of ABCp under S^[4] × {I, σ}

meric to each other. This type of stereoisogram set is designated by the symbol III/III/III, where a slash partitions isoskeletal isomers.

Stereoisogram sets (III/III/III) similar to Fig. 6 are obtained for ligand patterns such as ABCp, ABpq, Ap̄pq, Apqr, p̄pqr, and pqrs.

4 Various Stereoisogram Sets

The scheme deriving a stereoisogram set (e.g., Fig. 6) by starting from permutations of isomers (e.g., Fig. 3) is straightforward, although the initial definition requires the depiction of many molecules. Since the numbering of the top molecule in each part listed in Figs. 3 to 5 is common to every derivation, we can place ligands in accord with a respective function: $f(1)$, $f(2)$, $f(3)$, and $f(4)$. Thereby, representative molecules are

easily obtained without the full consideration of permutations, so as to construct such a stereoisogram set.

4.1 Setereoisogram Sets of I/I/I

Fig. 7 shows a set of three stereoisograms for the isoskeletal isomerism of ligands ABCD under the isoskeletal group $S^{[4]} \times \{I, \sigma\}$. According to the numbering listed in Figs. 3 to 5, we place ligands in accord with the function: $f(1) = A$, $f(2) = B$, $f(3) = C$, $f(4) = D$. Thereby we obtain representative molecules collected in Fig. 7.

The stereoisogram A of Fig. 7 belongs to Type I (achiral/*RS*-astereogenic), which is characterized by diagonal equality symbols. The action of $S^{[4]} \times \{I, \sigma\}$ generates stereoisograms B and C of Type I (chiral/*RS*-stereogenic) in addition to A, where they are different (isoskeletally isomeric) from each other. This type of stereoisogram set is designated by the symbol I/I/I.

Stereoisogram sets (I/I/I) similar to Fig. 7 are found for ligand patterns such as ABCD and p \overline{p} q \overline{q} .

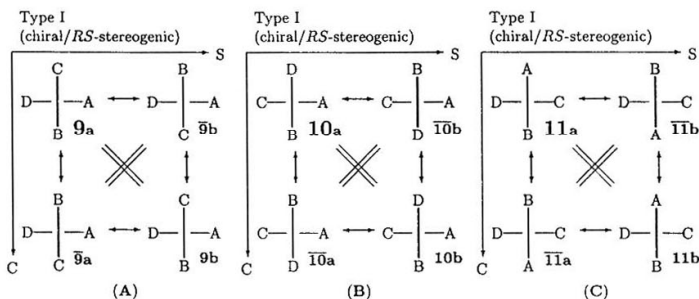


Figure 7: Stereoisogram set (I/I/I) containing three stereoisograms of Type I (A, B, and C) for isoskeletal isomerism of ABCD under $S^{[4]} \times \{I, \sigma\}$

4.2 Setereoisogram Sets of II/III²

Fig. 8 shows a set of three stereoisograms for isoskeletal isomerism of A^2p^2 under $S^{[4]} \times \{I, \sigma\}$. According to the numbering listed in Figs. 3 to 5, we place ligands in accord with the function: $f(1) = A$, $f(2) = p$, $f(3) = p$, $f(4) = B$. Thereby we obtain representative molecules collected in Fig. 8.

The stereoisogram A of Fig. 8 belongs to Type II (chiral/*RS*-astereogenic), which is characterized by horizontal equality symbols. On the other hand, the action of $S^{[4]} \times \{I, \sigma\}$ generates stereoisograms B and B' of type III (chiral/*RS*-stereogenic), which are essentially identical with each other. This type of stereoisogram set is designated by the symbol II/III². The superscript 2 represents the duplication of a stereoisogram.

Stereoisogram sets (II/III^2) similar to Fig. 8 are found for ligand patterns such as A^2p^2 , A^2Bp , A^2pq , ABp^2 , p^2q^2 , $\text{Ap}^2\bar{\text{p}}$, Ap^2q , $\text{p}^2\bar{\text{p}}\text{q}$, $\text{p}^2\text{q}\bar{\text{q}}$, and p^2qr .

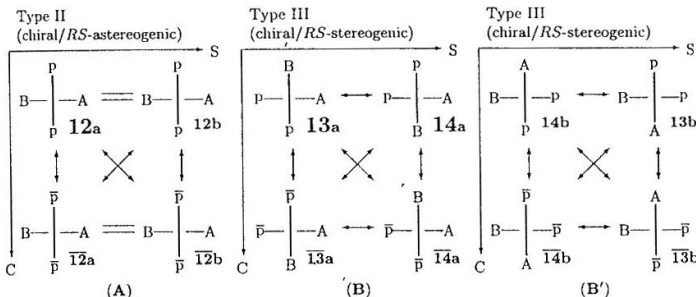


Figure 8: Stereoisogram set (II/III^2) of three stereoisograms (A, B, B') for isoskeletal isomerism of ABp^2 under $\text{S}^{[4]} \times \{I, \sigma\}$. The stereoisogram B is essentially identical with the stereoisogram B'.

4.3 Setereoisogram Sets of IV/I^2

Fig. 9 shows a set of three stereoisograms for isoskeletal isomerism of A^2B^2 under $\text{S}^{[4]} \times \{I, \sigma\}$. According to the numbering listed in Figs. 3 to 5, we place ligands in accord with the function: $f(1) = \text{A}$, $f(2) = \text{B}$, $f(3) = \text{B}$, $f(4) = \text{A}$. Thereby we obtain representative molecules collected in Fig. 9.

The stereoisogram A of Fig. 9 belongs to Type IV (achiral/ RS -astereogenic), which is characterized by equality symbols of every directions. The action of $\text{S}^{[4]} \times \{I, \sigma\}$ generates stereoisograms B and B' of Type I (chiral/ RS -stereogenic), which are essentially identical with each other. This type of stereoisogram set is designated by the symbol IV/I^2 .

Stereoisogram sets (IV/I^2) similar to Fig. 9 are found for ligand patterns such as A^2B^2 , $\text{p}^2\bar{\text{p}}^2$, A^2BC , and $\text{A}^2\text{p}\bar{\text{p}}$.

4.4 Setereoisogram Sets of V/III^2

Fig. 10 shows a set of three stereoisograms for the isoskeletal isomerism of ligands $\text{ABp}\bar{\text{p}}$ under $\text{S}^{[4]} \times \{I, \sigma\}$. According to the numbering listed in Figs. 3 to 5, we place ligands in accord with the function: $f(1) = \text{p}$, $f(2) = \text{B}$, $f(3) = \text{A}$, $f(4) = \bar{\text{p}}$. Thereby we obtain representative molecules collected in Fig. 10.

The stereoisogram A of Fig. 10 belongs to Type V (achiral/ RS -stereogenic), which is characterized by horizontal equality symbols. The stereodiagram A represents so-called pseudoasymmetric cases. The action of $\text{S}^{[4]} \times \{I, \sigma\}$ generates stereoisograms B and B' of Type III (chiral/ RS -stereogenic), which are essentially identical with each other. This type of stereoisogram set is designated by the symbol V/III^2 , where the ligand pattern $\text{ABp}\bar{\text{p}}$ is a sole example.

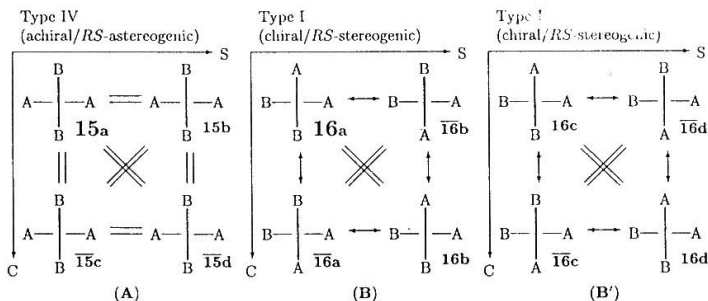


Figure 9: Stereoisogram set (IV/I²) of three stereoisograms (A, B, B') for isoskeletal isomerism of A²B² under S^[4] × {I, σ}. The stereoisogram B is essentially identical with the stereoisogram B'.

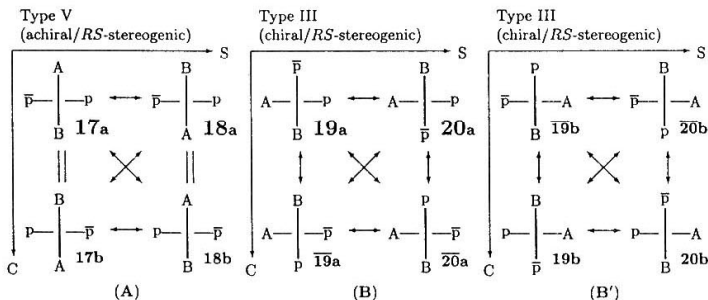


Figure 10: Stereoisogram set (V/III²) of three stereoisograms (A, B, B') for isoskeletal isomerism of ABp \bar{p} under S^[4] × {I, σ}.

4.5 Setereoisogram Sets of IV³

Fig. 11 shows a set of three stereoisograms for the isoskeletal isomerism of ligands A³B under S^[4] × {I, σ}. According to the numbering listed in Figs. 3 to 5, we place ligands in accord with the function: $f(1) = A$, $f(2) = A$, $f(3) = B$, $f(4) = A$. Thereby we obtain representative molecules collected in Fig. 11.

The stereoisogram A of Fig. 11 belongs to Type IV (achiral/RS-astereogenic), which is characterized by equality symbols of all directions. The action of S^[4] × {I, σ} generates identical stereoisograms A' and A'' of Type IV. This type of stereoisogram set is designated by the symbol IV³, where the superscript 3 represents the triplicate of a stereoisogram at issue. The ligand patterns A³B and A⁴ are obtained as examples.

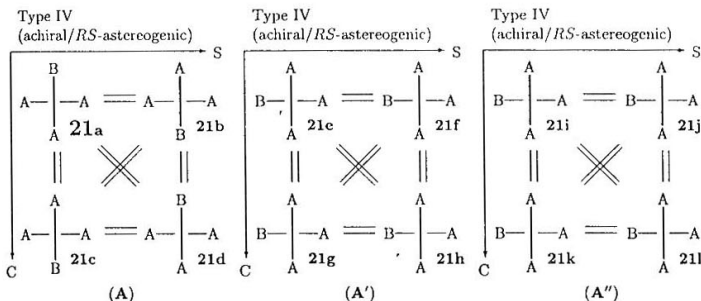


Figure 11: Stereoisogram set (IV^3) of three stereoisograms (A , A' , A'') for isoskeletal isomerism of A^3B under $S^{[4]} \times \{I, \sigma\}$

4.6 Stereoisogram Sets of II^3

Fig. 12 shows a set of three stereoisograms for the isoskeletal isomerism of ligands A^3p under $S^{[4]} \times \{I, \sigma\}$. According to the numbering listed in Figs. 3 to 5, we place ligands in accord with the function: $f(1) = A$, $f(2) = A$, $f(3) = p$, $f(4) = A$. Thereby we obtain representative molecules collected in Fig. 12.

The stereoisogram A of Fig. 12 belongs to Type II (chiral/ RS -astereogenic), which is characterized by horizontal equality symbols. The action of $S^{[4]} \times \{I, \sigma\}$ generates identical stereoisograms A' and A'' of Type II. This type of stereoisogram set is designated by the symbol II^3 .

Stereoisogram sets (II^3) similar to Fig. 12 are found for ligand patterns such as p^4 , A^3p , Ap^3 , $p^3\bar{p}$, and p^3q .

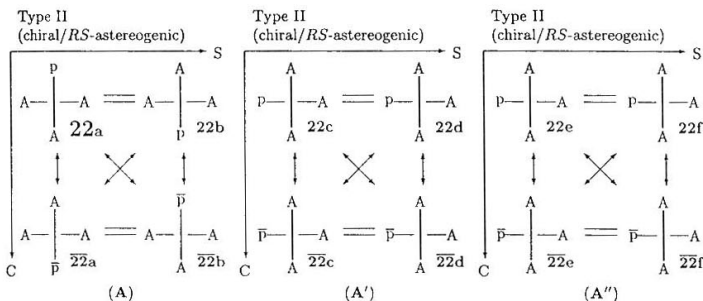


Figure 12: Stereoisogram set (II^3) of three stereoisograms (A , A' , A'') for isoskeletal isomerism of A^3p ($A^3\bar{p}$) under $S^{[4]} \times \{I, \sigma\}$

5 So-called Pseudoasymmetric Axes

In a previous paper [12], we have clarified that Types I, III, and V can be named by the *RS*-nomenclature proposed by Cahn, Ingold, and Prelog [5]. This means that Types I, III, and V can be used for classifying stereogenic units (centers, axes, and planes), e.g., Type I units, Type III units, and Type V units. The discussions described above have shown that allene derivatives recognized as stereogenic axes can be manipulated by this classification of stereogenic units. However, this classification is quite different from conventional ones, i.e., the classification into chiral and stereogenic units [14] or the classification into chiral and pseudoasymmetric units [5]. Among them, the term "pseudoasymmetry" is problematic as pointed out by Mislow [15]. Hence, we here apply the present approach to clarify the meaning of the term "pseudoasymmetry" in the characterization of allene derivatives so that we discuss the validity of the present classification of stereogenic units.

The stereoisogram of Type V (A) shown in Fig. 10 represents a pseudoasymmetric case, in which four different ligands $ABp\bar{p}$ placed on the vertices of an allene skeleton generate achiral molecules 17a and 18a. These molecules are diastereomeric (precisely *RS*-diastereomeric) to each other. The nature of being achiral corresponds to vertical equality symbols in the stereoisogram. If the ligand priority is presumed to be $A > B > p > \bar{p}$, the molecule 17a is determined to have *s*-configuration, while 18a is determined to have *r*-configuration. The lowercase letters *s* and *r* are used to designate pseudoasymmetry.

The other stereodiamgrams of Type III (B and B') are identical with each other and represent a usual case, in which four different ligands $ABp\bar{p}$ placed on the vertices of an allene skeleton generate chiral molecules 19a and 20a. These molecules are diastereomeric (precisely *RS*-diastereomeric) to each other. If the ligand priority is presumed to be $A > B > p > \bar{p}$, the molecule 19a is determined to have *S*-configuration, while 20a is determined to have *R*-configuration. The uppercase letters *S* and *R* are used to designate that these molecules are not related to pseudoasymmetry.

The latest IUPAC 1996 rule [16] has adopted extended pseudoasymmetry, where the term "pseudoasymmetric carbon atom" is defined as "The traditional name for a tetrahedrally coordinated carbon atom bonded to four different entities, two and only two of which have the same constitution but opposite chirality sense. The *r/s* descriptors of pseudoasymmetric carbon atoms are invariant on reflection in a mirror (i.e. *r* remains *r*, and *s* remains *s*), but are reversed by the exchange of any two entities (i.e. *r* becomes *s*, and *s* becomes *r*)." This definition and the term "pseudoasymmetric stereogenic units" described in the revised *RS*-nomenclature [5] force us to describe the first stereoisogram of Type III (A) of Fig. 13 as an example of such extended pseudoasymmetry, where four different ligands $Ap\bar{p}q$ are placed on the vertices of an allene skeleton to generate chiral molecules 23a and 24a. If the ligand priority is presumed to be $A > p > \bar{p} > q$, the molecule 23a is determined to have *S*-configuration, while 24a is determined to have *R*-configuration. The uppercase letters *S* and *R* should be replaced by the corresponding lowercase letters if we obey the revised *RS*-nomenclature. Note that all of the molecules contained in the stereoisogram (A) are chiral.

The other stereodiamgram of Type III (B or C) shown in Fig. 13 represents a usual case, in which four different ligands $Ap\bar{p}q$ placed on the vertices of an allene skeleton to generate chiral molecules 25a and 26a (or 27a and 28a). These molecules are diastereomeric (precisely *RS*-diastereomeric) to each other. If the ligand priority is presumed to be $A > p > \bar{p} > q$, the molecule 25a is determined to have *R*-configuration, while 26a

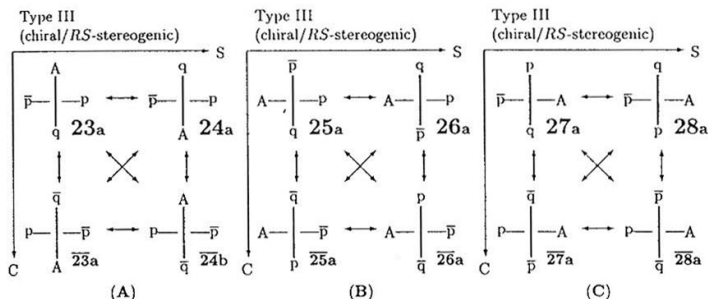


Figure 13: Three stereoisograms (III/III/III) for isoskeletal isomerism of $A p \bar{p} q$ under $S^{[4]} \times \{I, \sigma\}$

is determined to have S -configuration. Under the same priority, the molecule $27a$ is determined to have S -configuration, while $26a$ is determined to have R -configuration. The uppercase letters S and R are used to designate that these molecules are not related to pseudoasymmetry.

The latest IUPAC 1996 rule [16] and the revised RS -nomenclature [5] force us to discriminate the stereoisogram A of Fig. 13 from the other ones (B and C) of Fig. 13, although all of them belong to the same type (Type III). However, there is no reason to discriminate A from B (or C) of Fig. 13. The ligands $A p \bar{p} q$ are different in isolation and still remain different in molecule for the stereoisogram of A; this holds true for the other stereoisogram B (or C) of Fig. 13. It follows that the stereoisogram of A of Fig. 13 is not so an exceptional case to be discriminated from B (or C) of Fig. 13. Concretely speaking, the S -configuration of $23a$ and the S -configuration of $24a$ should be expressed by the uppercase letters, as they are, in disagreement with the latest IUPAC 1996 rule [16] and the revised RS -nomenclature [5].

In contrast, the ligands $A B p \bar{p}$ are different in isolation but p and \bar{p} become equivalent in molecule for the stereoisogram of A of Fig. 10 so that the resulting molecule $17a$ is found to be achiral. This means that the stereoisogram of A of Fig. 10 is an exceptional case to be discriminated from B (or C) of Fig. 10. It follows that the configurations of $17a$ and $18a$ are reasonably expressed by the lowercase letters in agreement with the latest IUPAC 1996 rule [16] and the revised RS -nomenclature [5].

6 Stereogenic Axes vs. Stereogenic Centers

It is worthwhile to compare groups for characterizing an allene skeleton (Fig. 1) with those for characterizing a tetrahedral skeleton (Fig. 14). For an allene skeleton (Fig. 1), we find the following hierarchy of groups: point groups \subset RS -stereogenic groups = stereogenic groups \subset isoskeletal groups. As a result, a stereoisogram based on RS -stereogenic groups is also effective to discuss stereoisomerism controlled by stereogenic groups. Since stereogenic groups are in general contained in isoskeletal groups, a stereoisogram set is

Table 1: Stereoisogram Set for Allenes vs. Stereoisogram for Tetrahedral Molecules

stereoisogram set for allene molecules		stereoisogram for tetrahedral molecules		example (ligand pattern)
I/I/I	(Fig. 7)	I		ABCD, $p\bar{p}q\bar{q}$
II/III ²	(Fig. 8)	II		A^2p^2 , A^2Bp , A^2pq , ABp^2 , p^2q^2 , $Ap^2\bar{p}$, Ap^2q , $p^2\bar{p}q$, $p^2q\bar{q}$, p^2qr
II ³	(Fig. 12)	II		p^4 , A^3p , Ap^3 , $p^3\bar{p}$, p^3q
III/III/III	(Fig. 6)	III		ABCP, ABpq, $Ap\bar{p}q$, $Apqr$, $p\bar{p}qr$, $pqrs$
IV/I ²	(Fig. 9)	IV		A^2B^2 , $p^2\bar{p}^2$, A^2BC , $A^2p\bar{p}$
IV ³	(Fig. 11)	IV		A^3B , A^4
V/III ²	(Fig. 10)	V		$ABp\bar{p}$

necessary to discuss the entire isomerism of allene derivatives.

For a tetrahedral skeleton (Fig. 14), on the other hand, we find the following hierarchy of groups: point groups \subset *RS*-stereogenic groups = stereogenic groups = isoskeletal groups. As a result, a stereoisogram is sufficient to discuss the entire isomerism concerning a tetrahedral skeleton. In other words, skeletal isomers defined in the present paper are unnecessary to be taken into consideration.

Table 1 shows the comparison between stereoisogram sets for Allenes (as stereogenic axes) and stereoisograms for tetrahedral molecules (as stereogenic centers). By replacing the central unit $C=C=C$ of an allene skeleton by a carbon atom, the allene skeleton can be conceptually converted into a tetrahedral skeleton. By replacing the central carbon atom of a tetrahedral skeleton by a unit $C=C=C$, the tetrahedral skeleton can be reversely converted into one of the three allene skeletons. Thus, either one of the three pairs of non-adjacent edges in the tetrahedral skeleton is separated by a unit $C=C=C$, so that a stereoisogram is converted into at most three stereoisogram sets.

The stereoisogram set (I/I/I) for Allenes corresponds to the stereoisogram of Type I for tetrahedral molecules. The stereoisogram sets (II/III² and II³) for Allenes correspond to the stereoisogram of Type II for tetrahedral molecules. The stereoisogram set (III/III/III) for Allenes corresponds to the stereoisogram of Type III for tetrahedral molecules. The stereoisogram sets (IV/I² and IV³) for Allenes correspond to the stereoisogram of Type IV for tetrahedral molecules. The stereoisogram set (V/III²) for Allenes corresponds to the stereoisogram of Type V for tetrahedral molecules.

7 Conclusion

Chirality, *RS*-stereogenicity, stereogenicity, and isoskeletal isomerism for allene derivatives have been comprehensively discussed by considering point groups, *RS*-permutation groups, *RS*-stereoisomeric groups, stereoisomeric groups, and isoskeletal groups. By taking the group-subgroup relationships between these groups into consideration, the isomerism of allene derivatives can be discussed in terms of enantiomers, holantimers, *RS*-diastereomers, diastereomers, and isoskeletal isomers. In the case of allene deriva-

		$S^{[4]}$	
		$S_{10}^{[4]}$	$(1)(2\ 3)(4)S_{10}^{[4]}$
\mathbf{T}	$I \sim$	$(1)(2)(3)(4)$	$(1)(2\ 3)(4)$
	$C_{2(1)} \sim$	$(1\ 2)(3\ 4)$	$(1\ 2\ 4\ 3)$
	$C_{2(2)} \sim$	$(1\ 3)(2\ 4)$	$(1\ 3\ 4\ 2)$
	$C_{2(3)} \sim$	$(1\ 4)(2\ 3)$	$(1\ 4)(2)(3)$
	$C_{3(1)} \sim$	$(1)(2\ 4\ 3)$	$(1)(2)(3\ 4)$
	$C_{3(3)} \sim$	$(1\ 2\ 3)(4)$	$(1\ 2)(3)(4)$
	$C_{3(2)} \sim$	$(1\ 3\ 4)(2)$	$(1\ 3\ 2\ 4)$
	$C_{3(4)} \sim$	$(1\ 4\ 2)(3)$	$(1\ 4\ 2\ 3)$
	$C_{3(1)}^2 \sim$	$(1)(2\ 3\ 4)$	$(1)(2\ 4)(3)$
	$C_{3(4)}^2 \sim$	$(1\ 2\ 4)(3)$	$(1\ 2\ 3\ 4)$
	$C_{3(3)}^2 \sim$	$(1\ 3\ 2)(4)$	$(1\ 3)(2)(4)$
	$C_{3(2)}^2 \sim$	$(1\ 4\ 3)(2)$	$(1\ 4\ 3\ 2)$
$\mathbf{T}\sigma_{d(1)}$	$\sigma_{d(1)} \sim$	$\overline{(1)(2\ 3)(4)}$	$\overline{(1)(2)(3)(4)}$
	$S_{4(3)} \sim$	$\overline{(1\ 2\ 4\ 3)}$	$\overline{(1\ 2)(3\ 4)}$
	$S_{4(3)}^3 \sim$	$\overline{(1\ 3\ 4\ 2)}$	$\overline{(1\ 3)(2\ 4)}$
	$\sigma_{d(6)} \sim$	$\overline{(1\ 4)(2)(3)}$	$\overline{(1\ 4)(2\ 3)}$
	$\sigma_{d(2)} \sim$	$\overline{(1)(2)(3\ 4)}$	$\overline{(1)(2\ 4\ 3)}$
	$\sigma_{d(4)} \sim$	$\overline{(1\ 2)(3)(4)}$	$\overline{(1\ 2\ 3)(4)}$
	$S_{4(1)} \sim$	$\overline{(1\ 3\ 2\ 4)}$	$\overline{(1\ 3\ 4)(2)}$
	$S_{4(1)}^3 \sim$	$\overline{(1\ 4\ 2\ 3)}$	$\overline{(1\ 4\ 2)(3)}$
	$\sigma_{d(3)} \sim$	$\overline{(1)(2\ 4)(3)}$	$\overline{(1)(2\ 3\ 4)}$
	$S_{4(2)}^3 \sim$	$\overline{(1\ 2\ 3\ 4)}$	$\overline{(1\ 2\ 4)(3)}$
	$\sigma_{d(5)} \sim$	$\overline{(1\ 3)(2)(4)}$	$\overline{(1\ 3\ 2)(4)}$
	$S_{4(2)} \sim$	$\overline{(1\ 4\ 3\ 2)}$	$\overline{(1\ 4\ 3)(2)}$
point group: $\mathbf{T}_d(/C_{3v})$			
RS -stereoisomeric group: $S^{[4]} \times \{I, \sigma\}$			
stereoisomeric group: $S^{[4]} \times \{I, \sigma\}$			
isoskeletal group: $S^{[4]} \times \{I, \sigma\}$			

Figure 14: Groups for characterizing a tetrahedral skeleton.

tives, *RS*-stereoisomeric groups have been clarified to coincide with stereoisomeric groups so that diastereomers are identical with *RS*-diastereomers. To discuss the relationship between (*RS*-)diastereomers and isoskeletal isomers, we have proposed the concept of *stereoisogram set*, which contains three stereoisograms categorized into five stereogenicity types (Types I–V) to represent isoskeletal isomerism. The term “pseudoasymmetry” for allene derivatives has been discussed in terms of stereoisograms.

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