

MOLECULAR CONNECTIVITY IN HÜCKEL'S MOLECULAR ORBITAL THEORY.

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ABSTRACT

The molecular connectivity consideration developed by Kier and Hall is suggested to be applied in the Hückel molecular orbital (HMO) theory. The h parameter for heteroatoms in the HMO model is defined in terms of atom connectivity indices and it is found to give as good results as its classical value obtained empirically. To exemplify, ionization potentials, electron affinities, bond orders and bond lengths are calculated for a series of hetero-aromatic systems. The connectivity indices are thus shown to rationalize the parametrization of h .

INTRODUCTION

Molecular connectivity, a topological characterization of molecular structure, has been extensively used in chemistry and drug research.¹ It encodes information about size, branching, cyclization, unsaturation, and heteroatom content.¹ The treatment of heteroatoms by Kier and Hall² introducing valence molecular connectivity brought molecular connectivity into the role of a structure quantitation method useful in structure-activity analyses of drug molecules.

Several investigators have employed valence molecular connectivity in quantitative structure-activity relationship (QSAR) studies on drugs.³

On the other hand the quantum chemistry has provided very powerful tools known as molecular orbital methods to encode the electronic structure of molecules. The simplest of them is the one developed by Hückel⁴ and known as Hückel molecular orbital (HMO) method. Though this method is very primitive and the crudest of all the molecular orbital methods, it has been extensively and successfully used in chemistry,⁵ biochemistry,⁶ and drug research,⁷ and it has been argued that for a comparative study on π -electron systems, this method is as good as any refined method.⁸⁻¹³

In HMO approximation, the coulomb integral (α_μ) for an atom μ , and the resonance integral ($\beta_{\mu\nu}$) for two bonded atoms μ and ν are given as,⁵⁻⁷

$$\alpha_\mu = \alpha_0 + h\beta_0 \quad (1)$$

$$\beta_{\mu\nu} = k\beta_0 \quad (2)$$

where α_0 and β_0 are the standard coulomb and resonance integrals for carbon atom and carbon-carbon bond, respectively, and h and k are the semi-empirical parameters under discussion; h is zero if μ is carbon atom and k is unity if μ and ν both are carbon atoms formally bonded to each other. For heteroatoms the values of h and k were obtained empirically, and because of the empiricism they varied from author to author. Now molecular connectivity consideration provides a

very simple way of rationalizing the evaluation of at least one of these parameters, i.e., h , as discussed below.

The empirical value of h_{μ} where μ is any heteroatom was based on the assumption that α_{μ} was proportional to empirical electronegativities¹⁴ and that h_{μ} was proportional to electronegativity difference.^{15,16} The proportionality constant was frequently taken as unity leading to

$$h_{\mu} = E_{\mu} - E_C \quad (3)$$

where E_{μ} represents the electronegativity of the heteroatom μ and E_C that of the carbon atom. Based on this concept, a uniform method can be found to evaluate h by the use of molecular connectivity indices. The most important in this respect are the first order simple connectivity index (${}^1\chi$) and the first order valence molecular connectivity index (${}^1\chi^v$). Both are calculated from the hydrogen suppressed graph of the molecule and are defined as,

$${}^1\chi = \sum (\delta_i \delta_j)^{-1/2} \quad (4)$$

$${}^1\chi^v = \sum (\delta_i^v \delta_j^v)^{-1/2} \quad (5)$$

where the sum is over all connections or edges in the hydrogen suppressed graph, and δ is the number of atoms adjacent or connected to any atom in the graph, while δ^v for any atom is defined as,

$$\delta^v = z^v - h \quad (6)$$

where z^v is the number of valence electrons of the atom and h the number of the hydrogen atoms attached to it.

Now δ essentially is a count of nonhydrogen sigma-bond electrons contributed by any atom, while δ^V is a more inclusive count of all valence electrons excluding those bonding hydrogen and including those sigma electrons (σ) enumerated by δ . Thus the relationship is³

$$\begin{aligned}\delta^V &= \delta + p + n \\ &= \sigma + p + n - h\end{aligned}\quad (7)$$

where p is the number of p-orbital electrons and n the number of lone-pair electrons on the atom. Thus δ^V and δ values are the counts of the electrons in sigma, pi, or lone-pair orbitals. The number and distribution of electrons certainly influence the electronic characteristics associated with atoms in the valence state. An obvious electronic property of a bonded atom in a molecule is electronegativity, and Kier and Hall³ found Mulliken's electronegativity to be related with $(\delta^V - \delta)$ as,

$$\begin{aligned}E_M &= 2.05 (\delta^V - \delta) + 6.99 \\ n &= 9, r = 0.989, s = 0.60, F = 305\end{aligned}\quad (8)$$

where n is the number of data points, r is the correlations coefficient, s is the standard deviation, and F is F-ratio between the variances of calculated and observed values. This correlation is excellent with the standard deviation being less than the estimate of Hinze and Jaffe¹⁷ due to uncertainties in the electron affinities. The intercept is close to the electronegativity of hydrogen (7.17 eV) which would have $(\delta^V - \delta)$ equal to zero. The electronegativity of an atom in its valence state is shown to be

closely related with the number of electrons in pi and lone-pair orbitals, since $(\delta^V - \delta) = p + n$ (Eq. 7).

Eq. 8 was obtained only for second-row atoms. The general expression relating the Mulliken electronegativities to δ values for 19 atoms of the first, second, and third rows was as

$$E_M = 7.99 \frac{(\delta^V - \delta)}{N^2} + 7.07$$

$$n = 19, r = 0.988, s = 0.48, F = 660 \quad (9)$$

where N is the principal quantum number. Now based on these discussions and using Eqs. 3 and 8 (or 9) we can define Hückel parameter h_μ as

$$h_\mu = a [(\delta^V - \delta)_\mu - (\delta^V - \delta)_c] \quad (10)$$

or

$$h_\mu = a' \left[\left(\frac{\delta^V - \delta}{N^2} \right)_\mu - \left(\frac{\delta^V - \delta}{N^2} \right)_c \right] \quad (11)$$

where a and a' are constants. Since these constants can be taken care of by the constant β_0 in Eq. 1, we can simply write

$$h_\mu = (\delta^V - \delta)_\mu - (\delta^V - \delta)_c \quad (12)$$

or

$$h_\mu = \left(\frac{\delta^V - \delta}{N^2} \right)_\mu - \left(\frac{\delta^V - \delta}{N^2} \right)_c \quad (13)$$

Now Eqs. 12 and 13 can be utilized to evaluate h to be used in HMO calculation. If, however, δ^V values for third-row atoms were calculated from the equation,^{18,19}

$$\delta^V = \frac{Z^V - h}{Z - Z^V} \quad (14)$$

where Z is the atomic number of the atom, the use of Eq. 13 can be avoided and Eq. 12 can be used for all types of atoms belonging to first, second or third row. For simplicity let us denote the h based on connectivity consideration as $h(\delta)$.

RESULTS AND DISCUSSIONS

HMO calculations are made on ionization potentials, electron affinities, and bond lengths of some heteroaromatic systems using $h(\delta)$ as well as empirical h .⁵ In either case the empirical k values were used, as the rationalization of k based on molecular connectivity consideration is somewhat complex. This may be taken up in a separate paper.

In Hückel theory the energy of any molecular orbital is given as,

$$E = \alpha + m\beta \quad (15)$$

According to Koopmann's theorem the energy of the highest occupied molecular orbital (HOMO) corresponds to the ionization potential, and that of the lowest unoccupied molecular orbital (LUMO) to the electron affinity.⁵ Hence we can write

$$I = \alpha + m(\text{HOMO})\beta \quad (16)$$

$$A = \alpha + m(\text{LUMO})\beta \quad (17)$$

where I and A stand for ionization potential and electron affinity, respectively. Now the values of α and β for both equations (16) and (17) can be separately evaluated by using some known values of ionization potential and

electron affinity. Thus we get

$$I = 6.448 + 2.932 m(\text{HOMO}) \quad (18)$$

$$A = 1.266 + 1.806 m(\text{LUMO}) \quad (19)$$

with the use of ionization potentials of benzene and naphthalene as 9.38 and 8.26 eV, respectively,⁵ and their electron affinities as -0.54 and 0.15 eV, respectively.²⁰ From either approach, the HMO gives $m(\text{HOMO})$ of benzene = 1.0, $m(\text{HOMO})$ of naphthalene = 0.618, $m(\text{LUMO})$ of benzene = -1.0, and $m(\text{LUMO})$ of naphthalene = -0.618. Now using Eqs. 18 and 19, the ionization potential and electron affinity calculated from both the approaches are listed in Tables I and II, respectively. Corresponding observed data are also given for the comparison. The comparison shows that use of $h(\delta)$ in HMO theory gives as good results as the use of empirical h .

Calculations were made on bond orders and bond lengths too. The bond orders and bond lengths calculated by the two approaches are listed in Table III. The bond length (r) is related with bond order (p) as,⁵

$$r = A - Bp \quad (20)$$

The values of A and B for the different types of bond were evaluated by Dewar and Gleicher,²¹ and are listed in Table IV. Using the bond order, calculated by the two approaches, in Eq. 20, bond lengths were calculated for various molecules. The comparison of calculated bond lengths with the observed ones again shows the validity of $h(\delta)$.

TABLE I : Calculated and Observed Ionization Potentials of Some Heteroaromatic Systems.

Molecule	m(HOMO)		Ionization Potential (eV)		
	a	b	Obs	Calcd ^a	Calcd ^b
Pyridine	1.000	1.000	9.28 ^c	9.38	9.38
Pyridazine	1.106	1.471	8.91 ^d	9.69	10.76
Pyrimidine	1.077	1.222	9.47 ^c	9.61	10.03
Pyrazine	1.000	1.000	9.36 ^d	9.38	9.38
Quinoline	0.703	0.857	8.62 ^d	8.51	8.96
Isoquinoline	0.646	0.684	8.54 ^d	8.34	8.45
Aniline	0.744	0.586	7.69 ^e	8.63	8.17
o-Phenylene diamine	0.604	0.405	7.69 ^f	8.22	7.64
m-Phenylene diamine	0.682	0.529	7.35 ^e	8.45	8.00
p-Phenylene diamine	0.576	0.390	7.25 ^f	8.14	7.59
Pyrrole	0.618	0.618	8.22 ^c	8.26	8.26
Phenol	0.827	0.901	8.50 ^g	8.87	9.09
Furan	0.618	0.613	8.77 ^c	8.26	8.25
Chlorobenzene	1.000	0.594	9.07 ^h	9.38	8.19
Bromobenzene	1.000	0.233	8.98 ^h	9.38	7.13

^a Using empirical h.

^b Using h(δ).

^c D.W. Turner, Adv. Phys. Org. Chem., 4, 31 (1966).

^d M.J.S. Dewar and A. Harget, Proc. Roy. Soc. (London), A-315, 457 (1970).

^e F.I. Vilesov and A.N. Terenin, Doklady Akad. Nauk, SSSR, 115, 744 (1957).

^f G. Briegleb and J. Czekalla, Z. Electrochem., 63, 6 (1959).

^g K. Watanabe et al., J. Quart. Spect. Rad. Transfer., 2, 369 (1959).

^h S.G. Lias and P. Ausloos, J. Am. Chem. Soc., 100, 6027 (1978).

TABLE II : Calculated and Observed Electron Affinities of
Some Heteroaromatic Systems.

Molecule	m(LUMO)		Electron affinity(eV)		
	a	b	Obs ^c	Calcd ^a	Calcd ^b
p-Benzoquinone	0.254	0.399	1.83	1.72	1.99
Fluorobenzoquinone	0.240	0.487	1.92	1.69	2.15
Chlorobenzoquinone	0.247	0.567	2.05	1.71	2.29
Bromobenzoquinone	0.248	0.227	2.01	1.71	1.68
Iodobenzoquinone	0.151	0.110	2.00	1.54	1.46
Cyanobenzoquinone	0.296	0.447	2.22	1.80	2.08
Nitrobenzoquinone	0.384	0.476	2.63	1.96	2.13
Nitrobenzene	0.154	0.204	1.19	1.54	1.63
1,2-Dinitrobenzene	0.372	0.862	1.07	1.94	2.82
Phthalamide	-0.186	0.141	1.34	0.93	1.52
Maleic anhydride	0.000	0.123	1.33	1.27	1.49
Dichlorophthalic anhydride ^d	-0.193	0.095	1.50	0.92	1.44
Tetrachlorophthalic anhydride	-0.204	0.036	1.72	0.90	1.33
4-Nitrophthalic anhydride	0.219	0.463	1.80	1.66	2.10

a,b See footnotes of Table I.

c E.C.M. Chen and W.E. Wentworth, J. Chem. Phys., 63
3183 (1975).

d The positions of chlorines were not mentioned in the source for the observed value, calculation was however made for 3,4-dichlorophthalic anhydride.

TABLE III : Calculated and Observed Bond Lengths and Bond Orders of Some Hetero-Aromatic Systems.

Compound	Bond	<u>π-Bond Order</u>		<u>Bond Length (Å)</u>		
		a	b	Obs ^c	Calcd ^a	Calcd ^b
Pyridine	1-2	0.654	0.520	1.340	1.331	1.354
	2-3	0.669	0.697	1.400	1.396	1.391
	3-4	0.665	0.647	1.390	1.397	1.400
Pyrimidine	1-2	0.658	0.562	1.335	1.330	1.347
	3-4	0.649	0.496	1.355	1.331	1.359
	4-5	0.667	0.676	1.395	1.397	1.395
Pyrazine	1-2	0.660	0.576	1.334	1.330	1.344
	2-3	0.661	0.599	1.388	1.398	1.409
Pyrrole	1-2	0.439	0.503	1.383	1.369	1.357
	2-3	0.790	0.762	1.371	1.375	1.380
	3-4	0.553	0.573	1.429	1.417	1.413
Furan	1-2	0.385	0.316	1.371	1.331	1.343
	2-3	0.811	0.830	1.354	1.371	1.368
	3-4	0.537	0.528	1.440	1.420	1.421

a,b See footnotes of Table I.

c Ref. 21.

TABLE IV : Values of A and B.²¹

Bond	A (Å)	B (Å)
C-C	1.514	0.176
C-N	1.445	0.175
C-O	1.396	0.169
N-N	1.419	0.179

We therefore find that $h(\delta)$ can be successfully applied in the HMO theory. This provides a convenient theoretical way to parametrize the Hückel theory which can be successfully applied in chemistry and drug research.

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