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FUZZY CLASSIFICATION AND COMPARISON OF SOME ROMANIAN AND AMERICAN COALS*

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Abstract. Different fuzzy clustering algorithms, namely hierarchical fuzzy clustering, hierarchical and horizontal samples and characteristics clustering and a new clustering technique, namely fuzzy hierarchical cross-classification were applied to the study of several Romanian and American coals using data obtained by applying the two-phase model of coal.

The characteristics clustering technique produces fuzzy partitions of coals properties involved and thus is a useful tool for studying (dis)similarities between different features. The cross-classification algorithm provides not only a fuzzy partition of the coals analysed, but also a fuzzy partition of the characteristics considered. In this way it is possible, for example, to identify which regression parameters or other physico-chemical features are responsible for the similarities or differences observed between different groups of coals.

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*Dedicated to Professor Alexandru T. Balaban on the occasion of his 70[th] anniversary

INTRODUCTION

The "two phase model of coals" considers that macerals on the one hand and minerals on the other hand are well-mixed each of them and behaving as a homogeneous phases. In this case the only location dependent characteristic is the ratio of the two phases, maceral and mineral, respectively. Moreover, the ash content is a measure of this ratio and, as a consequence, any property of a given coal sample appears as a weighted average of the properties of the two specific phases to each coal sample.

The linear relationship between some well-known characteristics of coal and ash content was demonstrated using many samples from different sites and countries [1,2]. The intercept of the ash axis, for example, by the organic carbon straight line, represents the ash content of the "pure mineral phase". By the other way, the intercept by any exclusively mineral property, silica content, for example, will estimate the ash originating in the "pure maceral phase", i.e. organic matter and inorganic ions and complexes. The ash content will vary only between these two limits. Alternatively, using extrapolation to the two limiting values, the estimation of any characteristic specific to each of the two phases could be practically obtained.

Rather than using analytical data of particular samples, advantage was taken of the two phase model of coal, in order to reduce the number of data subjected to fuzzy clustering analysis. The regression lines allow a reliable characterisation of a given coal seam by means of the intercept and the slope for each of the properties being taken into consideration, i.e. by as much as two parameters for each property.

The proposed procedure should be more reliable than the conventional way of using matrices of individual data (see for instance [3, 4]), since each straight line results from many of these, and errors are controlled by fitting to the straight line and rejecting the obviously erroneous data. In addition, the size of matrices can be reduced by about one order of magnitude.

It is the goal of the present paper to check, by the use of fuzzy methods of data classification, whether from chemical point of view there is a significant difference between different Romanian and American coals and, additionally, which are the most specific factors influencing the quality of them. In addition, the fuzzy partition may be successfully used, for instance, in the studying the origin and quality of coals.

FUZZY CLUSTERING

Fuzzy clustering is an important tool to identify the structure in data. In general, a fuzzy clustering algorithm with objective function can be formulated as follows: let $X = \{ x^1, \dots , x^p\} \subset R^s$ be a finite set of feature vectors, where p is the number of measurements and s is the dimension of the input variables, $x_k^j = [x_1^j, x_2^j, \dots , x_s^j]^T$ and $L = (L^1, L^2, \dots , L^n)$ be a n-tuple of prototypes (supports) each of which characterizes one of the n clusters; a partition of X into n fuzzy clusters will be performed by minimizing the objective function [5-7]

$$J(P,L) = \sum_{i=1}^{n}\sum_{j=1}^{p}(A_i(x^j))^2 d^2(x^j, L^i) \qquad (1)$$

where $P = \{ A_1, \dots , A_n \}$ is the fuzzy partition, $A_i(x^j) \in [0,1]$ represents the membership degree of feature point x^j to cluster A_i, $d(x^j, L^i)$ is the distance from a feature point x^j to cluster A_i, defined by the Euclidean distance norm

$$d(x^j, L^i) = \left\| x^j - L^i \right\| = \left[\sum_{k=1}^{s} (x_k{}^j - L^i{}_k)^2 \right]^{1/2} \qquad (2)$$

Since the elements in the set have s features (coordinates) to describe their location in feature space, each cluster center also requires s features to determine its location in the same space.

If L^i is from X we may suppose that L^i has the greatest membership degree to A_i, that is:

$$A_i(L^i) = \max_{x \in X} A_i(x) \qquad (3)$$

The optimal fuzzy set will be determined by using an iterative method where J is successively minimized with respect to A and L.

Supposing that L is given, the minimum of the function $J(\bullet, L)$ is obtained for:

$$A_i(x^j) = \frac{C(x^j)}{\sum_{k=1}^{n} \frac{d^2(x^j, L^i)}{d^2(x^j, L^k)}}, i = 1, \ldots, n \tag{4}$$

For a given P, the minimum of the function $J(P,\bullet)$ is obtained for:

$$L^i = \frac{\sum_{j=1}^{p} A_i(x^j)^2 x^j}{\sum_{j=1}^{p} A_i(x^j)^2}, i = 1, \ldots, n \tag{5}$$

The above formula allows to compute each of the s components of L^i (the center of the cluster i). Each component k is thus a weighted "mean" of the corresponding k-th component of all the elements in the data set. Elements with a high degree of membership in cluster i (i.e. close to cluster i's center) will contribute significantly to this weighted average, while elements with a low degree of membership (far from the center) will contribute almost nothing.

The iterative procedure for obtaining the cluster substructure of the fuzzy class C is called generalized fuzzy n-means (GFNM). The method is firstly useful when the number of classes is unknown.

Fuzzy hierarchical cross-classification algorithm

The method described below is the straightforward way of developing a hierarchical algorithm that should use at each node of the classification tree the algorithm described in [8-11]. We will first show the way of building the classification binary tree. The nodes of the tree are labeled with a pair (C, D), where C is a fuzzy set from a fuzzy partition of samples and D is a fuzzy set from a fuzzy partition of characteristics. The root node corresponds to the pair (X, Y). In the first step the two sub-nodes (A_1, B_1) and respectively (A_2, B_2) will be computed by using the cross-classification algorithm. Of course, these two nodes will be effectively built only if the fuzzy partitions $\{A_1, A_2\}$ and $\{B_1, B_2\}$ describe real clusters. For each of the terminal nodes of the tree we try to determine partitions having the form $\{A_1, A_2\}$ and $\{B_1, B_2\}$. In this way the binary classification tree is extended with two new nodes, (A_1, B_1) and (A_2, B_2). The processes continues until for any terminal node we are not able to determine a structure of real clusters, either for the set of samples, or for the set of characteristics. The final fuzzy partitions will contain the fuzzy sets corresponding to the terminal nodes of the binary classification tree. This algorithm, that we have called FHCC algorithm, seems to be suitable for application where the important idea is to get most of the relationships between different classes of samples and different classes of characteristics [12-15].

Characteristics classification

In what follows we consider that in the classification processes essentially appear two sets: a set X of samples and a set Y of characteristics. As usually, we denote by $X = \{x^1, \dots, x^p\} \subset \mathbf{R}^d$ the set of samples to be classified. A characteristic may be specified by its values corresponding to the p samples. So, we may say that $Y = \{y^1, \dots, y^d\} \subset \mathbf{R}^p$ is the set of characteristics. y_j^k is the value of the characteristic k with respect to the sample j, so we may write

$$y_j^k = x_k^j.$$

Here we discuss the problem of characteristics clustering. This may be useful in many situations of analytical chemistry or environment [16]. For example, the dimensionality reduction may be considered as a classification process of characteristics. The characteristics in the same class (which are, consequently, very similar to each other) will realize a reduced discrimination among the samples. On the contrary, the more distant the classes that contain two different characteristics are, the greater their discrimination power is. If the classes of characteristics are homogenous and well separated, a class may be replaced by the most representative characteristic. This characteristic represents an average of the proprieties of the class. The more compact the class is, the smaller the loss of information produced by this replacement is. In this way we realize a dimensionality reduction.

RESULTS AND DISCUSSION

Fuzzy hierarchical and horizontal classification of the coals

The hard partition corresponding to the fuzzy successive partition of the 31 coals produced by using the autoscaled data in Table 1 are presented in Table 2. Comparing the classes in Table 2 and the membership degrees (MD) to the final fuzzy partition obtained using the GFNM algorithm, we can observe that the results obtained are in good agreement with the quality and origin of coals. Thus, for example, it appears clear that the Pennsylvanian bituminous coals form a very well defined group in the right part of the fuzzy tree separated from the oldest Romanian bituminous coals (Cz, An, Se and Po) and from Romanian Jiu Valley bituminous coals (Ur, Vu, Pe, Lo and Lu). It is also easy to observe that the oldest Romanian bituminous coals are different from others.

TABLE 1. Regression parameters a and b concerning the estimation of carbon, hydrogen and higher heating value for 31 Romanian and American coals

Coal	Code	Carbon		Hydrogen		HHV	
		a*	b*	a*	b*	a*	b*
San Juan	Sj	79.60	-0.866	5.91	-0.059	33.13	-0.369
Waynesburg.Gr.	Wg	84.67	-0.940	5.60	-0.052	35.07	-0.394
Pittsburgh A&I	Pa	85.22	-0.946	5.39	-0.045	35.07	-0.379
Pittsburg Wa.	Pw	82.50	-0.895	5.66	-0.059	34.46	-0.385
Drider	D	87.70	-1.041	5.44	-0.058	36.14	-0.415
Upp. Freeport Cl.	Uf	87.24	-0.941	5.08	-0.047	35.75	-0.390
Low Freeport Cl.	Lfc	87.71	-0.994	5.29	-0.049	35.86	-0.394
Low Freeport Jef.	Lfj	86.63	-1.051	5.33	-0.059	35.49	-0.441
Low Kittaning Cl.	Lk	87.05	-0.934	5.32	-0.048	36.08	-0.394
Mercer Cl.	Mc	87.14	-0.957	5.28	-0.049	36.10	-0.397
Mercer Jef.	Mj	83.28	-0.882	5.13	-0.042	34.97	-0.375
Roşiuţa	Rs	60.47	-0.636	5.12	-0.043	25.54	-0.277
Zăuan	Za	66.50	-0.717	5.06	-0.045	28.09	-0.312
Ţebea	Ţe	73.67	-0.790	5.54	-0.057	29.02	-0.315
Lonea	Lo	77.53	-0.848	5.33	-0.054	32.32	-0.362

TABLE 1. (continued)

Câmpulung	Cl	68.49	-0.721	5.33	-0.046	27.78	-0.291
Ponor	Po	83.90	-0.919	4.45	-0.036	34.67	-0.387
Secu-Doman	Se	87.20	-0.955	4.55	-0.039	35.18	-0.375
Caransebeş	Ca	65.00	-0.700	4.80	-0.040	26.69	-0.285
Ceptura	Ce	62.80	-0.664	5.76	-0.062	26.20	-0.267
Ojasca	Oj	64.20	-0.675	5.76	-0.062	27.02	-0.287
Filipeşti	Fi	64.40	-0.702	5.55	-0.059	26.11	-0.277
Comăneşti	Co	69.50	-0.735	5.50	-0.056	28.66	-0.296
Popeşti-Vzi.	Pv	65.00	-0.734	5.35	-0.059	27.61	-0.306
Rovinari	Rv	63.13	-0.668	5.36	-0.048	27.51	-0.306
Lupeni	Lu	79.32	-0.850	5.39	-0.058	33.66	-0.363
Petrila	Pe	76.50	-0.838	5.18	-0.053	32.66	-0.365
Cozla	Cz	83.70	-0.943	4.94	-0.044	36.26	-0.408
Anina	An	82.50	-0.940	4.76	-0.042	34.80	-0.391
Uricani	Ur	80.20	-0.877	5.18	-0.047	35.36	-0.409
Vulcan	Vu	79.80	-0.897	5.18	-0.049	33.68	-0.362

a* and b* represent the intercept and slope of the straight line for the estimation of each of the property mentioned in the table from ash content

This remarkable individuality is determined by a high content of carbon and hydrogen and also a higher heating value. In the opposite side of the fuzzy tree appear Oltenia and Transylvania lignites – Western part of Romania – (Rs, Rv, and Ca, Za) and Muntenia and Moldova lignites – Eastern part of Romania – (Ce, Oj, Fi, Pv and Co).

TABLE 2. The membership degrees corresponding to the hierarchical final fuzzy partition

Coal	Code	A_{111}	A_{1121}	A_{1122}	A_{1211}	A_{1212}	A_{12211}	A_{12212}
San Juan	Sj	0.052	0.045	0.041	0.114	0.058	0.021	0.027
Waynesburg.Gr.	Wg	0.018	0.014	0.012	0.021	0.013	0.004	0.005
Pittsburgh A&I	Pa	0.009	0.007	0.005	0.008	0.005	0.001	0.002
Pittsburg Wa.	Pw	0.037	0.029	0.026	0.062	0.033	0.011	0.013
Drider	D	0.025	0.017	0.015	0.027	0.018	0.006	0.007
Upp. Freeport Cl.	Uf	0.005	0.003	0.003	0.004	0.003	0.001	0.001
Low Freeport Cl.	Lfc	0.007	0.005	0.004	0.006	0.004	0.001	0.001
Low Freeport Jef.	Lfj	0.029	0.019	0.017	0.029	0.019	0.006	0.008
Low Kittaning Cl.	Lk	0.004	0.003	0.003	0.004	0.002	0.001	0.001
Mercer Cl.	Mc	0.004	0.003	0.002	0.004	0.002	0.001	0.001
Mercer Jef.	Mj	0.021	0.014	0.010	0.012	0.009	0.002	0.002
Roşiuţa	Rs	**0.541**	0.090	0.117	0.023	0.037	0.010	0.010
Zăuan	Za	**0.492**	0.194	0.131	0.010	0.013	0.002	0.002
Ţebea	Ţe	0.046	0.059	0.050	**0.619**	0.015	0.010	0.014
Lonea	Lo	0.049	0.045	0.037	0.081	0.037	0.008	0.010
Câmpulung	Cl	0.063	**0.765**	0.009	0.025	0.040	0.005	0.006
Ponor	Po	0.079	0.038	0.032	0.029	0.026	0.007	0.008
Secu-Doman	Se	0.059	0.031	0.025	0.025	0.021	0.006	0.007
Caransebeş	Ca	**0.565**	0.047	0.038	0.026	0.034	0.008	0.008
Ceptura	Ce	0.044	0.035	0.044	0.016	0.031	**0.644**	0.001
Ojasca	Oj	0.028	0.024	0.031	0.015	0.030	0.001	**0.721**
Filipeşti	Fi	0.022	0.021	0.027	0.025	0.081	0.005	0.005
Comăneşti	Co	0.020	0.031	0.031	0.055	**0.775**	0.007	0.010
Popeşti-Vzi.	Pv	0.034	0.033	0.043	0.102	**0.484**	0.027	0.037
Rovinari	Rv	0.105	0.007	**0.673**	0.033	0.069	0.011	0.012
Lupeni	Lu	0.042	0.036	0.031	0.081	0.038	0.009	0.012
Petrila	Pe	0.053	0.044	0.036	0.059	0.035	0.007	0.008
Cozla	Cz	0.017	0.010	0.008	0.010	0.007	0.002	0.002
Anina	An	0.036	0.020	0.016	0.017	0.014	0.004	0.004
Uricani	Ur	0.010	0.007	0.006	0.007	0.005	0.001	0.001
Vulcan	Vu	0.016	0.012	0.009	0.013	0.008	0.002	0.002

TABLE 2. The membership degrees corresponding to the hierarchical final fuzzy partition (continued)

Coal	Code	A_{1222}	A_{211}	A_{212}	A_{221}	A_{2221}	A_{2222}
San Juan	Sj	0.048	**0.343**	0.102	0.103	0.017	0.028
Waynesburg.Gr.	Wg	0.010	**0.448**	0.383	0.057	0.005	0.009
Pittsburgh A&I	Pa	0.004	0.105	**0.479**	0.327	0.014	0.033
Pittsburg Wa.	Pw	0.026	**0.562**	0.076	0.092	0.012	0.021
Drider	D	0.014	0.302	**0.389**	0.128	0.019	0.033
Upp. Freeport Cl.	Uf	0.002	0.033	0.182	**0.685**	0.018	0.060
Low Freeport Cl.	Lfc	0.003	0.033	**0.669**	0.223	0.013	0.030
Low Freeport Jef.	Lfj	0.016	0.273	**0.348**	0.159	0.027	0.048
Low Kittaning Cl.	Lk	0.002	0.036	**0.674**	0.243	0.008	0.019
Mercer Cl.	Mc	0.002	0.015	**0.710**	0.226	0.008	0.021
Mercer Jef.	Mj	0.006	0.041	0.095	**0.629**	0.039	0.120
Roşiuţa	Rs	0.032	0.036	0.030	0.038	0.016	0.019
Zăuan	Za	0.007	0.037	0.030	0.044	0.017	0.021
Ţebea	Ţe	0.037	0.063	0.029	0.037	0.008	0.012
Lonea	Lo	0.023	**0.375**	0.120	0.160	0.019	0.036
Câmpulung	Cl	0.020	0.019	0.014	0.019	0.006	0.008
Ponor	Po	0.019	0.064	0.106	0.075	**0.468**	0.048
Secu-Doman	Se	0.016	0.055	0.102	0.056	**0.557**	0.038
Caransebeş	Ca	0.025	0.054	0.050	0.068	0.036	0.038
Ceptura	Ce	0.066	0.041	0.026	0.030	0.009	0.012
Ojasca	Oj	0.037	0.041	0.024	0.028	0.008	0.011
Filipeşti	Fi	**0.755**	0.020	0.013	0.015	0.005	0.006
Comăneşti	Co	0.038	0.012	0.007	0.009	0.002	0.003
Popeşti-Vzi.	Pv	0.191	0.017	0.010	0.012	0.004	0.005
Rovinari	Rv	0.043	0.014	0.010	0.013	0.004	0.005
Lupeni	Lu	0.026	**0.479**	0.078	0.124	0.016	0.028
Petrila	Pe	0.019	**0.285**	0.153	0.221	0.027	0.054
Cozla	Cz	0.005	0.018	0.055	**0.502**	0.047	0.315
Anina	An	0.010	0.025	0.053	0.048	0.024	**0.727**
Uricani	Ur	0.003	0.116	0.262	**0.518**	0.015	0.048
Vulcan	Vu	0.005	0.213	**0.260**	0.385	0.020	0.052

The same main conclusions are strongly supported by the results obtained by applying horizontal classification of coals for 2 and 3 respectively predefined classes as is shown in the Table 3: a high similarity between Pennsylvanian and Romanian bituminous coals.

TABLE 3. The membership degrees corresponding to the horizontal fuzzy clustering with 2 (A_1-A_2) and 3 (A_1-A_3) respectively predefined classes

Coal	Code	A_1	A_2	A_1	A_2	A_3
San Juan	Sj	0.406	**0.593**	**0.408**	0.406	0.185
Waynesburg.Gr.	Wg	0.097	**0.903**	0.097	**0.826**	0.076
Pittsburgh A&I	Pa	0.041	**0.959**	0.035	**0.926**	0.038
Pittsburg Wa.	Pw	0.236	**0.764**	0.249	**0.609**	0.143
Drider	D	0.129	**0.870**	0.125	**0.778**	0.097
Upp. Freeport Cl.	Uf	0.022	**0.978**	0.016	**0.965**	0.019
Low Freeport Cl.	Lfc	0.031	**0.969**	0.024	**0.952**	0.023
Low Freeport Jef.	Lfj	0.144	**0.856**	0.136	**0.754**	0.110
Low Kittaning Cl.	Lk	0.020	**0.980**	0.015	**0.970**	0.015
Mercer Cl.	Mc	0.019	**0.981**	0.014	**0.973**	0.013
Mercer Jef.	Mj	0.076	**0.924**	0.061	**0.851**	0.088
Roşiuţa	Rs	**0.861**	0.139	0.150	0.048	**0.802**
Zăuan	Za	**0.850**	0.150	0.016	0.006	**0.977**
Ţebea	Ţe	**0.850**	0.150	**0.826**	0.060	0.113
Lonea	Lo	0.289	**0.711**	0.265	**0.542**	0.193
Câmpulung	Cl	**0.932**	0.068	0.125	0.029	**0.846**
Ponor	Po	0.238	**0.761**	0.152	**0.588**	0.259
Secu-Doman	Se	0.191	**0.809**	0.132	**0.663**	0.205
Caransebeş	Ca	**0.754**	0.246	0.118	0.067	**0.815**
Ceptura	Ce	**0.882**	0.118	**0.781**	0.056	0.163
Ojasca	Oj	**0.887**	0.113	**0.843**	0.043	0.114
Filipeşti	Fi	**0.941**	0.059	**0.856**	0.029	0.115
Comăneşti	Co	**0.967**	0.033	**0.944**	0.012	0.044
Popeşti-Vzi.	Pv	**0.952**	0.048	**0.864**	0.027	0.110
Rovinari	Rv	**0.954**	0.046	0.228	0.038	**0.733**
Lupeni	Lu	0.276	**0.724**	0.283	**0.552**	0.164
Petrila	Pe	0.261	**0.739**	0.220	**0.571**	0.209
Cozla	Cz	0.062	**0.938**	0.049	**0.886**	0.064
Anina	An	0.122	**0.877**	0.091	**0.769**	0.139
Uricani	Ur	0.040	**0.960**	0.038	**0.918**	0.044
Vulcan	Vu	0.069	**0.931**	0.072	**0.845**	0.083

Fuzzy hierarchical and horizontal classification of the characteristics

The characteristics hierarchical clustering, considering regression parameters a and b concerning the estimation of carbon, hydrogen and higher heating value for 31 Romanian and American coals with the scaling of data gave the final partition presented in Table 4.

TABLE 4. The membership degrees corresponding to the hierarchical final fuzzy partition and horizontal fuzzy clustering with 3 predefined classes (A_1 – A_3) of the characteristics of coals

Code	A_{111}	A_{112}	A_{12}	A_{211}	A_{212}	A_{22}	A_1	A_2	A_3
a (carbon)	**0.967**	0.005	0.006	0.004	0.004	0.014	**0.975**	0.010	0.015
b (carbon)	0.005	0.005	0.016	**0.957**	0.003	0.014	0.007	**0.977**	0.016
a (hydrogen)	0.088	0.087	0.121	0.001	0.001	**0.701**	0.001	0.002	**0.996**
b (hydrogen)	0.001	0.001	**0.681**	0.098	0.096	0.123	**0.530**	0.292	0.177
a (HHV)	0.004	**0.968**	0.007	0.004	0.004	0.013	**0.978**	0.008	0.013
b (HHV)	0.005	0.005	0.016	0.003	**0.956**	0.015	0.007	**0.975**	0.018

The first features separated from the others are the intercepts corresponding to the carbon content and the higher heating value respectively. By the other way the class A_{21} contains the slope of the carbon content and the slope of the higher heating value again. It appears clear that the higher heating of coals is strongly affected by carbon content (as it is well known) and the main conclusion is that the algorithm works well.

The horizontal characteristics clustering with a predefined number of three classes (see table 4) illustrates the same aspect i.e. the high similarity of the regression parameters a and b concerning the estimation of carbon and higher heating value respectively corresponding to 31 Romanian and American coals.

Fuzzy hierarchical cross-classification

The classification hierarchy produced in this way for the samples (31 coals) and the characteristics (regression parameters a and b concerning the estimation of carbon, hydrogen and higher heating value) using scaling data is presented in Table 5. The partitioning of the coals in different classes is similar with those obtained above by the usual fuzzy clustering algorithms and the main conclusion is that also this algorithm works well. What is relatively different but very useful is the partitioning of the characteristics and their association with different types of coals. The regression parameter associated to the class A_{11} including mainly Oltenia and Transylivania lignites (Rs, Za, Cl, Rv) are intercept corresponding to the estimation of carbon and the intercept resulted from the estimation of the higher heating value. By the other way the regression parameter associated to Muntenia and Moldova lignites (Ce, Oj, Fi, Co, Pv) is the slope corresponding to the estimation of hydrogen. The oldest Romanian bituminous coals and the majority of Pennsylvanian bituminous coals are characterized by a very close content of carbon and as a consequence a high higher heating value.

TABLE 5 The cross-classification of the coals (**c**) and regression parameter(**p**) produced with the scaling data in Table 1.

Fuzzy class	Coals (**c**) and regression parameter associated (**p**)
A_{111}	(**c**) Rs, Za, Ca (**p**) a (HHV)
A_{112}	(**c**) Cl, Rv (**p**) a (carbon)
A_{12}	(**c**) Ţe, Ce, Oj, Fi, Co, Pv (**p**) b (hydrogen)
A_{211}	(**c**) Sj, Wg, Pw, Lo, Lu, Pe (**p**) b (HHV)
A_{212}	(**c**) Pa, D, Lfc, Lfj, Lk, Mc, Vu (**p**) b (carbon)
A_{22}	(**c**) Uf, Mj, Po, Se, Cz, An, Ur (**p**) a (hydrogen)

CONCLUSIONS

Different fuzzy clustering algorithms were applied to the study of some Romanian and American coals using the regression parameters a and b concerning the estimation of carbon, hydrogen and higher heating value resulted from two phase model of coals. The results obtained indicated a good performance in terms of classification and prediction for all the fuzzy clustering algorithms applied. However, the new fuzzy approach namely, fuzzy cross-classification algorithm allows the qualitative and quantitative identification of the characteristics (carbon and hydrogen content and also higher heating value) responsible for the observed similarities and dissimilarities between the coals considered. The regression parameters concerning the estimation of carbon, hydrogen and higher heating value provided sufficient information to enable classification rules to be developed for identifying coals according with their origin and quality. As a main conclusion of this study it appeared clear that the oldest Romanian bituminous coals and the majority of Pennsylvanian bituminous coals are characterized by a very close content of carbon and a high higher heating value and as a consequence they are very similar.

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