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MATHEMATICAL MODELING OF POLYMERS. PART II.[1]
IRREDUCIBLE SEQUENCES IN n-ARY COPOLYMERS

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Abstract. *The number of irreducible sequences in n-ary copolymers ($n = 2,3,4$) is shown to be related to the graph-theoretical necklace problem, with supplementary conditions involving presence of all n monomer types at least once, invariance to relabelling of vertices, and impossibility of decomposition into smaller repeating subsequences. It is shown that by combining the Polya-de Bruijn theorem with the Möbius function of the lattice of divisors one may solve this problem. A general formula is also presented for the number of irreducible sequences in n-ary copolymers. A computer program was devised for generating and enumerating all irreducible sequences involving $n = M$ types of comonomers wherein the repeating irreducible sequence contains $m = N$ monomer units.*

Introduction

In the previous part of this series,[1] we defined and discussed irreducible sequences in binary copolymers, and showed that the same sequences would be found in stereoregular homopolymers or polybutenamers ; these irreducible sequences were formed from a binary alphabet. The present paper generalizes the problem to irreducible sequences corresponding to n-ary copolymers, wherein the alphabet consists of n letters. In chemical practice,[2] higher alphabets than $n = 5$ are not interesting.

The necklace problem and irreducible sequences

In Part I of the present series [1] it was shown that the enumeration of irreducible sequences in binary copolymers is related to the enumeration of diastereomeric cycloalkanes bearing one type of substituent at each carbon atom (ignoring enantiomerism), and to the "necklace problem" with beads of two colors.

A hierarchical order of the colors is accepted (corresponding to priority rules for letters of the alphabet, namely $R < S < T < U$), giving rise to a lexicographically ordered list of all irreducible sequences. The enumeration of irreducible sequences in n-ary copolymers as discussed above is equivalent to enumerating the distinct necklaces formed from beads of n colors, provided that three supplementary conditions are added to the well--known[3] necklace problem :

1) Two or more non-isomorphic necklaces correspond to one and the same irreducible sequence if, on relabeling the colors, one obtains the same necklace (i.e. if on permuting the letters of the given alphabet one obtains the same sequence). The necessary and sufficient condition for the existence of such distinct necklaces which give rise to the same irreducible sequence is that they contain at least two types of differently colored beads in equal numbers, with the exception of necklaces containing three types of beads with two single beads of different colors.

For instance the partition R^2S^2T of $m = 5$ beads into $n = 3$ colors gives rise according to Polya's theorem to four necklaces which according to the lexicographic order can be linearised as RRSST, RSRST, RSRST and RSSRT ; the fourth necklace is converted, however, into the third one by permutation (RS)(T) so that only the first three necklaces correspond to irreducible sequences. As an exception, all necklaces with n beads of 3 colors partitioned into 1,1, and n-2 beads of the same color correspond to irreducible sequences.

The special position for this partition $R^{n-2}ST$ is connected to the facts that the two faces of such necklaces correspond to a permutation of the S and T symbols, and that the macromolecular chain of repeating sequences has no privileged direction from one end to the other.

2) No irreducible sequence should consist of smaller repeating subsequences. Whenever m is not a prime number, such repeating subsequences may be possible. Thus, there are two necklaces with m = four beads of n = two colors, but only one irreducible sequence, namely RRSS, because the sequence RSRS can be decomposed into two repeating subsequences. Similarly, RSRSRS, RRSRRS and RSTRST are not irreducible sequences with m = 6 because they can be decomposed into smaller, repeated subsequences.

3) Any sequence of m symbols which does not contain all n symbols of the alphabet is reducible to a sequence corresponding to a lower alphabet.

When all these conditions for non-equivalence are fulfilled, we call such chemically non-equivalent sequences :"irreducible sequences".

The necklace problem can be solved by means of Polya's theorem.[4] A diagram with the number of necklaces with up to three colors and eight beads may be found in Table 13 of reference[5]. For the general case, the number of necklaces with r_1 beads of color 1, r_2 beads of color 2, ..., r_n beads of color n ($\sum_i r_i = m$), is given by coefficient of the term $x_1^{r_1}x_2^{r_2}\ldots x_n^{r_n}$ in the polynomial obtained, according to Polya's theorem, on substituting the figure-counting series

$$y_k = \sum_{i=1}^{n} x_i^k$$

into the cycle index of the dihedral group:

$$Z(D_m) = \tfrac{1}{2}Z(C_m) + \tfrac{1}{2}y_1 y_2^{(m-1)/2} \quad \text{for m odd}$$

$$Z(D_m) = \tfrac{1}{2}Z(C_m) + \tfrac{1}{4}(y^{m/2} + y_1^2 y_2^{(m-2)/2}) \quad \text{for m even}$$

where the cycle index of the cyclic group is :

$$Z(C_m) = \frac{1}{m}\sum_{k|m} \phi(k)\, y_k^{m/k}$$

The symmetry operation y_k permutes k points of the m-gon ; $\phi(k)$ denotes the Euler ϕ-function (i.e. the number of positive integers less than k and relatively prime to k) ; $k|m$ indicates that k divides into m.

Results are presented in Table 1.

TABLE 1. Cycle indices of dihedral groups.

$Z(D_4) = (y_1^4 + 2y_1^2y_2 + 3y_2^2 + 2y_4)/8$

$Z(D_5) = (y_1^5 + 5y_1y_2^2 + 4y_5)/10$

$Z(D_6) = (y_1^6 + 4y_2^3 + 2y_3^2 + 3y_1^2y_2^2 + 2y_6)/12$

$Z(D_7) = (y_1^7 + 7y_1y_2^3 + 6y_7)/14$

$Z(D_8) = (y_1^8 + 2y_4^2 + 4y_8 + 5y_2^4 + 4y_1^2y_2^3)/16$

$Z(D_9) = (y_1^9 + 2y_3^3 + 9y_1y_2^4 + 6y_9)/18$

A list of configuration-counting series (polynomials in y_k for various values of m) is to be found in references[6,7]. The coefficient of each term $x_1^r x_2^s \ldots x_n^v$ (where $m = \sum_i r_i = r + s + \ldots + v$ is the number of beads, some of the terms s,..., v may be zero, and n is the number of colors, $n \leqslant m$), is the number of distinct necklaces.

In Table 2 one can see the numbers NK of necklaces (left-hand figures for each partition of m), according to Polya's theorem. The numbers IS of irreducible sequences for each partition, obtained according to the computer program (last section of this paper) and the total number N(m, n) of irreducible sequences obtained either according to the Polya -de Bruijn theorem (following section of this paper) or to a generalized formula (presented in a subsequent section) are also displayed in Table 2 for $3 \leqslant m \leqslant 8$ and $2 \leqslant n \leqslant 5$. Results for binary copolymers (n = 2) agree with those described in Part I of the present series.[1] One should note that, in agreement with condition 1 indicated above, partition R^2S^2TU may stand for R^2ST^2U, or $R^3S^2T^2$ for $R^2S^3T^2$, etc. All irreducible sequences corresponding to the same partition are isomeric with one another. A list of irreducible sequences with n = 2, 3, and 4, and m = 2 through 7, arranged according to partitions, is to be found in Part III of the present series.[8]

TABLE 2. Numbers NK of necklaces (left-hand figure) and irreducible sequences IS (right-hand figure) for each partition $R^rS^s...V^v$ $(r + s + ... + v = m)$, and total number N(m,n) of irreducible sequences for given values of the number n of monomer types and the length m of the sequence.

n → / m ↓	2				3				4				5			
m	NK	Partition	IS	N(m,n)	NK	Partition	IS	N(m,n)	NK	Partition	IS	N(m,n)	NK	Partition	IS	N(m,n)
2	1	RS	1	1	-	-	-	-	-	-	-	-	-	-	-	-
3	1	R^2S	1	1	1	RST	1	1	-	-	-	-	-	-	-	-
4	1	R^3S	1	2	2	R^2ST	2	2	3	RSTU	1	1	-	-	-	-
	2	R^2S^2	1													
5	1	R^4S	1	3	2	R^3ST	2	5	6	R^2STU	2	2	12	RSTUV	1	1
	2	R^3S^2	2		4	R^2S^2T	3									
6	1	R^5S	1	5	3	R^4ST	3	13	10	R^3STU	3	11	30	R^2STUV	3	3
	3	R^4S^2	2		6	R^3S^2T	6		16	R^2S^2TU	8					
	3	R^3S^3	2		11	$R^2S^2T^2$	4									
7	1	R^6S	1	8	3	R^5ST	3	31	15	R^4STU	4	33	60	R^3STUV	4	16
	3	R^5S^2	3		9	R^4S^2T	9		30	R^3S^2TU	17		90	R^2S^2TUV	12	
	4	R^4S^3	4		10	R^3S^3T	7		48	$R^2S^2T^2U$	12					
					18	$R^3S^2T^2$	12									
8	1	R^7S	1	14	4	R^6ST	4	80	21	R^5STU	5	136	105	R^4STUV	8	85
	4	R^6S^2	4		12	R^5S^2T	12		54	R^4S^2TU	33		210	R^3S^2TUV	38	
	5	R^5S^3	5		19	R^4S^3T	19		70	R^3S^3TU	25		1155	$R^2S^2T^2UV$	39	
	8	R S	4		33	$R^4S^2T^2$	20		108	$R^3S^2T^2U$	57					
					38	$R^3S^3T^2$	25		171	$R^2S^2T^2U^2$	16					

A formula for calculating the number $N(m,n)$ of irreducible sequences based upon the Polya-de Bruijn theorem

Let $n \geqslant 2$ and $m \geqslant n$ be two natural numbers, and S_n the symmetric group of order n. For any two natural numbers a and b, we denote by $[a,b]$ the range of natural numbers in this interval. The set of surjections $\alpha : [0,a-1] \to$ $\to [0,b-1]$ is denoted by $F(a,b)$. This set $F(a,b)$ may be considered as the set of sequences with length a over an alphabet containing b symbols such that all these symbols are contained at least once in any sequence. In this context we shall consider a sequence as a function. The cardinal of this set is :

$$s_{a,b} = |F(a,b)| = \sum_{i=0}^{b-1} (-1)^i \binom{b}{i} (b-i)^a$$

For any natural number $p \geqslant n$ which divides the number m, we denote by A_p the set of functions in n^m having p as primitive period (A_m is the set of non-periodic functions).

Let X and Y be two finite sets : $X = \{a_1, \ldots, a_m\}$ and $Y = \{b_1, \ldots, b_n\}$. The set X may represent the objects and the set Y may represent their colors. Let G be a permutation group on X and H a permutation group on Y. A function $f : X \to Y$ is called a coloring of objects in X with colors in Y. On the set Y^X of functions defined on X and with values in Y we define the equivalence relationship (denoted by $\div$) as follows : $f,g \in Y^X$, $f \div g$ iff there exists a permutation $\pi \in G$ and a permutation $\sigma \in H$ and such that $g = \sigma f \pi$, i.e. for any $a \in X$, we have $g(a) = \sigma(f(\pi(a)))$. We observe that the equivalence $\div$ depends on the choice of permutation groups G and H.

We denote with Λ the set of classes of $\div$ equivalence, called the set of coloring schemes ; only the structure of coloring matters, and not the actual colors.

de Bruijn, by generalizing Polya's results, obtained the so-called "Polya-de Bruijn theorem" (formulated here for the particular case of finite sets and of weights equal to one).[9]

The number of coloring schemes (defined as above) is :

$$B_{G,H}(m,n) = |\Lambda| = P_G\left(\frac{\partial}{\partial z_1}, \frac{\partial}{\partial z_2}, \ldots, \frac{\partial}{\partial z_m}\right) P_H(\rho_1, \rho_2, \ldots, \rho_n)$$

evaluated in $z_1 = z_2 = \ldots = z_m = 0$, where

$\rho_i = \exp\{i(z_i + z_{2i} + z_{3i} + \ldots)\}$, and P_G and P_H are the configuration-counting series of the permutation groups G and H, respectively. For simplifying the notation, whenever G and H are evident from the context, we shall abbreviate $B_{G,H}(m,n)$ to $B(m,n)$.

Example. Let $|X| = 6$ and $|Y| = 3$, therefore $G = D_6$, the dihedral group of order six, and $H = S_3$, the symmetric group of order 3.

The corresponding configuration-counting series are :

$$Z(D_6) = \tfrac{1}{2}\left[\tfrac{1}{6}\sum_{k|6} \phi(k) y_k^{6/k} + \tfrac{1}{2}(y_2^3 + y_1^2 y_2^2)\right]$$

leading to the expression from Table 1.

From the expression of the $Z(S_k)$:

$$Z(S_k) = \sum_{\lambda_1 + 2\lambda_2 + \ldots + k\lambda_k = k} \frac{1}{\lambda_1!\lambda_2!\ldots\lambda_k!\,1^{\lambda_1}2^{\lambda_2}\ldots k^{\lambda_k}} \cdot y_1^{\lambda_1} y_2^{\lambda_2}\ldots y_k^{\lambda_k}$$

we obtain for $k = 3$

$$Z(S_3) = y_1^3/6 + y_1 y_2/2 + y_3/3.$$

On applying the Polya-de Bruijn theorem one obtains :

$$B(6,3) = \tfrac{1}{12}\left(\frac{\partial^6}{\partial z_1^6} + 4\frac{\partial^3}{\partial z_2^3} + 3\frac{\partial^2}{\partial z_1^2}\frac{\partial^2}{\partial z_2^2} + 2\frac{\partial^2}{\partial z_3^2} + \frac{\partial}{\partial z_6}\right) \cdot$$

$$\cdot\left(\tfrac{1}{6}e^{3(z_1+z_2+\ldots+z_6)} + \tfrac{1}{2}e^{z_1+3z_2+z_3+3z_4+z_5+3z_6} + \tfrac{1}{3}e^{3z_3+3z_6}\right)$$

for $z_1 = z_2 = \ldots = z_6 = 0$.

We obtain, therefore :

$$B(6,3) = \tfrac{1}{12}\left(\frac{3^6}{6} + \frac{1}{2} + 0 + \frac{4\cdot 3^3}{6} + \frac{4\cdot 3^3}{2} + 0 + \frac{3\cdot 3^2\cdot 3^2}{6} + \frac{3\cdot 3^2}{2} + 0 + \frac{2\cdot 3^2}{6} + \frac{2}{2} + \right.$$

$$\left. + \frac{2\cdot 3^2}{3} + \frac{2\cdot 3}{6} + \frac{2\cdot 3}{2} + \frac{2\cdot 3}{3}\right) = 22.$$

If, in the Polya-de Bruijn theorem, we consider the particular case $G = D_m$ and $H = S_n$, then we observe that the restriction of the $\div$ relationship to the set of non-periodic and surjective functions $f \in Y^X$, denoted by $F(m,n) \cap A_m$, is exactly the chemical equivalence defined earlier.[1]

We observe that the equivalence class of a noperiodic or a surjective function is formed only from nonperiodic or surjective functions, respectively. Furthermore, the property of surjectivity is transmitted to any restriction to a period. Therefore we may write (denoting by $N'(m)$ the number of ÷ equivalence classes from $F(m,n)$, and by $N(m)$ the number of ÷ equivalence classes from $F(m,n) \cap A_n$, i.e. the number of classes of chemical equivalence we wish to obtain) :

$$N'(m,n) = B_{D_m,S_n}(m,n) - B_{D_m,S_{n-1}}(m,n-1)$$

$$N'(m,n) = \sum_{q|n} N(q)$$

By using the Möbius inversion theorem (described in the previous Part[1]) we obtain :

$$N(m,n) = \sum_{q|m} \mu(q,m) \left[B_{D_q,S_n}(q,n) - B_{D_q,S_{n-1}}(q,n-1)\right]$$

where $\mu(q,m)$ is the Möbius function of the lattice of divisors.

Example. For $m = 6$ and $n = 3$, we calculate first by means of the Polya-de Bruijn theorem $B(6,2) = 8$; $B(3,3) = 3$; $B(3,2) = B(2,2) = B(2,3) = 2$; $B(1,2) = B(1,3) = 1$. On using the result of the previous example, $B(6,3) = 22$, and on performing the calculations, we obtain

$$N(6,3) = (22 - 8) - (3 - 2) - (2 - 2) + (1 - 1) = 13.$$

This value coincides with that obtained for irreducible sequences of length 6 in ternary copolymers according to the computer program presented below ; the same value is displayed in Table 2.

An explicit formula (based on the Möbius inversion theorem) for computing the number $N(m,n)$ of irreducible sequences

We denote by P_k the set of permutations π with order k in S_n, where k is a natural number. If e denotes the identity permutation, we have : $\pi^k = e$, and $\pi^u \neq e$ for any $u \in [1,k-1]$. It may be observed that k is the lowest common multiple of the lengths of cycles in the unique decomposition of π into disjoint cycles.

A permutation is said to be of type $(\lambda_1,\lambda_2,\ldots,\lambda_n)$ if it contains λ_i ($i \in [1,n]$) cycles of length i in its decompositon into disjoint cycles.

Evidently, $\sum_{i=1}^{n} i\lambda_i = n$.

According to Cauchy's formula, the number $h(\lambda_1,\lambda_2,\dots,\lambda_n)$ of permutations of type $(\lambda_1,\lambda_2,\dots,\lambda_n)$ is :

$$h(\lambda_1,\lambda_2,\dots,\lambda_n) = \frac{n!}{\lambda_1!\lambda_2!\dots\lambda_n!\ 1^{\lambda_1}2^{\lambda_2}\dots n^{\lambda_n}}$$

As a simplified notation, whenever n is known from the context, we shall write $h(\lambda_1,\lambda_2)$ instead of $h(\lambda_1,\lambda_2,0,\dots,0)$.

The set of n-uples indicating the permutation types from S_n which have order k is denoted by W_k. The cardinal $|W_k|$ is the number of permutation types with order k, and the number of permutations with order k is :

$$\sum_{(\lambda_1,\dots,\lambda_n) \in W_k} h(\lambda_1,\dots,\lambda_n)$$

Given two functions $\alpha : [0,a-1] \to [0,b-1]$ and $\beta : [0,c-1] \to [0,b-1]$, we define another function $\alpha\beta$ called "concatenation of α with β" as follows :

$\alpha\beta : [0,a+c-1] \to [0,b-1]$

$$(\alpha\beta)(i) = \begin{cases} \alpha(i) \text{ if } i \in [0,a-1] \\ \beta(i-a) \text{ if } i \in [a,a+c-1] \end{cases}$$

As an example, let for an alphabet of n = 3 symbols the two functions be : $\alpha = 1021$ and $\beta = 21$. Then we have the concatenations $\alpha\beta = 102121$ and $\beta\alpha = 211021$.

We shall use definitions and notation employed in the first part of this series. Thus, cyclic equivalence will be denoted by $\simeq$. If a function α remains in the same class of cyclic equivalence after composition with another function (e.g. a permutation), we say that α is cyclically invariant with respect to this function. We denote by M_k the set of non-periodic functions from F(m,n) which are invariant with respect to a permutation from P_k. In mathematical form,

$$M_k = \{\alpha \in F(m,n) \cap A_m | \ \exists\ \pi\ P_k \text{ such that } \alpha \simeq \pi(\alpha)\}$$

The set of functions in M_k which do not belong to M_p (for any p, multiple of k) is denoted by L_k. A composition of α with the permutation

$\pi = (a-1, a-2, \ldots, 0)$ represents a function, which will be denoted by $\hat{\alpha}$, that we call the symmetrical transpose of α. We define L'_k as the set of functions in L_k such that their symmetrical transpose are cyclically invariant with respect to a permutation. In mathematical form,

$L'_k = \{\alpha \in L_k \mid \exists\ \pi \in S_n$ such that $\pi(\alpha) \simeq \hat{\alpha}\}$

As an example, $\alpha = 012102 \in F(6,3)$; $\pi = (1,0,2) \in P_2$; $\pi(\alpha) = 102012 \simeq$ $\simeq \alpha$, hence $\alpha \in M_2$. Taking into account that $P_{2k} = \emptyset$ for any $k \geqslant 2$ (according to Fermat's theorem from the theory of finite groups, the order of any permutation from S_n must divide $n!$), α must belong also to L_2. Now we examine the symmetrical transpose of α :

$\hat{\alpha} = 201210$; $\pi(\alpha) = 102012 \simeq \hat{\alpha}$

Therefore we find that $\alpha \in L'_2$.

Observations.

1. If $\alpha \in L_k$, then $\pi(\alpha) \in L_k$ for any $\pi \in S_n$.
2. $\{P_k\}_{k|n!}$ is a partition of S_n.
3. If $\alpha \in F(m,n)$, for any two different permutations π and σ from S_n we have $\pi(\alpha) \neq \sigma(\alpha)$.
4. If $\alpha \in L_k$ then there exists a unique permutation $\pi \in P_k$ such that $\pi^0(\alpha) \simeq \pi(\alpha) \simeq \ldots \simeq \pi^{k-1}(\alpha)$.

For any $\sigma \in S_n$ such that $\sigma^k \neq e$, α is not cyclically equivalent to $\sigma(\alpha)$, and we have : $\sigma(\alpha) \simeq \sigma\pi(\alpha) \simeq \ldots \simeq \sigma\pi^{k-1}(\alpha)$. Thus for $\alpha \in L_k$, the set $\{\pi(\alpha) \mid \mid \pi \in S_n\}$, having cardinal $n!$, may be partitioned in disjoint subsets of k elements each. Each of these subsets is included in a class of cyclic equivalence. As an example, again for an alphabet of three symbols, we define the function $\alpha = 011220 \in F(6,3)$, and we observe that $\alpha \in L_3$. The symmetric group of order 3 consists of $e = (0,1,2)$; $\pi_1 = (1,0,2)$;$\pi_2 = (2,1,0)$; $\pi_3 =$ $= (0,2,1)$; $\sigma_1 = (1,2,0)$ and $\sigma_2 = (2,0,1)$. The set $\{\xi(\alpha) \mid \xi \in S_n\}$ is partitioned into subsets $\{e(\alpha), \sigma_1(\alpha), \sigma_2(\alpha)\}$, $\{\pi_1(\alpha), \pi_2(\alpha), \pi_3(\alpha)\}$ such that : $e(\alpha) = 011220 \simeq \sigma_1(\alpha) = 122001 \simeq \sigma_2(\alpha) = 200112$, and $\pi_1(\alpha) = 100221 \simeq \pi_2(\alpha) =$ $= 211002 \simeq \pi_3(\alpha) = 200110$.

Proposition 1. The number, denoted by $N(m,n)$, of non-periodic, chemically non-equivalent sequences with length m over an alphabet with n symbols is :

$$N(m,n) = \frac{1}{2m \cdot n!} \sum_{k|m} k(|L_k| + |L'_k|) \tag{1}$$

Proof. Let k be a natural number which divides m. For $\alpha \in L_k$ we have the set $\{\xi(\alpha) \mid \xi \in S_n\}$ which may be partitioned into n!/k subsets from different classes of cyclic equivalence (see Observation 4). Thus the set L_k contains $|L_k|/m$ classes of cyclic equivalence. For obtaining the cardinal of a system of representatives which are independent of the S_n permutation group (i.e. which cannot be obtained from one another by a permutation within S_n and, possibly, by a cyclic permutation) we have to divide further by n!/k.

Taking into account that $L'_k \subseteq L_k$, it results that $|L_k - L'_k| = |L_k| - |L'_k|$.

In order to make the representatives of classes of chemical equivalence independent relative to symmetry, we have to consider that :

(i) in L'_k the number of classes of chemical equivalence is $|L'_k| \cdot k/(m \cdot n!)$.

(ii) in L_k we have $(|L_k| - |L'_k|) \cdot k/(2m \cdot n!)$ classes.

Therefore in L_k we have $(|L_k| + |L'_k|) \cdot k/(2m \cdot n!)$ classes. By summing over $k|m$, we obtain the formula (1). An example for n = 2 and m = 4 with $\pi = (1,0)$ is :

$\alpha_1 = 0001$; $\pi(\alpha_1) = 1110$; $\hat{\alpha}_1 = 1000$; $\pi(\hat{\alpha}_1) = 0111$
$\alpha_2 = 0010$; $\pi(\alpha_2) = 1101$; $\hat{\alpha}_2 = 0100$; $\pi(\hat{\alpha}_2) = 1011$
$\alpha_3 = 0011$; $\pi(\alpha_3) = 1100$; $\hat{\alpha}_3 = 1100$; $\pi(\hat{\alpha}_3) = 0011$
$\alpha_4 = 0100$; $\pi(\alpha_4) = 1011$; $\hat{\alpha}_4 = 0010$; $\pi(\hat{\alpha}_4) = 1101$
$\alpha_5 = 0110$; $\pi(\alpha_5) = 1001$; $\hat{\alpha}_5 = 0110$; $\pi(\hat{\alpha}_5) = 1001$
$\alpha_6 = 0111$; $\pi(\alpha_6) = 1000$; $\hat{\alpha}_5 = 1110$; $\pi(\hat{\alpha}_6) = 0001$
$\alpha_7 = 1000$; $\pi(\alpha_7) = 0111$; $\hat{\alpha}_7 = 0001$; $\pi(\hat{\alpha}_7) = 1110$
$\alpha_8 = 1001$; $\pi(\alpha_8) = 0110$; $\hat{\alpha}_{88} = 1001$; $\pi(\hat{\alpha}_8) = 0110$
$\alpha_9 = 1011$; $\pi(\alpha_9) = 0100$; $\hat{\alpha}_9 = 1101$; $\pi(\hat{\alpha}_9) = 0010$
$\alpha_{10} = 1100$; $\pi(\alpha_{10}) = 0011$; $\hat{\alpha}_{10} = 0011$; $\pi(\hat{\alpha}_{10}) = 1100$
$\alpha_{11} = 1101$; $\pi(\alpha_{11}) = 0010$; $\hat{\alpha}_{11} = 1011$; $\pi(\hat{\alpha}_{11}) = 0100$
$\alpha_{12} = 1110$; $\pi(\alpha_{12}) = 0001$; $\hat{\alpha}_{12} = 0111$; $\pi(\hat{\alpha}_{12}) = 1000$
$L'_1 = \{\alpha_1, \alpha_2, \alpha_4, \alpha_6, \alpha_7, \alpha_9, \alpha_{11}, \alpha_{12}\} = L_1$
$L'_2 = \{\alpha_3, \alpha_5, \alpha_8, \alpha_{10}\} = L_2$

According to the above formulas, we obtain that in L'_1 there is $|L'_1|/2 \cdot 4 = 1$ class of chemical equivalence, and that in L'_2 there is $|L'_2| \cdot 2/2 \cdot 4 = 1$ class of chemical equivalence, therefore formula (1) yields N(4,2) = 2.

Lemma 1. If $\alpha \in L_k$, then k divides into m and there exist : permutation $\pi \in P_k$ and subsequence γ of length m/k such that $\alpha = \pi^0(\gamma)\pi^1(\gamma)\ldots\pi^{k-1}(\gamma)$.

For the above-mentioned example $\alpha = 011220 \in F(6,3)$ we observe that

$\alpha \in L_3$, $\alpha = \pi^0(\gamma)\pi^1(\gamma)\pi^2(\gamma)$ where $\alpha = 01$, and $\pi = (1,2,0) \in P_3$.

Lemma 2.

$$|F(m,n) \cap A_m| = \sum_{k|m} |L_k|$$

The proof is based on reductio ad absurdum, by supposing the existence of $\alpha \in L_i \cap L_j$, $i < j$. From the definition of L_k it results that i is not a divisor of j, and including also Lemma 1 one obtains a contradiction.

For the sake of the next Lemma we make use of the set of triples $H(m,n) = \{(\alpha,\pi,t) \in (F(m,n) \cap A_m) \times S_n \times [0,m-1]$ with the property that $\alpha(i) = \pi(\alpha(t \oplus (-i)))$ for any $i \in [0,m-1]\}$.

The operation $\oplus$ (sum modulo m) is :

$$t \oplus (-i) = \begin{cases} t - i \text{ for } i \in [0,t] \\ m + t - i \text{ for } i \in [t+1,m-1] \end{cases}$$

For example, if $\alpha = 010020 \in F(6,3) \cap A_6$, $\pi = (0,2,1) \in S_3$; we choose $t_1 = 2$, $t_2 = 5$. Hence, $\{(\alpha,e,t_1),(\alpha,\pi,t_2)\} \subset H(6,3)$. We observe that $\alpha \in L_2'$.

Lemma 3.

$$|H(m,n)| = \sum_{k|m} k|L_k'|$$

The proof consists in showing that for each $\alpha \in L_k'$ there exist exactly k pairs (π,t) that satisfy the condition $(\alpha,\pi,t) \in H(m,n)$.

Lemma 4. For any natural number k

$$|M_k| = \sum_{k|d} |L_d|$$

Proof. From the definition of M_k it results that $\{L_d\}_{k|d} = \{L_d$ with the property that k divides into d$\}$ is a partition of M_k.

Proposition 2.

$$(2)\quad N(m,n) = \frac{1}{2m\cdot n!}\left(|F(m,n) \cap A_m| + |H(m,n)| + \sum_{\substack{k|n\\k\neq 1}} (k-1) \sum_{\substack{p|m\\k|p}} \mu(k,p)|M_p|\right)$$

where μ is the Möbius function of the lattice of divisors.

Proof. From Lemmas 2 and 3 and formula (1) we obtain :

$$(3)\quad N(m,n) = \frac{1}{2m\cdot n!}\left(|F(m,n) \cap A_m| + |H(m,n)| + \sum_{\substack{k|m\\k\neq 1}} (k-1)|L_k|\right)$$

With the help of the μ function of the lattice of divisors, a new Möbius function μ^*, is introduced such that :

$\mu^* : [1,m] \times [1,m] \to \{-1,0,1\}$, $\mu^*(p,q) = \mu(q,p)$

for any q and p from the set of the divisors of m. One observes that $[1,m]$ is a locally finite set, which is partially ordered according to the order relationship defined as : $x,y \in [1,m]$, $x \leqslant y$ iff $y|x$. The universal minorant is m.

One demonstrates easily that the function μ^* verifies the conditions of Möbius functions.

From Lemmas 1 and 4 we obtain $|M_k| = \sum_{\substack{k|p \\ p|m}} |L_p|$, therefore $|M_k| = \sum_{m \leqslant p \leqslant k} |L_p|$, where $\leqslant$ denotes the partial order relationship defined above on $[1,m]$. By applying the Möbius inversion theorem we obtain :

$$|L_k| = \sum_{\substack{k|p \\ p|m}} \mu^*(p,k) \qquad\qquad |M_p| = \sum_{\substack{k|p \\ p|m}} \mu(k,p)|M_p|$$

On replacing this result in formula (3) we obtain formula (2).

Lemma 5.

$$|F(m,n) \cap A_m| = \sum_{q|m} \mu(q,m)\cdot s_{q,n}$$

As indicating earlier, $\mu(q,m)$ is the Möbius function of the lattice of divisors, and $s_{q,n}$ is the number of surjections from a set with q elements to a set with n elements.

Proof. Let P(m,n) be a set of functions with a property Ξ ; let this property be transmitted to any restriction of the function to a period. Then we may compute the number of non-periodic functions which possess property Ξ by using the Möbius function μ and Möbius's inversion theorem:

$$|P(m,n) \cap A_m| = \sum_{q|m} \mu(q,m) \; |P(q,n)|$$

If we make the assignment $P(m,n) = F(m,n)$, and if we assume that property Ξ is surjectivity (knowing that a surjective function restricted to one period is also a surjective function), we obtain Lemma 5.

In order to obtain the next lemma, we start from a natural number r, and from the n-uple of natural numbers $(\lambda_1,\dots,\lambda_n)$. We assume that there exist λ_i boxes, each containing i distinct numbers from $[0,r-1]$. The number of functions $\gamma : [0,r-1] \to [0,n-1]$ with the property that the set of values of these functions contains at least one element from each box is denoted by $v(r ; \lambda_1,\dots,\lambda_n)$.

Lemma 6.

$$v(r\ ;\ \lambda_1,\dots,\lambda_n) = n^r - \sum_{l=1}^{z} (-1)^{l+1} \sum_{\substack{j_1+\dots+j_n=l \\ 0 \leq j_i \leq \lambda_i}} \binom{\lambda_1}{j_1}\dots\binom{\lambda_n}{j_n}\left(n - \sum_{i=1}^{n} i j_i\right)^r$$

where $z = \sum_{i=1}^{n} \lambda_i$.

Proof. One calculates the cardinal of the set of functions with the property that there exists at least one box whose elements are not present in the set of values of functions, then one may obtain the cardinal $v(r\ ;\ \lambda_1,\dots,\lambda_n)$ of the complement of this set.

Lemma 7. For two natural numbers p and m (p divides into m, i.e. $p|m$, we have :

$$|M_p| = \sum_{\substack{p|q|m \\ q \nmid \frac{m}{p}}} \mu(q,m) \sum_{(\lambda_1,\dots,\lambda_n) \in W_p} h(\lambda_1,\dots,\lambda_n)\cdot v(\tfrac{q}{p}\ ;\ \lambda_1,\dots,\lambda_n)$$

where $p|q|m$ is read as $p|q$, and $q|m$.

Proof. When $p|q$, then $\alpha \in M_p$ iff there exists a unique permutation $\alpha \in P_p$, and if there exists a function $\gamma : [0,q/p-1] \to [0,n-1]$ such that $\alpha =$ $= \pi^0(\gamma)\pi^1(\gamma)\dots\pi^{p-1}(\gamma)$, and $\alpha \in A_m$. We therefore have a bijection between M_k and the set of pairs (π,γ) from the previous sentence ; the numbers of functions $\boldsymbol{\gamma}$, which, for a given permutation $\pi \in P_p$ of type $(\lambda_1,\lambda_2,\dots,\lambda_n)$ have the property that $\pi^0(\gamma)\pi^1(\gamma)\dots\pi^{p-1}(\gamma)$ is a surjection, is exactly $v(q/p$; ; $\lambda_1,\dots,\lambda_n)$. On following the reasoning from the proof of Lemma 6 for the elimination of periodic sequences by means of the Möbius function μ, we obtain Lemma 7.

Lemma 8. Let $\alpha \in F(m,n) \cap A_m$. If there exists a permutation $\pi \in S_n$ such that $\pi(\alpha) \simeq \hat{\alpha}$, then $\pi^2 = e$.

Proof. If $\pi(\alpha) \simeq \hat{\alpha}$, then there exists $t \in [0,m-1]$ such that $\alpha(i) =$ $= \pi(\alpha(t \oplus (-i)))$ for any $i \in [0,m-1]$. Hence $\alpha(i) = \pi^2(\alpha(i))$ for any $i \in$ $\in [0,m-1]$. Since α is a surjection, it results that $\pi^2 = e$.

Lemma 9.

$$|H(m,n)| = \sum_{\lambda_1+2\lambda_2=r} h(\lambda_1,\lambda_2)\cdot m\left[\sum_{q|p} \mu(q,m)\lambda_1 v\left(\left\lfloor\frac{q-1}{2}\right\rfloor\ ;\ \lambda_1-1,\lambda_2\right) + \right.$$

$$\left. + \tfrac{1}{2}\sum_{2|q|m} \mu(q,m)\left(v(\tfrac{q}{2}\ ;\ \lambda_1,\lambda_2) - \lambda_1 v(\tfrac{q}{2} - 1\ ;\ \lambda_1-1,\lambda_2) + \lambda_1(\lambda_1-1)v(\tfrac{q}{2} - 1;\lambda_1-2,\lambda_2)\right)\right]$$

We have denoted by $v(r\ ;\ \lambda_1,\lambda_2)$ the cardinal $v(r\ ;\ \lambda_1,\lambda_2,0,\ldots,0)$.

Proof. By using Lemma 8 and the transmittance, over any restriction to one period, of the invariance to a permutation from S_n and the symmetry, we obtain :

$$(4)\quad |H(m,n)| = \sum_{q|m} \mu(q,m) \sum_{\lambda_1+2\lambda_2=n} h(\lambda_1,\lambda_2)\,\frac{m}{q}\, v'(q\ ;\ \lambda_1,\lambda_2)$$

where $v'(q\ ;\ \lambda_1,\lambda_2)$ represents the number of triplets $(\alpha,\pi,t) \in F(q,n) \times S_n \times [0,q-1]$ with the property that π is of type $(\lambda_1,\lambda_2,0,\ldots,0)$ and $\alpha(i) = \pi(\alpha(t \oplus (-i)))$ for any $i \in [0,q-1]$. We have two cases to consider, namely :

(A) if q is even,

$$v'(q\ ;\ \lambda_1,\lambda_2) = \frac{q}{2}\Big[v(\frac{q}{2}\ ;\ \lambda_1,\lambda_2) + \lambda_1 v(\frac{q}{2} - 1\ ;\ \lambda_1-1,\lambda_2) + \lambda_1(\lambda_1-1)\cdot v(\frac{q}{2} - 1\ ;\ \lambda_1-2,\lambda_2)\Big]$$

(B) if q is odd,

$$v'(q\ ;\ \lambda_1,\lambda_2) = q\lambda_1 v(\frac{q-1}{2}\ ;\ \lambda_1-1,\lambda_2).$$

On replacing in (4) we obtain Lemma 9.

Theorem. For $n \geqslant 2$ and $m \geqslant n$, the number $N(m,n)$ of non-periodic, chemically non-equivalent sequences with length m over an alphabet of n symbols is (for $|H(m,n)|$ Lemma 9 gives its explicit form) :

$$N(m,n) = \frac{1}{2m\cdot n!}\Big[\sum_{q|m} \mu(q,m) s_{q,n} + |H(m,n)| + \sum_{k|m} (k-1)\cdot \sum_{k|p|m} \mu(k,p)\cdot \sum_{\substack{p|q|m \\ q \nmid \frac{m}{p}}} (q,m) \cdot \sum_{(\lambda_1,\ldots,\lambda_n) \in W_p} h(\lambda_1,\ldots,\lambda_n)\, v(\frac{q}{p}\ ;\ \lambda_1,\ldots,\lambda_n)\Big]$$

The proof is obtained by replacing in formula (2) the results of Lemmas 5, 7, and 9.

Corollary. For the particular case when m is odd,

$$N(m,n) = \frac{1}{2m\cdot n!}\Big[\sum_{q|m} \mu(q,m)\Big(s_{q,n} + m \sum_{\lambda_1+2\lambda_2=n} h(\lambda_1,\lambda_2)\lambda_1 v(\frac{q-1}{2}\ ;\ \lambda_1-1,\lambda_2)\Big) + \sum_{k|m} (k-1) \sum_{k|p|m} \mu(k,p) \sum_{\substack{p|q|m \\ q \nmid \frac{m}{p}}} \mu(q,n) \sum_{(\lambda_1,\ldots,\lambda_n)\in W_p} h(\lambda_1,\ldots,\lambda_n) v(\frac{q}{p};\lambda_1,\ldots\lambda_n)\Big]$$

We shall now examine a few particular cases for binary, ternary and quaternary copolymers.

1°. The case n = 2 (binary copolymers, and other isomorphic sequences such as stereoregular homopolymers or elastomers, which were examined in the first part of this series[1]).

$$N(m,n) = \frac{1}{4m}\left(\sum_{q|m} \mu(q,m)2^{q} + m \sum_{q|m} \mu(q,m)2^{\lceil q/2\rceil} + m \sum_{2|q|m} \mu(q,m)2^{q/2}\right.$$

$$\left. + \sum_{\substack{2|q|m \\ q \nmid \frac{m}{2}}} \mu(q,m)2^{q/2}\right)$$

One obtains thus the result of Theorem 1 from Part 1 of the present series, in slightly different notation. As in the corollaries of that paper, one may further particularize the formula for even or odd m values.

2°. The case n = 3 (ternary copolymers) :

$$N(m,n) = \frac{1}{12m}\left[\sum_{q|m} \mu(q,m)(s_{q,3} + m(2\cdot 3^{\lceil q/2\rceil} - 3\cdot 2^{\lceil q/2\rceil})) + \right.$$

$$+ \sum_{2|q|m} \mu(q,m)m(2\cdot 3^{q/2} - 3\cdot 2^{q/2}) + 4 \sum_{\substack{3|q|m \\ q \nmid \frac{m}{3}}} \mu(q,m)\cdot 3^{q/2} +$$

$$\left. + 3 \sum_{\substack{2|q|m \\ q \nmid \frac{m}{2}}} (q,m)(3^{q/2} - 2^{q/2} - 1)\right]$$

3°. The case n = 4 (quaternary copolymers) :

$$N(m,n) = \frac{1}{48m}\left[\sum_{q|m} \mu(q,m)\left[s_{q,4} + 4m(4^{\lceil q/2\rceil} - 2\cdot 3^{\lceil q/2\rceil} + 2)\right] + \right.$$

$$+ 2 \sum_{2|q|m} \mu(q,m)(3\cdot 4^{q/2} - 4\cdot 3^{q/2}) + 3 \sum_{\substack{2|q|m \\ q \nmid \frac{m}{2}}} \mu(q,m)(3\cdot 4^{q/2} - 4\cdot 3^{q/2} - 2\cdot 2^{q/2} + 4) +$$

$$+ 16 \sum_{\substack{3|q|m \\ q \nmid \frac{m}{3}}} \mu(q,m)\ (4^{q/3} - 3^{q/3} - 1) + 12 \sum_{\substack{4|q|m \\ q \nmid \frac{m}{4}}} \mu(q,m)\cdot 4^{q/4} \Big]$$

In comparison with applying the Polya-de Bruijn theorem, the above expressions, which do not contain partial derivatives, present the advantage that they may be easily implemented by means of a pocket calculator.

For computing values of $s_{m,n}$ (the number of surjections from a set with m elements to a set with n elements), one may use the recurrence relationship :

$$s_{m+1} = ns_{m,n-1} + s_{m,n}$$

(where $s_{m,1} = 1$, and $s_{m,n} = m!$).

This relationship is obtained from the two formulas indicated below where S(m,n) is the Stirling number of the second kind, i.e. the number of partitions of a set with m elements into n classes :

$$S(m,n) = \frac{1}{n!} s_{m,n}$$

$$S(m+1,n) = S(m,n-1) = nS(m,n)$$

(where S(m,1) = S(m,m) = 1).

Asymptotic behaviour of the numbers N(m,n) of irreducible sequences

In the preceding part[1] it was shown that for $m \to \infty$ the asymptotic value for N(m,2) is $2^{m-2}/m$.

One can easily demonstrate that in the general case the asymptotic limit for N(m,n) is :

$$\lim_{n \to \infty} = n^m/2m\cdot n!$$

One sees that for n = 2 this formula becomes identical to that indicated above.

Computer program for generating an ordered list of irreducible sequences

The SEQV program, written in PASCAL using structured programming, and displayed in Table 3, generates a system of representatives of classes of chemical equivalence for sequences of length $N \leqslant NMAX$ which contain symbols from a set with cardinal M. It will be observed that from now onwards the letters M and N have been permuted relatively to the previous text and formulas (where n and m, respectively, had been used)* ; the total number of irreducible sequences will therefore be $N(m,n) = N(N,M)$.

The program was implemented on a Roumanian-made microcomputer M-216 (IBM-PC comparable) based upon the INTEL-8086 microprocessor.

The main program calls the following procedures :

INCREASE, which furnishes the next sequence (in lexicographic order) which is a candidate for being an irreducible sequence.

MINIMAL, which tests if the sequence being analysed is minimal in its equivalence class (from all sequences of this equivalence class, only one representative, namely the minimal one, is displayed).

PERIOD, which tests if the sequence is decomposable into repeating subsequences (i.e. if it is periodical).

DISPLAY which prints or displays the representatives.

The algorithm selects all minimal sequences in their equivalence classes, and displays them as representatives of these classes in lexicographic order ; it considers as alphabet the set $\{1,2,\dots,M\}$ with the order relationship from the set of the first M natural numbers.

Only those sequences will be considered which contain at least one from each symbol of the alphabet, i.e. only surjections will be considered. Consequently, the search of irreducible sequences will have as range $N \in [M, NMAX]$. For this purpose the array IND is used ; it indicates the position (rank) of the first appearance of symbols in the sequence.

Taking into account that all irreducible sequences start with a string

---

*We wish to conserve in the computer print-out (Table 4) the notation N for the length of the sequence, as in Part I[1] ; on the other hand, in the title of the present paper we wished to refer to n-ary copolymers, hence the correspondence $n = M$ and $m = N$.

(block) of ℓ symbols 1 (translated as RR...R), the sequences which contain a longer block of any other symbol cannot act as representatives because a permutation of the corresponding symbols folowed by a circular permutation of the positions leads to a smaller sequence. The array BLOCK is used for memorizing the position of subsequences forming blocks of length ℓ (when all but the first blocks have smaller lengths than ℓ, the sequence is irreducible).

By using the arrays IND and BLOCK, the procedure INCREASE provides to the main program the next sequence having the property that it is surjective and that all its blocks have lengths smaller than, or equal to, ℓ. Evidently, no irreducible sequence may end in 1 (corresponding to R). Finally the program ignores (by means of the PERIOD procedure) any periodical sequence.

On reaching a sequence which contains in its first M positions all ordered symbols of the alphabet, the algorithm goes on to sequences with length N+1 (if N < NMAX) or stops (if N = NMAX).

In Table 4 we present the results of the program for $N \in [3,10]$ and $M = 3$; in Table 5 for $N \in [4,9]$ and $M = 4$; and in Table 6 for $N \in [5,9]$ and $M = 5$.

The upper limit NMAX is not inherent in the program and may be easily increased above 10, to any desired number. Also, the M value may be increased. Of course, such increases will lead to enhanced execution times. A comparison of the present program SEQV with the previous program[1] (which was written in FORTRAN-IV and had M = 2) shows that the execution time of SEQV is lower, and that its algorithm is more performing.

TABLE 3. Computer program SEQV

```
program seqv;
const     nmax=10;
type      s1= array [1..10] of integer;
          i1= array [1..10] of integer;
          ds= array [1..6] of integer;
          data=text;
 var      ddd:data;
          seqv,block:s1;
          ind:i1;
          divis:ds;
          test,ovrfl: boolean;
          aux,c,i,ii,j,k,l,ll,m,n,nr,p,q,s,t: integer;
 procedure increase(var k,p:integer);
    var l,i: integer;
 begin
    if seqv[k]=m then
    begin while seqv[k]=m do k:=k-1;
          if ind[m]=k+1 then s:=s-1;aux:=seqv[k];
          while ind[aux]=k do
          begin k:=k-1;s:=s-1;aux:=seqv[k]
          end
    end;
    seqv[k]:=seqv[k]+1;aux:=seqv[k];
    if ind[aux]>k then
    begin s:=s+1;ind[aux]:=k end;
    if seqv[k]=seqv[k-1] then
    begin i:=1;
          while i<=p do
          begin if block[i]<k-1
                then i:=i+1
                else if block[i]=k-1 then
                     begin ovrfl:=false;i:=p+1 end
                     else p:=i-1
          end;
          l:=1;j:=k;
          while seqv[j]=seqv[j-1] do
          begin l:=l+1;j:=j-1 end;
          if l=block[1] then begin p:=p+1;block[p]:=k end
    end
    else
    begin i:=1;
          while i<=p do
          begin if block[i]<k then i:=i+1
                else begin if block[i]=k then
                           begin if i=1 then
                                 begin block[1]:=k-1;p:=1 end
                                 else p:=i-1
                           end
                           else p:=i-1;
                           i:=p+1
                     end
          end
    end;
    if block[1]=1 then
    begin p:=p+1;block[p]:=k;l:=1;i:=k+1;
          while i<=n-(m-s+1) do
          begin seqv[i]:=l;p:=p+1;block[p]:=i;
```

```
            l:=l mod 2+1;i:=i+1
        end;
        l:=s;
        while i<=n do
        begin ind[l]:=i;seqv[i]:=l;
            l:=l+1;p:=p+1;block[p]:=i;i:=i+1
        end;
        if seqv[n]=1 then
        begin if seqv[n-1]=2 then if ovrfl then
                                  begin ovrfl:=false;k:=n end;
            seqv[n]:=2
        end
    end
    else
    begin l:=0;i:=k+1;
        while i<=n-(m-s+1) do
        begin seqv[i]:=1;l:=l+1;
            if l=block[1] then
            begin l:=0;p:=p+1;block[p]:=i;
                if i<>n-(m-s+1) then
                begin i:=i+1;seqv[i]:=2 end
            end;
            i:=i+1
        end;
        l:=s;
        while i<=n do
        begin ind[l]:=i;seqv[i]:=l;i:=i+1;l:=l+1 end;
        if seqv[n]=1 then
        begin if (block[1]=2) and (seqv[n-1]=2) then
            begin if block[p]=n-1 then
                begin if ovrfl then begin ovrfl:=false;k:=n end
                end
                else begin p:=p+1;block[p]:=n end
            end
            else if block[p]=n then p:=p-1;
            seqv[n]:=2
        end
    end;
    if ovrfl then k:=n+s-m-1
end;{increase}
procedure minimal(c,dir,j:integer);
    var aux,i,niv:integer;
        perm: array[1..5] of integer;
begin for i:=1 to m do perm[i]:=0;
      niv:=0;
      while c<=n do
      begin aux:=seqv[j];
          if perm[aux]=0 then
          begin niv:=niv+1;perm[aux]:=niv end;
          if perm[aux]=seqv[c] then
          begin c:=c+1;j:=(j+n-1+dir) mod n+1 end
          else
          begin if perm[aux]<seqv[c] then test:=false;
              c:=n+1
          end
       end
end;{minimal}
```

```
procedure period(indc:integer;d:ds);
   var aux,i,k,q,r:integer;
begin if indc<=d[ll] then
      begin i:=1;while d[i]<indc do i:=i+1;
            while (i<=ll) and (test=true) do
            begin test:=false;r:=1;aux:=d[i];
                  while r<=aux do
                  begin q:=1;k:=n div aux;
                        while q<k do
                        begin if seqv[r]=seqv[aux*q+r] then q:=q+1
                              else
                              begin test:=true;q:=k;r:=aux end
                        end;
                        r:=r+1
                  end;
                  i:=i+1
            end
      end
end;{period}
procedure display(var ddd:data;var ii:integer);
   var alph:array[1..10] of char;
       i:integer;
begin for i:=1 to n do
      begin if seqv[i]=1 then write(ddd,'R')
            else if seqv[i]=2 then write(ddd,'S')
                 else if seqv[i]=3 then write(ddd,'T')
                      else if seqv[i]=4 then write(ddd,'U')
                           else write(ddd,'V')
      end;if ii=5 then begin ii:=0;writeln(ddd);
                             write(ddd,'           ')
                       end
          else write(ddd,'    ')
end;{display}
begin{main program}
      assign(ddd,'rez.dta');
      rewrite(ddd);
      read(m);writeln;
      for n:=m to nmax do
      begin write(ddd,'     N=',n);writeln(ddd);
            for i:=1 to n-m+1 do seqv[i]:=1;
            for i:=2 to m do
            begin seqv[n-m+i]:=i;ind[i]:=n-m+i end;
            ind[1]:=1;p:=1;block[1]:=n-m+1;ii:=1;
            write(ddd,'           ');display(ddd,ii);
            s:=2;test:=true;ovrfl:=true;ll:=0;aux:=n div 2;k:=n-m+1;
            for q:=m+1 to aux do
            begin if n div q*q=n then
                  begin ll:=ll+1;divis[ll]:=q end
            end;
            while ind[m]>m do
            begin increase(k,p);
                  if ovrfl then
                  begin minimal(block[1],-1,1);
                        if test then
                        begin nr:=2;
                              while nr<=p do
                              begin c:=block[1];
```

```
                              j:=block[nr]-block[1]+1;
                              minimal(c,-1,j);
                              if test then
                              minimal(block[1],1,block[nr])
                              else nr:=p;
                              nr:=nr+1
                          end;
                      end;
                      if test then
                      begin if l1<>0 then period(ind[m],divis);
                            if test then begin ii:=ii+1;
                                               display(ddd,ii)
                                         end
                            else test:=true
                      end
                      else test:=true
                 end
                 else ovrfl:=true
             end;
             if ii<5 then writeln(ddd);writeln(ddd);writeln(ddd)
        end;close(ddd)
    end.{main program}
```

TABLE 4. IRREDUCIBLE SEQUENCES FOR AN ALPHABET OF THREE LETTERS: R,S,T.

N=3

RST

N=4

RRST	RSRT

N=5

RRRST	RRSRT	RRSST	RRSTS	RSRST

N=6

RRRRST	RRRSRT	RRRSST	RRRSTS	RRSRRT
RRSRST	RRSRTS	RRSRTT	RRSSTT	RRSTST
RRSTTS	RSRSRT	RSRTST		

N=7

RRRRRST	RRRRSRT	RRRRSST	RRRRSTS	RRRSRRT
RRRSRST	RRRSRTS	RRRSRTT	RRRSSST	RRRSSTS
RRRSSTT	RRRSTST	RRRSTTS	RRSRRST	RRSRRTT
RRSRSRT	RRSRSST	RRSRSTS	RRSRSTT	RRSRTRS
RRSRTSS	RRSRTST	RRSRTTS	RRSSRTS	RRSSRTT
RRSTRST	RRSTRTS	RRSTSTS	RSRSRST	RSRSTRT
RSRTRST				

N=8

RRRRRRST	RRRRRSRT	RRRRRSST	RRRRRSTS	RRRRSRRT
RRRRSRST	RRRRSRTS	RRRRSRTT	RRRRSSST	RRRRSSTS
RRRRSSTT	RRRRSTST	RRRRSTTS	RRRSRRRT	RRRSRRST
RRRSRRTS	RRRSRRTT	RRRSRSRT	RRRSRSST	RRRSRSTS
RRRSRSTT	RRRSRTRS	RRRSRTSS	RRRSRTST	RRRSRTTS
RRRSRTTT	RRRSSRST	RRRSSRTS	RRRSSRTT	RRRSSSTT
RRRSSTSS	RRRSSTST	RRRSSTTS	RRRSTRST	RRRSTRTS
RRRSTSST	RRRSTSTS	RRRSTTTS	RRSRRSRT	RRSRRSST
RRSRRSTS	RRSRRSTT	RRSRRTRT	RRSRRTST	RRSRSRST
RRSRSRTS	RRSRSRTT	RRSRSSRT	RRSRSSTT	RRSRSTRS
RRSRSTRT	RRSRSTSS	RRSRSTST	RRSRSTTS	RRSRTRSS
RRSRTRST	RRSRTRTS	RRSRTSRT	RRSRTSST	RRSRTSTS
RRSRTSTT	RRSRTTRS	RRSRTTSS	RRSRTTST	RRSSRRST
RRSSRRTT	RRSSRTST	RRSSRTTS	RRSSTRST	RRSTRRTS
RRSTRSST	RRSTRSTS	RRSTRTST	RRSTSTST	RSRSRSRT
RSRSRTRT	RSRSRTST	RSRSTRST	RSRSTSRT	RSRTSRST

N=9

RRRRRRRST	RRRRRRSRT	RRRRRRSST	RRRRRRSTS	RRRRRSRRT
RRRRRSRST	RRRRRSRTS	RRRRRSRTT	RRRRRSSST	RRRRRSSTS

RRRRRSSTT	RRRRRSTST	RRRRRSTTS	RRRRSRRRT	RRRRSRRST
RRRRSRRTS	RRRRSRRTT	RRRRSRSRT	RRRRSRSST	RRRRSRSTS
RRRRSRSTT	RRRRSRTRS	RRRRSRTSS	RRRRSRTST	RRRRSRTTS
RRRRSRTTT	RRRRSSRST	RRRRSSRTS	RRRRSSRTT	RRRRSSSST
RRRRSSSTS	RRRRSSSTT	RRRRSSTSS	RRRRSSTST	RRRRSSTTS
RRRRSTRST	RRRRSTRTS	RRRRSTSST	RRRRSTSTS	RRRRSTTTS
RRRSRRRST	RRRSRRRTT	RRRSRRSRT	RRRSRRSST	RRRSRRSTS
RRRSRRSTT	RRRSRRTRS	RRRSRRTRT	RRRSRRTSS	RRRSRRTST
RRRSRRTTS	RRRSRRTTT	RRRSRSRST	RRRSRSRTS	RRRSRSRTT
RRRSRSSRT	RRRSRSSST	RRRSRSSTS	RRRSRSSTT	RRRSRSTRS
RRRSRSTRT	RRRSRSTSS	RRRSRSTST	RRRSRSTTS	RRRSRSTTT
RRRSRTRSS	RRRSRTRST	RRRSRTRTS	RRRSRTRTT	RRRSRTSRT
RRRSRTSSS	RRRSRTSST	RRRSRTSTS	RRRSRTSTT	RRRSRTTRS
RRRSRTTSS	RRRSRTTST	RRRSRTTTS	RRRSSRRST	RRRSSRRTS
RRRSSRRTT	RRRSSRSST	RRRSSRSTS	RRRSSRSTT	RRRSSRTSS
RRRSSRTST	RRRSSRTTS	RRRSSRTTT	RRRSSSRTS	RRRSSSTTT
RRRSSTRST	RRRSSTRTS	RRRSSTSST	RRRSSTSTS	RRRSSTSTT
RRRSSTTSS	RRRSSTTST	RRRSSTTTS	RRRSTRRST	RRRSTRRTS
RRRSTRSST	RRRSTRSTS	RRRSTRTST	RRRSTRTTS	RRRSTSSTS
RRRSTSTST	RRRSTSTTS	RRRSTTSST	RRSRRSRRT	RRSRRSRST
RRSRRSRTS	RRSRRSRTT	RRSRRSSRT	RRSRRSSTS	RRSRRSSTT
RRSRRSTRT	RRSRRSTST	RRSRRSTTS	RRSRRTRST	RRSRRTRTT
RRSRRTSST	RRSRRTSTT	RRSRSRRST	RRSRSRRTT	RRSRSRSRT
RRSRSRSST	RRSRSRSTS	RRSRSRSTT	RRSRSRTRS	RRSRSRTRT
RRSRSRTSS	RRSRSRTST	RRSRSRTTS	RRSRSSRST	RRSRSSRTS
RRSRSSRTT	RRSRSSTRT	RRSRSTRSS	RRSRSTRST	RRSRSTRTS
RRSRSTRTT	RRSRSTSRS	RRSRSTSRT	RRSRSTSST	RRSRSTSTS
RRSRSTSTT	RRSRSTTRS	RRSRSTTSS	RRSRSTTST	RRSRTRRSS
RRSRTRRST	RRSRTRRTS	RRSRTRSRT	RRSRTRSST	RRSRTRSTS
RRSRTRSTT	RRSRTRTRS	RRSRTRTSS	RRSRTRTST	RRSRTSRSS
RRSRTSRST	RRSRTSRTS	RRSRTSRTT	RRSRTSSRT	RRSRTSSTS
RRSRTSSTT	RRSRTSTRS	RRSRTSTST	RRSRTSTTS	RRSRTTRSS
RRSRTTRST	RRSRTTSST	RRSSRRSST	RRSSRRSTS	RRSSRRSTT
RRSSRRTST	RRSSRTRST	RRSSRTRTS	RRSSRTSST	RRSSTRRST
RRSSTRSTT	RRSSTRTST	RRSTRRSTS	RRSTRSRST	RRSTRSRTS
RRSTRSTST	RRSTRSTTS	RRSTRTRTS	RRSTRTSST	RRSTRTSTS
RRSTSRSTS	RRSTSRTST	RRSTSTSTS	RRSTTRSST	RSRSRSRST
RSRSRSTRT	RSRSRTRST	RSRSTRSRT	RSRSTRTST	RSRSTSTRT
RSRTSRTST				

N=10

RRRRRRRRST	RRRRRRRSRT	RRRRRRRSST	RRRRRRRSTS	RRRRRRSRRT
RRRRRRSRST	RRRRRRSRTS	RRRRRRSRTT	RRRRRRSSST	RRRRRRSSTS
RRRRRRSSTT	RRRRRRSTST	RRRRRRSTTS	RRRRRSRRRT	RRRRRSRRST
RRRRRSRRTS	RRRRRSRRTT	RRRRRSRSRT	RRRRRSRSST	RRRRRSRSTS
RRRRRSRSTT	RRRRRSRTRS	RRRRRSRTSS	RRRRRSRTST	RRRRRSRTTS
RRRRRSRTTT	RRRRRSSRST	RRRRRSSRTS	RRRRRSSRTT	RRRRRSSSST
RRRRRSSSTS	RRRRRSSSTT	RRRRRSSTSS	RRRRRSSTST	RRRRRSSTTS
RRRRRSTRST	RRRRRSTRTS	RRRRRSTSST	RRRRRSTSTS	RRRRRSTTTS
RRRRSRRRRT	RRRRSRRRST	RRRRSRRRTS	RRRRSRRRTT	RRRRSRRSRT
RRRRSRRSST	RRRRSRRSTS	RRRRSRRSTT	RRRRSRRTRS	RRRRSRRTRT
RRRRSRRTSS	RRRRSRRTST	RRRRSRRTTS	RRRRSRRTTT	RRRRSRSRST
RRRRSRSRTS	RRRRSRSRTT	RRRRSRSSRT	RRRRSRSSST	RRRRSRSSTS
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RRRRSRSTTS	RRRRSRSTTT	RRRRSRTRSS	RRRRSRTRST	RRRRSRTRTS
RRRRSRTRTT	RRRRSRTSRT	RRRRSRTSSS	RRRRSRTSST	RRRRSRTSTS

RRRRSRTSTT	RRRRSRTTRS	RRRRSRTTSS	RRRRSRTTST	RRRRSRTTTS
RRRRSRTTTT	RRRRSSRRST	RRRRSSRRTS	RRRRSSRRTT	RRRRSSRSST
RRRRSSRSTS	RRRRSSRSTT	RRRRSSRTSS	RRRRSSRTST	RRRRSSRTTS
RRRRSSRTTT	RRRRSSSRST	RRRRSSSRTS	RRRRSSSSTT	RRRRSSSTSS
RRRRSSSTST	RRRRSSSTTS	RRRRSSSTTT	RRRRSSTRST	RRRRSSTRTS
RRRRSSTSST	RRRRSSTSTS	RRRRSSTSTT	RRRRSSTTSS	RRRRSSTTST
RRRRSSTTTS	RRRRSTRRST	RRRRSTRRTS	RRRRSTRSST	RRRRSTRSTS
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RRRRSTSTTS	RRRRSTTSST	RRRRSTTTTS	RRRSRRRSRT	RRRSRRRSST
RRRSRRRSTS	RRRSRRRSTT	RRRSRRRTRT	RRRSRRRTST	RRRSRRRTTT
RRRSRRSRRT	RRRSRRSRST	RRRSRRSRTS	RRRSRRSRTT	RRRSRRSSRT
RRRSRRSSST	RRRSRRSSTS	RRRSRRSSTT	RRRSRRSTRS	RRRSRRSTRT
RRRSRRSTSS	RRRSRRSTST	RRRSRRSTTS	RRRSRRSTTT	RRRSRRTRRS
RRRSRRTRSS	RRRSRRTRST	RRRSRRTRTS	RRRSRRTRTT	RRRSRRTSRS
RRRSRRTSRT	RRRSRRTSSS	RRRSRRTSST	RRRSRRTSTS	RRRSRRTSTT
RRRSRRTTRS	RRRSRRTTRT	RRRSRRTTSS	RRRSRRTTST	RRRSRRTTTS
RRRSRSRRST	RRRSRSRRTS	RRRSRSRRTT	RRRSRSRSRT	RRRSRSRSST
RRRSRSRSTS	RRRSRSRSTT	RRRSRSRTRS	RRRSRSRTRT	RRRSRSRTSS
RRRSRSRTST	RRRSRSRTTS	RRRSRSRTTT	RRRSRSSRST	RRRSRSSRTS
RRRSRSSRTT	RRRSRSSSRT	RRRSRSSSTT	RRRSRSSTRS	RRRSRSSTRT
RRRSRSSTSS	RRRSRSSTST	RRRSRSSTTS	RRRSRSSTTT	RRRSRSTRSS
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RRRSRSTSSS	RRRSRSTSST	RRRSRSTSTS	RRRSRSTSTT	RRRSRSTTRS
RRRSRSTTSS	RRRSRSTTST	RRRSRSTTTS	RRRSRTRRSS	RRRSRTRRST
RRRSRTRRTS	RRRSRTRRTT	RRRSRTRSRT	RRRSRTRSSS	RRRSRTRSST
RRRSRTRSTS	RRRSRTRSTT	RRRSRTRTRS	RRRSRTRTSS	RRRSRTRTST
RRRSRTRTTS	RRRSRTSRSS	RRRSRTSRST	RRRSRTSRTS	RRRSRTSRTT
RRRSRTSSRT	RRRSRTSSST	RRRSRTSSTS	RRRSRTSSTT	RRRSRTSTRS
RRRSRTSTSS	RRRSRTSTST	RRRSRTSTTS	RRRSRTSTTT	RRRSRTTRSS
RRRSRTTRST	RRRSRTTRTS	RRRSRTTRTT	RRRSRTTSSS	RRRSRTTSST
RRRSRTTSTS	RRRSRTTSTT	RRRSRTTTRS	RRRSRTTTSS	RRRSRTTTST
RRRSSRRRST	RRRSSRRRTT	RRRSSRRSST	RRRSSRRSTS	RRRSSRRSTT
RRRSSRRTSS	RRRSSRRTST	RRRSSRRTTS	RRRSSRRTTT	RRRSSRSRST
RRRSSRSRTS	RRRSSRSRTT	RRRSSRSSTS	RRRSSRSSTT	RRRSSRSTSS
RRRSSRSTST	RRRSSRSTTS	RRRSSRSTTT	RRRSSRTRSS	RRRSSRTRST
RRRSSRTRTS	RRRSSRTSSS	RRRSSRTSST	RRRSSRTSTS	RRRSSRTSTT
RRRSSRTTSS	RRRSSRTTST	RRRSSRTTTS	RRRSSSRRTS	RRRSSSRTST
RRRSSSRTTS	RRRSSSRTTT	RRRSSSTRST	RRRSSTRRST	RRRSSTRRTS
RRRSSTRSST	RRRSSTRSTS	RRRSSTRSTT	RRRSSTRTSS	RRRSSTRTST
RRRSSTRTTS	RRSSSTSRST	RRRSSTSRTS	RRRSSTSSTS	RRRSSTSSTT
RRRSSTSTSS	RRRSSTSTST	RRRSSTSTTS	RRRSSTTRST	RRRSSTTRTS
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RRRSTRRSTS	RRRSTRRTST	RRRSTRRTTS	RRRSTRSRST	RRRSTRSRTS
RRRSTRSSST	RRRSTRSSTS	RRRSTRSTST	RRRSTRSTTS	RRRSTRTRTS
RRRSTRTSST	RRRSTRTSTS	RRRSTRTTST	RRRSTSRSST	RRRSTSRSTS
RRRSTSRTST	RRRSTSRTTS	RRRSTSSTST	RRRSTSSTTS	RRRSTSTSST
RRRSTSTSTS	RRRSTTRSST	RRRSTTRTTS	RRRSTTSTTS	RRSRRSRRST
RRSRRSRRTT	RRSRRSRSRT	RRSRRSRSST	RRSRRSRSTS	RRSRRSRSTT
RRSRRSRTRS	RRSRRSRTRT	RRSRRSRTSS	RRSRRSRTST	RRSRRSRTTS
RRSRRSSRRT	RRSRRSSRST	RRSRRSSRTS	RRSRRSSRTT	RRSRRSSTRT
RRSRRSSTSS	RRSRRSSTST	RRSRRSSTTS	RRSRRSTRRT	RRSRRSTRST
RRSRRSTRTS	RRSRRSTRTT	RRSRRSTSRT	RRSRRSTSST	RRSRRSTSTS
RRSRRSTSTT	RRSRRSTTRT	RRSRRSTTST	RRSRRTRRST	RRSRRTRSRT
RRSRRTRSST	RRSRRTRSTT	RRSRRTRTRT	RRSRRTRTST	RRSRRTSRST
RRSRRTSRTT	RRSRRTSSTT	RRSRRTSTST	RRSRRTTSTT	RRSRSRRSRT
RRSRSRRSST	RRSRSRRSTS	RRSRSRRSTT	RRSRSRRTRT	RRSRSRRTST
RRSRSRSRST	RRSRSRSRTS	RRSRSRSRTT	RRSRSRSSRT	RRSRSRSSTT

RRSRSRSTRS	RRSRSRSTRT	RRSRSRSTSS	RRSRSRSTST	RRSRSRSTTS
RRSRSRTRSS	RRSRSRTRST	RRSRSRTRTS	RRSRSRTRTT	RRSRSRTSRS
RRSRSRTSRT	RRSRSRTSST	RRSRSRTSTS	RRSRSRTSTT	RRSRSRTTRS
RRSRSRTTRT	RRSRSRTTSS	RRSRSRTTST	RRSRSSRRST	RRSRSSRRTS
RRSRSSRRTT	RRSRSSRSRT	RRSRSSRSTS	RRSRSSRSTT	RRSRSSRTST
RRSRSSRTTS	RRSRSSTRST	RRSRSSTRTT	RRSRSSTSRT	RRSRSSTSTT
RRSRSTRRSS	RRSRSTRRST	RRSRSTRRTS	RRSRSTRRTT	RRSRSTRSRT
RRSRSTRSST	RRSRSTRSTS	RRSRSTRSTT	RRSRSTRTRS	RRSRSTRTSS
RRSRSTRTST	RRSRSTRTTS	RRSRSTSRSS	RRSRSTSRST	RRSRSTSRTS
RRSRSTSRTT	RRSRSTSSRT	RRSRSTSSTS	RRSRSTSSTT	RRSRSTSTRS
RRSRSTSTRT	RRSRSTSTST	RRSRSTSTTS	RRSRSTTRSS	RRSRSTTRST
RRSRSTTRTS	RRSRSTTSRS	RRSRSTTSRT	RRSRSTTSST	RRSRSTTSTS
RRSRTRRSST	RRSRTRRSTS	RRSRTRRTRS	RRSRTRRTSS	RRSRTRSRST
RRSRTRSRTS	RRSRTRSRTT	RRSRTRSSRT	RRSRTRSSTS	RRSRTRSSTT
RRSRTRSTRS	RRSRTRSTSS	RRSRTRSTST	RRSRTRSTTS	RRSRTRTRSS
RRSRTRTRST	RRSRTRTRTS	RRSRTRTSRT	RRSRTRTSST	RRSRTRTSTS
RRSRTRTSTT	RRSRTSRRSS	RRSRTSRRST	RRSRTSRRTS	RRSRTSRRTT
RRSRTSRSST	RRSRTSRSTS	RRSRTSRSTT	RRSRTSRTSS	RRSRTSRTST
RRSRTSRTTS	RRSRTSSRST	RRSRTSSRTS	RRSRTSSRTT	RRSRTSSTRS
RRSRTSSTTS	RRSRTSTRSS	RRSRTSTRST	RRSRTSTRTS	RRSRTSTSRT
RRSRTSTSTS	RRSRTSTTRS	RRSRTSTTSS	RRSRTTRRSS	RRSRTTRRST
RRSRTTRRTS	RRSRTTRRTT	RRSRTTRSST	RRSRTTRSTS	RRSRTTRSTT
RRSRTTRTSS	RRSRTTSRSS	RRSRTTSRST	RRSRTTSRTS	RRSRTTSRTT
RRSRTTSSRT	RRSRTTSSTT	RRSSRRSSTT	RRSSRRSTST	RRSSRRSTTS
RRSSRRTSST	RRSSRTRSST	RRSSRTRSTS	RRSSRTRSTT	RRSSRTRTST
RRSSRTSRST	RRSSRTSRTS	RRSSRTSRTT	RRSSRTSSTT	RRSSRTSTST
RRSSRTTRST	RRSSRTTSST	RRSSTRRSTT	RRSSTRRTST	RRSSTRSRST
RRSSTRSSTT	RRSSTRSTST	RRSTRRSTST	RRSTRRSTTS	RRSTRRTSTS
RRSTRSRSTS	RRSTRSRTST	RRSTRSRTTS	RRSTRSSRTS	RRSTRSTRST
RRSTRSTRTS	RRSTRSTSTS	RRSTRTRSST	RRSTRTRSTS	RRSTRTRTST
RRSTRTSRST	RRSTRTSTST	RRSTSRRTST	RRSTSRSTST	RRSTSRSTTS
RRSTSRTSTS	RRSTSTSTST	RSRSRSRSRT	RSRSRSRTRT	RSRSRSRTST
RSRSRSTRST	RSRSRSTSRT	RSRSRTRSRT	RSRSRTRTST	RSRSRTSRST
RSRSRTSTST	RSRSTRSTRT	RSRSTRSURT	RSRSTRTSRT	RSRSTRUSRT
RSRTRSRTST	RSRTRSTRST	RSRTSRTRST		

TABLE 5. IRREDUCIBLE SEQUENCES FOR AN ALPHABET OF FOUR LETTERS: R,S,T,U.

N=4

RSTU

N=5

RRSTU	RSRTU

N=6

RRRSTU	RRSRTU	RRSSTU	RRSTSU	RRSTTU
RRSTUS	RSRSTU	RSRTRU	RSRTSU	RSRTUT
RSTRSU				

N=7

RRRRSTU	RRRSRTU	RRRSSTU	RRRSTSU	RRRSTTU
RRRSTUS	RRSRRTU	RRSRSTU	RRSRTRU	RRSRTSU
RRSRTTU	RRSRTUS	RRSRTUT	RRSRTUU	RRSSRTU
RRSSTTU	RRSSTUT	RRSTRSU	RRSTRTU	RRSTRUS
RRSTSTU	RRSTSUS	RRSTSUT	RRSTTSU	RRSTUST
RRSTUTS	RSRSRTU	RSRSTRU	RSRSTUT	RSRTRSU
RSRTSTU	RSRTSUT	RSTRSTU		

N=8

RRRRRSTU	RRRRSRTU	RRRRSSTU	RRRRSTSU	RRRRSTTU
RRRRSTUS	RRRSRRTU	RRRSRSTU	RRRSRTRU	RRRSRTSU
RRRSRTTU	RRRSRTUS	RRRSRTUT	RRRSRTUU	RRRSSRTU
RRRSSSTU	RRRSSTSU	RRRSSTTU	RRRSSTUS	RRRSSTUT
RRRSSTUU	RRRSTRSU	RRRSTRTU	RRRSTRUS	RRRSTSSU
RRRSTSTU	RRRSTSUS	RRRSTSUT	RRRSTTSU	RRRSTTTU
RRRSTTUS	RRRSTUST	RRRSTUTS	RRSRRSTU	RRSRRTRU
RRSRRTSU	RRSRRTTU	RRSRRTUT	RRSRSRTU	RRSRSSTU
RRSRSTRU	RRSRSTSU	RRSRSTTU	RRSRSTUS	RRSRSTUT
RRSRSTUU	RRSRTRSU	RRSRTRTU	RRSRTRUS	RRSRTRUT
RRSRTRUU	RRSRTSRU	RRSRTSSU	RRSRTSTU	RRSRTSUS
RRSRTSUT	RRSRTSUU	RRSRTTRU	RRSRTTSU	RRSRTTUS
RRSRTTUT	RRSRTTUU	RRSRTURS	RRSRTUSS	RRSRTUST
RRSRTUSU	RRSRTUTS	RRSRTUTT	RRSRTUTU	RRSRTUUS
RRSRTUUT	RRSSRRTU	RRSSRTSU	RRSSRTTU	RRSSRTUS
RRSSRTUT	RRSSRTUU	RRSSTRSU	RRSSTRTU	RRSSTRUT
RRSSTTUU	RRSSTUTU	RRSSTUUT	RRSTRRSU	RRSTRRTU
RRSTRSSU	RRSTRSTU	RRSTRSUS	RRSTRSUT	RRSTRTSU
RRSTRTUS	RRSTRTUT	RRSTRUST	RRSTRUSU	RRSTRUTS
RRSTRUTU	RRSTRUUS	RRSTRUUT	RRSTSTSU	RRSTSTUS
RRSTSTUT	RRSTSUST	RRSTSUTS	RRSTSUTU	RRSTSUUS
RRSTSUUT	RRSTUSTU	RRSTUSUT	RRSTUTUS	RRSTUUST
RRSTUUTS	RSRSRSTU	RSRSRTRU	RSRSRTSU	RSRSRTUT
RSRSTRSU	RSRSTRTU	RSRSTRUT	RSRSTSRU	RSRSTSTU
RSRSTSUT	RSRSTUTU	RSRTRSRU	RSRTRSTU	RSRTRSUT
RSRTRUSU	RSRTSRSU	RSRTSRTU	RSRTSRUT	RSRTSUST
RSRTSUTU	RSRTURTU	RSRTUSTU	RSRTUSUT	RSTRUSTU
RSTRUTSU				

N=9

RRRRRRSTU	RRRRRSRTU	RRRRRSSTU	RRRRRSTSU	RRRRRSTTU
RRRRRSTUS	RRRRSRRTU	RRRRSRSTU	RRRRSRTRU	RRRRSRTSU
RRRRSRTTU	RRRRSRTUS	RRRRSRTUT	RRRRSRTUU	RRRRSSRTU
RRRRSSSTU	RRRRSSTSU	RRRRSSTTU	RRRRSSTUS	RRRRSSTUT
RRRRSSTUU	RRRRSTRSU	RRRRSTRTU	RRRRSTRUS	RRRRSTSSU
RRRRSTSTU	RRRRSTSUS	RRRRSTSUT	RRRRSTTSU	RRRRSTTTU
RRRRSTTUS	RRRRSTUST	RRRRSTUTS	RRRSRRRTU	RRRSRRSTU
RRRSRRTRU	RRRSRRTSU	RRRSRRTTU	RRRSRRTUS	RRRSRRTUT
RRRSRRTUU	RRRSRSRTU	RRRSRSSTU	RRRSRSTRU	RRRSRSTSU
RRRSRSTTU	RRRSRSTUS	RRRSRSTUT	RRRSRSTUU	RRRSRTRSU
RRRSRTRTU	RRRSRTRUS	RRRSRTRUT	RRRSRTRUU	RRRSRTSRU
RRRSRTSSU	RRRSRTSTU	RRRSRTSUS	RRRSRTSUT	RRRSRTSUU
RRRSRTTRU	RRRSRTTSU	RRRSRTTTU	RRRSRTTUS	RRRSRTTUT
RRRSRTTUU	RRRSRTURS	RRRSRTUSS	RRRSRTUST	RRRSRTUSU
RRRSRTUTS	RRRSRTUTT	RRRSRTUTU	RRRSRTUUS	RRRSRTUUT
RRRSRTUUU	RRRSSRRTU	RRRSSRSTU	RRRSSRTSU	RRRSSRTTU
RRRSSRTUS	RRRSSRTUT	RRRSSRTUU	RRRSSSRTU	RRRSSSTTU
RRRSSSTUT	RRRSSTRSU	RRRSSTRTU	RRRSSTRUS	RRRSSTRUT
RRRSSTSSU	RRRSSTSTU	RRRSSTSUS	RRRSSTSUT	RRRSSTSUU
RRRSSTTSU	RRRSSTTTU	RRRSSTTUS	RRRSSTTUT	RRRSSTTUU
RRRSSTUSS	RRRSSTUST	RRRSSTUSU	RRRSSTUTS	RRRSSTUTU
RRRSSTUUS	RRRSSTUUT	RRRSTRRSU	RRRSTRRTU	RRRSTRRUS
RRRSTRSSU	RRRSTRSTU	RRRSTRSUS	RRRSTRSUT	RRRSTRTSU
RRRSTRTTU	RRRSTRTUS	RRRSTRTUT	RRRSTRUST	RRRSTRUSU
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RRRSTSSUS	RRRSTSSUT	RRRSTSTSU	RRRSTSTTU	RRRSTSTUS
RRRSTSTUT	RRRSTSUST	RRRSTSUTS	RRRSTSUTU	RRRSTSUUS
RRRSTSUUT	RRRSTTSSU	RRRSTTSTU	RRRSTTSUT	RRRSTTTSU
RRRSTTUST	RRRSTTUTS	RRRSTTUUS	RRRSTUSTU	RRRSTUSUT
RRRSTUTUS	RRRSTUUST	RRRSTUUTS	RRSRRSRTU	RRSRRSSTU
RRSRRSTRU	RRSRRSTSU	RRSRRSTTU	RRSRRSTUS	RRSRRSTUT
RRSRRSTUU	RRSRRTRRU	RRSRRTRSU	RRSRRTRTU	RRSRRTRUT
RRSRRTRUU	RRSRRTSSU	RRSRRTSTU	RRSRRTSUT	RRSRRTSUU
RRSRRTTUT	RRSRRTTUU	RRSRRTUTU	RRSRRTUUT	RRSRSRRTU
RRSRSRSTU	RRSRSRTRU	RRSRSRTSU	RRSRSRTTU	RRSRSRTUS
RRSRSRTUT	RRSRSRTUU	RRSRSSRTU	RRSRSSTRU	RRSRSSTTU
RRSRSSTUT	RRSRSTRSU	RRSRSTRTU	RRSRSTRUS	RRSRSTRUT
RRSRSTRUU	RRSRSTSRU	RRSRSTSSU	RRSRSTSTU	RRSRSTSUS
RRSRSTSUT	RRSRSTSUU	RRSRSTTRU	RRSRSTTSU	RRSRSTTUS
RRSRSTTUT	RRSRSTTUU	RRSRSTURS	RRSRSTURT	RRSRSTURU
RRSRSTUSS	RRSRSTUST	RRSRSTUSU	RRSRSTUTS	RRSRSTUTT
RRSRSTUTU	RRSRSTUUS	RRSRSTUUT	RRSRTRRSU	RRSRTRRTU
RRSRTRRUU	RRSRTRSRU	RRSRTRSSU	RRSRTRSTU	RRSRTRSUS
RRSRTRSUT	RRSRTRSUU	RRSRTRTRU	RRSRTRTSU	RRSRTRTUS
RRSRTRTUT	RRSRTRTUU	RRSRTRURS	RRSRTRUSS	RRSRTRUST
RRSRTRUSU	RRSRTRUTS	RRSRTRUTT	RRSRTRUTU	RRSRTRUUS
RRSRTRUUT	RRSRTSRSU	RRSRTSRTU	RRSRTSRUS	RRSRTSRUT
RRSRTSRUU	RRSRTSSRU	RRSRTSSTU	RRSRTSSUS	RRSRTSSUT
RRSRTSSUU	RRSRTSTRU	RRSRTSTSU	RRSRTSTTU	RRSRTSTUS
RRSRTSTUT	RRSRTSTUU	RRSRTSURS	RRSRTSURT	RRSRTSUSS
RRSRTSUST	RRSRTSUSU	RRSRTSUTS	RRSRTSUTT	RRSRTSUTU
RRSRTSUUS	RRSRTSUUT	RRSRTTRSU	RRSRTTRUS	RRSRTTRUT
RRSRTTRUU	RRSRTTSRU	RRSRTTSSU	RRSRTTSTU	RRSRTTSUS

RRSRTTSUT	RRSRTTSUU	RRSRTTURS	RRSRTTUSS	RRSRTTUST
RRSRTTUSU	RRSRTTUTS	RRSRTTUUS	RRSRTTUUT	RRSRTURSS
RRSRTURST	RRSRTURSU	RRSRTURTS	RRSRTURTT	RRSRTURTU
RRSRTURUS	RRSRTURUT	RRSRTUSRT	RRSRTUSST	RRSRTUSSU
RRSRTUSTS	RRSRTUSTT	RRSRTUSTU	RRSRTUSUS	RRSRTUSUT
RRSRTUSUU	RRSRTUTRS	RRSRTUTSS	RRSRTUTST	RRSRTUTSU
RRSRTUTTS	RRSRTUTUS	RRSRTUTUT	RRSRTUUSS	RRSRTUUST
RRSRTUUTS	RRSRTUUTT	RRSSRRSTU	RRSSRRTSU	RRSSRRTTU
RRSSRRTUT	RRSSRTRSU	RRSSRTRTU	RRSSRTRUS	RRSSRTRUT
RRSSRTRUU	RRSSRTSSU	RRSSRTSTU	RRSSRTSUT	RRSSRTSUU
RRSSRTTSU	RRSSRTTUS	RRSSRTTUU	RRSSRTUST	RRSSRTUSU
RRSSRTUTS	RRSSRTUTU	RRSSRTUUT	RRSSTRRSU	RRSSTRRTU
RRSSTRRUT	RRSSTRSTU	RRSSTRSUT	RRSSTRSUU	RRSSTRTSU
RRSSTRTUT	RRSSTRTUU	RRSSTRUST	RRSSTRUTU	RRSSTRUUT
RRSSTTRUT	RRSSTURTU	RRSSTURUT	RRSSTUTUT	RRSTRRSTU
RRSTRRSUS	RRSTRRSUT	RRSTRRTSU	RRSTRRTUS	RRSTRRUSU
RRSTRSRSU	RRSTRSRTU	RRSTRSRUS	RRSTRSRUT	RRSTRSSTU
RRSTRSTSU	RRSTRSTTU	RRSTRSTUS	RRSTRSTUT	RRSTRSUST
RRSTRSUSU	RRSTRSUTS	RRSTRSUTU	RRSTRSUUS	RRSTRSUUT
RRSTRTRSU	RRSTRTRTU	RRSTRTRUS	RRSTRTSSU	RRSTRTSTU
RRSTRTSUS	RRSTRTSUT	RRSTRTUST	RRSTRTUSU	RRSTRTUTS
RRSTRTUTU	RRSTRTUUS	RRSTRTUUT	RRSTRURST	RRSTRURTS
RRSTRUSST	RRSTRUSSU	RRSTRUSTS	RRSTRUSTU	RRSTRUSUS
RRSTRUSUT	RRSTRUTST	RRSTRUTSU	RRSTRUTTS	RRSTRUTUS
RRSTRUTUT	RRSTRUUST	RRSTRUUTS	RRSTSRSTU	RRSTSRSUS
RRSTSRSUT	RRSTSRTSU	RRSTSRTUS	RRSTSRTUT	RRSTSRUST
RRSTSRUTS	RRSTSRUTU	RRSTSRUUS	RRSTSRUUT	RRSTSTSTU
RRSTSTSUS	RRSTSTSUT	RRSTSTUST	RRSTSTUSU	RRSTSTUTS
RRSTSTUTU	RRSTSTUUS	RRSTSUSTS	RRSTSUSTU	RRSTSUSUT
RRSTSUTST	RRSTSUTSU	RRSTSUTUS	RRSTSUUST	RRSTTRSSU
RRSTTRSUT	RRSTTRUUS	RRSTTSUUS	RRSTURSTU	RRSTURSUT
RRSTURTUS	RRSTURUST	RRSTURUTS	RRSTUSTSU	RRSTUSTUS
RRSTUSUST	RRSTUSUTS	RRSTUTSTU	RRSTUTUTS	RSRSRSRTU
RSRSRSTRU	RSRSRSTUT	RSRSRTRSU	RSRSRTRTU	RSRSRTRUT
RSRSRTSTU	RSRSRTSUT	RSRSRTUTU	RSRSTRSRU	RSRSTRSTU
RSRSTRSUT	RSRSTRTRU	RSRSTRTSU	RSRSTRTUT	RSRSTRURT
RSRSTRUST	RSRSTRUTU	RSRSTSRTU	RSRSTSRUT	RSRSTSTRU
RSRSTSURT	RSRSTSUTU	RSRSTURTU	RSRSTURUT	RSRTRSRTU
RSRTRSTRU	RSRTRSTSU	RSRTRSTUT	RSRTRSUTU	RSRTRURSU
RSRTRUSTU	RSRTRUTSU	RSRTSRSTU	RSRTSRTSU	RSRTSRTUT
RSRTSRUST	RSRTSRUSU	RSRTSRUTU	RSRTSTRUT	RSRTSTUSU
RSRTSURTU	RSRTSURUT	RSRTSUTSU	RSRTURSTU	RSRTURSUT
RSTRSTRSU	RSTRSURTU	RSTRUSRTU		

TABLE 6. IRREDUCIBLE SEQUENCES FOR AN ALPHABET OF FIVE LETTERS: R,S,T,U,V.

N=5

RSTUV				

N=6

RRSTUV	RSRTUV	RSTRUV		

N=7

RRRSTUV	RRSRTUV	RRSSTUV	RRSTRUV	RRSTSUV
RRSTTUV	RRSTUSV	RRSTUTV	RRSTUVS	RSRSTUV
RSRTRUV	RSRTSUV	RSRTUTV	RSRTUVT	RSTRSUV
RSTRUSV				

N=8

RRRRSTUV	RRRSRTUV	RRRSSTUV	RRRSTRUV	RRRSTSUV
RRRSTTUV	RRRSTUSV	RRRSTUTV	RRRSTUVS	RRSRRTUV
RRSRSTUV	RRSRTRUV	RRSRTSUV	RRSRTTUV	RRSRTURV
RRSRTUSV	RRSRTUTV	RRSRTUUV	RRSRTUVS	RRSRTUVT
RRSRTUVU	RRSRTUVV	RRSSRTUV	RRSSTRUV	RRSSTTUV
RRSSTUTV	RRSSTUUV	RRSSTUVT	RRSTRRUV	RRSTRSUV
RRSTRTUV	RRSTRUSV	RRSTRUTV	RRSTRUUV	RRSTRUVS
RRSTRUVT	RRSTRUVU	RRSTSTUV	RRSTSUSV	RRSTSUTV
RRSTSUUV	RRSTSUVS	RRSTSUVT	RRSTSUVU	RRSTTSUV
RRSTTUSV	RRSTUSTV	RRSTUSUV	RRSTUSVT	RRSTUSVU
RRSTUTSV	RRSTUTUV	RRSTUTVS	RRSTUUSV	RRSTUUTV
RRSTUVST	RRSTUVTS	RSRSRTUV	RSRSTRUV	RSRSTSUV
RSRSTUTV	RSRSTUVT	RSRTRSUV	RSRTRURV	RSRTRUSV
RSRTRUVU	RSRTSRUV	RSRTSTUV	RSRTSUSV	RSRTSUTV
RSRTSUVT	RSRTSUVU	RSRTURTV	RSRTURVT	RSRTUSTV
RSRTUSUV	RSRTUSVT	RSRTUVTU	RSRTUVUT	RSTRSTUV
RSTRSUTV	RSTRUSTV	RSTRUSVU	RSTRUTSV	RSTURSTV

N=9

RRRRRSTUV	RRRRSRTUV	RRRRSSTUV	RRRRSTRUV	RRRRSTSUV
RRRRSTTUV	RRRRSTUSV	RRRRSTUTV	RRRRSTUVS	RRRSRRTUV
RRRSRSTUV	RRRSRTRUV	RRRSRTSUV	RRRSRTTUV	RRRSRTURV
RRRSRTUSV	RRRSRTUTV	RRRSRTUUV	RRRSRTUVS	RRRSRTUVT
RRRSRTUVU	RRRSRTUVV	RRRSSRTUV	RRRSSSTUV	RRRSSTRUV
RRRSSTSUV	RRRSSTTUV	RRRSSTUSV	RRRSSTUTV	RRRSSTUUV
RRRSSTUVS	RRRSSTUVT	RRRSSTUVU	RRRSSTUVV	RRRSTRRUV
RRRSTRSUV	RRRSTRTUV	RRRSTRUSV	RRRSTRUTV	RRRSTRUUV
RRRSTRUVS	RRRSTRUVT	RRRSTRUVU	RRRSTSSUV	RRRSTSTUV
RRRSTSUSV	RRRSTSUTV	RRRSTSUUV	RRRSTSUVS	RRRSTSUVT
RRRSTSUVU	RRRSTTSUV	RRRSTTTUV	RRRSTTUSV	RRRSTTUTV
RRRSTTUUV	RRRSTTUVS	RRRSTTUVT	RRRSTUSTV	RRRSTUSUV
RRRSTUSVT	RRRSTUSVU	RRRSTUTSV	RRRSTUTUV	RRRSTUTVS
RRRSTUUSV	RRRSTUUTV	RRRSTUUVS	RRRSTUVST	RRRSTUVTS
RRSRRSTUV	RRSRRTRUV	RRSRRTSUV	RRSRRTTUV	RRSRRTUTV

RRSRRTUUV	RRSRRTUVT	RRSRSRTUV	RRSRSSTUV	RRSRSTRUV
RRSRSTSUV	RRSRSTTUV	RRSRSTURV	RRSRSTUSV	RRSRSTUTV
RRSRSTUUV	RRSRSTUVS	RRSRSTUVT	RRSRSTUVU	RRSRSTUVV
RRSRTRRUV	RRSRTRSUV	RRSRTRTUV	RRSRTRURV	RRSRTRUSV
RRSRTRUTV	RRSRTRUUV	RRSRTRUVS	RRSRTRUVT	RRSRTRUVU
RRSRTRUVV	RRSRTSRUV	RRSRTSSUV	RRSRTSTUV	RRSRTSURV
RRSRTSUSV	RRSRTSUTV	RRSRTSUUV	RRSRTSUVS	RRSRTSUVT
RRSRTSUVU	RRSRTSUVV	RRSRTTRUV	RRSRTTSUV	RRSRTTURV
RRSRTTUSV	RRSRTTUTV	RRSRTTUUV	RRSRTTUVS	RRSRTTUVT
RRSRTTUVU	RRSRTTUVV	RRSRTURSV	RRSRTURTV	RRSRTURUV
RRSRTURVS	RRSRTURVT	RRSRTURVU	RRSRTURVV	RRSRTUSRV
RRSRTUSSV	RRSRTUSTV	RRSRTUSUV	RRSRTUSVS	RRSRTUSVT
RRSRTUSVU	RRSRTUSVV	RRSRTUTRV	RRSRTUTSV	RRSRTUTTV
RRSRTUTUV	RRSRTUTVS	RRSRTUTVT	RRSRTUTVU	RRSRTUTVV
RRSRTUUSV	RRSRTUUTV	RRSRTUUVS	RRSRTUUVT	RRSRTUUVV
RRSRTUVRS	RRSRTUVSS	RRSRTUVST	RRSRTUVSU	RRSRTUVSV
RRSRTUVTS	RRSRTUVTT	RRSRTUVTU	RRSRTUVTV	RRSRTUVUS
RRSRTUVUT	RRSRTUVUU	RRSRTUVUV	RRSRTUVVS	RRSRTUVVT
RRSRTUVVU	RRSSRRTUV	RRSSRTRUV	RRSSRTSUV	RRSSRTTUV
RRSSRTUSV	RRSSRTUTV	RRSSRTUUV	RRSSRTUVS	RRSSRTUVT
RRSSRTUVU	RRSSRTUVV	RRSSTRRUV	RRSSTRSUV	RRSSTRTUV
RRSSTRUSV	RRSSTRUTV	RRSSTRUUV	RRSSTRUVT	RRSSTRUVU
RRSSTRUVV	RRSSTTRUV	RRSSTTUUV	RRSSTTUVU	RRSSTURTV
RRSSTURUV	RRSSTURVT	RRSSTURVU	RRSSTUTUV	RRSSTUTVT
RRSSTUTVU	RRSSTUUTV	RRSSTUUVT	RRSSTUVTU	RRSSTUVUT
RRSTRRSUV	RRSTRRTUV	RRSTRRUSV	RRSTRRUVU	RRSTRSRUV
RRSTRSSUV	RRSTRSTUV	RRSTRSUSV	RRSTRSUTV	RRSTRSUUV
RRSTRSUVS	RRSTRSUVT	RRSTRSUVU	RRSTRTRUV	RRSTRTSUV
RRSTRTUSV	RRSTRTUTV	RRSTRTUUV	RRSTRTUVS	RRSTRTUVT
RRSTRTUVU	RRSTRURSV	RRSTRURTV	RRSTRURVS	RRSTRUSSV
RRSTRUSTV	RRSTRUSUV	RRSTRUSVS	RRSTRUSVT	RRSTRUSVU
RRSTRUTSV	RRSTRUTTV	RRSTRUTUV	RRSTRUTVS	RRSTRUTVT
RRSTRUTVU	RRSTRUUSV	RRSTRUUTV	RRSTRUUVS	RRSTRUUVT
RRSTRUVST	RRSTRUVSU	RRSTRUVSV	RRSTRUVTS	RRSTRUVTU
RRSTRUVTV	RRSTRUVUS	RRSTRUVUT	RRSTRUVUV	RRSTRUVVS
RRSTRUVVT	RRSTRUVVU	RRSTSRSUV	RRSTSRTUV	RRSTSRUSV
RRSTSRUTV	RRSTSRUUV	RRSTSRUVS	RRSTSRUVT	RRSTSRUVU
RRSTSTSUV	RRSTSTUSV	RRSTSTUTV	RRSTSTUUV	RRSTSTUVS
RRSTSTUVT	RRSTSTUVU	RRSTSUSTV	RRSTSUSUV	RRSTSUSVS
RRSTSUSVT	RRSTSUSVU	RRSTSUTSV	RRSTSUTUV	RRSTSUTVS
RRSTSUTVT	RRSTSUTVU	RRSTSUUSV	RRSTSUUTV	RRSTSUVST
RRSTSUVSU	RRSTSUVTS	RRSTSUVTU	RRSTSUVTV	RRSTSUVUS
RRSTSUVUT	RRSTSUVVS	RRSTSUVVT	RRSTSUVVU	RRSTTRSUV
RRSTTRUSV	RRSTTRUUV	RRSTTRUVS	RRSTTRUVT	RRSTTSUUV
RRSTTSUVS	RRSTTUSVU	RRSTURSTV	RRSTURSUV	RRSTURSVT
RRSTURSVU	RRSTURTSV	RRSTURTUV	RRSTURTVS	RRSTURUSV
RRSTURUTV	RRSTURUVS	RRSTURVST	RRSTURVTS	RRSTUSTSV
RRSTUSTUV	RRSTUSTVS	RRSTUSTVU	RRSTUSUSV	RRSTUSUTV
RRSTUSUVS	RRSTUSUVT	RRSTUSVST	RRSTUSVSU	RRSTUSVTS
RRSTUSVTU	RRSTUSVUT	RRSTUTSTV	RRSTUTSUV	RRSTUTSVU
RRSTUTUSV	RRSTUTUTV	RRSTUTUVS	RRSTUTVSU	RRSTUTVTS
RRSTUTVUS	RRSTUUSTV	RRSTUUSVT	RRSTUUTSV	RRSTUUTVS
RRSTUVSTU	RRSTUVSUT	RRSTUVTUS	RRSTUVUST	RRSTUVUTS
RSRSRSTUV	RSRSRTRUV	RSRSRTSUV	RSRSRTUTV	RSRSRTUVT
RSRSTRSUV	RSRSTRTUV	RSRSTRURV	RSRSTRUSV	RSRSTRUTV
RSRSTRUVT	RSRSTRUVU	RSRSTSRUV	RSRSTSTUV	RSRSTSURV
RSRSTSUTV	RSRSTSUVT	RSRSTSUVU	RSRSTURTV	RSRSTURUV

RSRSTURVT	RSRSTURVU	RSRSTUTUV	RSRSTUTVT	RSRSTUTVU
RSRSTUVTU	RSRSTUVUT	RSRTRSRUV	RSRTRSTUV	RSRTRSURV
RSRTRSUSV	RSRTRSUTV	RSRTRSUVT	RSRTRSUVU	RSRTRURSV
RSRTRUSTV	RSRTRUSUV	RSRTRUSVU	RSRTRUTSV	RSRTRUTUV
RSRTRUTVU	RSRTSRSUV	RSRTSRTUV	RSRTSRUSV	RSRTSRUTV
RSRTSRUVT	RSRTSRUVU	RSRTSTRUV	RSRTSTUSV	RSRTSTUVU
RSRTSURTV	RSRTSURUV	RSRTSURVT	RSRTSURVU	RSRTSUSTV
RSRTSUSVT	RSRTSUTSV	RSRTSUTUV	RSRTSUTVT	RSRTSUTVU
RSRTSUVST	RSRTSUVTU	RSRTSUVTV	RSRTSUVUT	RSRTURSTV
RSRTURSUV	RSRTURSVT	RSRTURSVU	RSRTURTUV	RSRTURTVT
RSRTURTVU	RSRTURVTU	RSRTURVTV	RSRTURVUT	RSRTUSTUV
RSRTUSTVU	RSRTUSUTV	RSRTUSVTU	RSRTUSVTV	RSRTUSVUT
RSRTUVTUV	RSTRSTRUV	RSTRSTUSV	RSTRSURSV	RSTRSURTV
RSTRSURVU	RSTRSUTVU	RSTRUSRTV	RSTRUSTUV	RSTRUSTVU
RSTRUSVTU	RSTRUTSUV	RSTRUTSVU	RSTURSTUV	

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