match no. 2 pp. 123-137 1976

INFORMATION STUDY OF ATOMIC NUCLEI. INFORMATION FOR PROTON-NEUTRON COMPOSITION

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(received: January 1976)

Abstract

Equations have been derived, expressing the information content of the atomic nuclei as a function of the mass number, the number of neutrons, and the isodifferent number. Some possible applications to the nuclei systematics and the study of the analogy between nuclear and electron structure have been shown. A correlation has been found between the information content and the binding energy of the isodifferent atomic nuclei.

1. Introduction

Any system possessing a determined structure contains some quantity of information. The information content of different systems is usually determined by the Shannon's equation [1]:

$$\bar{I} = -\sum p_i \log_2 p_i \quad \text{bits/element} \qquad (1.1)$$

where $\bar{I}$ is the average quantity of information in bits, corresponding to an element of the examined system; $p_i = N_i/N$ is the probability of a randomly chosen element of the examined system, consisting of N elements, to be in a definite (i-th) group of elements, N_i in number.

The "structural" information of the nonliving systems (molecules, atoms, atomic nuclei) introduced by equation (1.1) plays a role of an additional statistical characteristic connected with the degree of complexity of the systems.

This makes it particularly useful for different classificational aims, as well as in establishing analogies between systems of different nature.

No data is available in literature for information investigations of atomic nuclei. However, there are no principle obstacles for such investigations, since atomic nuclei possess a determined structure, and they are constructed of two distinct types of structural elements - protons and neutrons.

In the light of aforementioned the aim of the present work is to apply the information theory in investigation of atomic nuclei, and in particular, in deriving some analytical expressions for the information for proton-neutron composition carried by each atomic nucleus. This information quantity may be useful in analyzing various problems, and, above all, in the interpretation of some nuclear properties, as well as in the attemps at a nuclei systematics by analogy with the periodic table.

2. Information for Atomic Nuclei Proton-Neutron Composition. General Analysis.

The probability for a randomly chosen elementary particle, from a nucleus having mass number A and possessing z protons and n neutrons being a proton, is $p_z = \frac{z}{A} = \frac{A - \beta}{2A}$,

and alternatively the probability of being a neutron is $p_n = \frac{n}{A} = \frac{A + \beta}{2A}$, where $\beta = n - z = A - 2z$. Then each nucleon will carry an average quantity of information for proton-neutron composition of the atomic nuclei:

$$\bar{I} = -\frac{z}{A}\lg_2\frac{z}{A} - \frac{n}{A}\lg_2\frac{n}{A} \quad \text{bits/nucleon} \qquad \ldots(2.1)$$

The information defined according to equation (2.1) was calculated for all known 2β -stable nuclides up to $^{209}_{83}Bi$, taken from the chart of Nuclides [2].

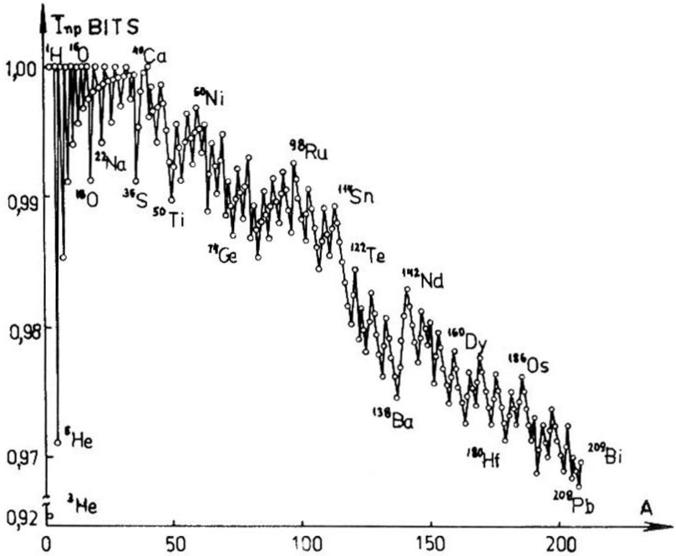

Fig.1 Dependence of the average information for proton-neutron composition per nucleon on the mass number A of the 2β -stable nuclei

As can be seen from fig.1, the average information for proton-neutron composition alters regularly with increase of A. The general form of the curve is zigzag, but the values of $\bar{I}_{np}$ fluctuate around a mean value, near to 1 bit. The average information carried by one nucleon is exactly 1 bit for all atomic nuclei having equal number of protons and neutrons. The total information for proton-neutron composition expressed in bits for these 15 2β -stable nuclei:

$$I_{np} = A.\bar{I}_{np} = A\lg_2 A - z\lg_2 z - n\lg_2 n \quad \text{bits/nucleus...(2.2)}$$

will be exactly equal to the mass number A:

$$\text{for } z = n,\ I_{np} = A \text{ bits/nucleus; } \bar{I}_{np} = 1 \text{ bit/nucleon..(2.3)}$$

Since the number of the neutrons grows with the increase of the mass number more rapidly than that of the protons, $\bar{I}_{np}$ deviates from the whole number value 1 bit remaining close to it. The maximums and minimums in fig.1 correspond to a smallest, respectively to a largest relative excess of neutrons for each series of several 2β-stable nuclei. This reflects fairly the periodic filling up of the nucleons' states and manifests the "slowing down" of this process for protons.

It can be also shown that some series of nuclei in fig.1 can be described by general equations. The investigation of these "information analogues" may be of some interest for systematics of the atomic nuclei.

The total information for proton_neutron composition, calculated by equation (2.2), increases relatively monotonously with the increase of the mass number. That is why it seams advisably to introduce the "differential" information characteristic, $\triangle I_{np}$, which expresses the increase of the information content between two isotopes with mass numbers respectively A and A - 1:

$$\triangle I_{np} = I_{np}^{A} - I_{np}^{A-1} \qquad .....(2.4)$$

This quantity is shown in fig.2 as a function of the mass number A for the 209 studied 2β-stable nuclides.

It may be stated that the differential information alters harmoniously with the increase of the mass number. It fluctuates around the average value 1 bit and two branches on fig.2 with positive and respectively negative deviations from this value may be distinguished.

There are some sharply expressed minimums and one maximum in fig.2, too. The last is due to the absence of 2β-stable isotope of Tc, while the minimums are connected with disturbances in the order of filling up the nuclei with protons and neutrons. These are the cases when a nucleus with a smaller number of protons is inserted between the nuclei with a larger one: $^{36}_{16}S$ between $^{35}_{17}Cl$ and $^{37}_{17}Cl$; $^{40}_{18}Ar$ between $^{39}_{19}K$ and $^{41}_{19}K$ etc.

Several areas may be set apart in fig.2 with different

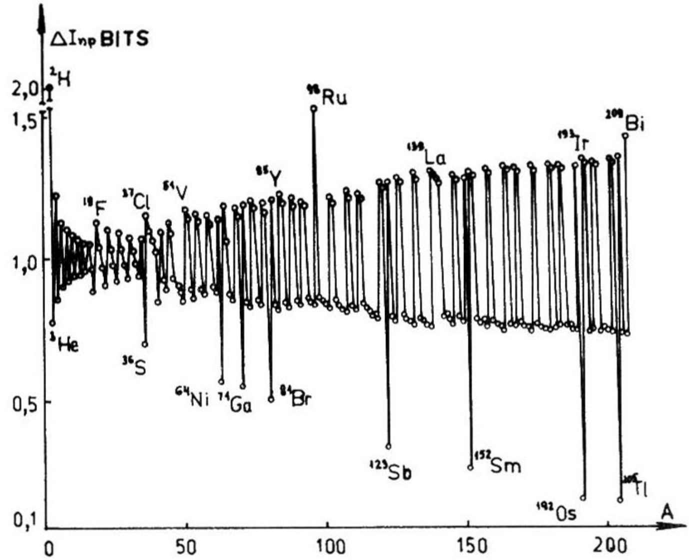

Fig.2 Dependence of the differential information for proton-neutron composition of 2β -stable nuclei on their mass number.

form of the function ΔI_{np} depending on the type of filling in the 2β -stable nuclei - deutron type (n, p) at A<20 and helium type (2n, 2p and 4n, 2p) at higher values of the mass number.

A conclusion can be made, that the information quantities introduced by equations (2.1), (2.2) and (2.4) reflect fairly the main features of the proton-neutron structure of the atomic nuclei.

3. Deduction of Some Basic Equations

In a previous paper, where the information for proton-neutron composition [3] was defined for the first time, empirical equations for I_{np} and ΔI_{np} were obtained for eight groups of nuclei. The present study deals with an alternative method for deriving from the general information equations (2.1) and (2.2) simplified equations for $\bar{I}_{np}$ and $I_{\bar{n}p}$ which include in principal all atomic nuclei, stable or radioactive, known or still unknown. Meanwhile, as it will be shown, these expressions are sufficiently accurate.

Let's substitute in equation (2.1) the neutrons and the protons in a given nucleus through their difference β and the mass number A. After some transformations we obtain:

$$\bar{I}_{np} = 1 - \frac{1}{2\ln 2}\ln\left[1-\left(\frac{\beta}{A}\right)^2\right] - \frac{\beta}{2A\ln 2}\ln\left[\frac{1+\frac{\beta}{A}}{1-\frac{\beta}{A}}\right] \qquad ....(3.1)$$

Since $A > \beta$, the expansion of the two logarithmic functions into McLauren's series is possible, by which one obtains:

$$\bar{I}_{np} = 1 - \frac{1}{\ln 2}\left[\frac{1}{2}\left(\frac{\beta}{A}\right)^2 + \frac{1}{12}\left(\frac{\beta}{A}\right)^4 + \frac{1}{30}\left(\frac{\beta}{A}\right)^6 + ...\right] \qquad ...(3.2)$$

The third and the following terms in this series can be ignored, since the error introduced in this way is smaller than 0,01%. By ignoring the second term the relative error is relatively great for $^{3}_{2}He$ only ($\varepsilon \approx 0,15\%$), but for all the remaining 2β-stable nuclei it does not exceed 0,02% (for 43 nuclei it is 0,02%, for 28 nuclei - 0,01% and for the remaining 137 nuclei it is less than 0,01%). This picture does not alter essentially when one takes into account the radioactive nuclei [4]. For nuclides with moderate and high values of the mass number $\varepsilon < 0,05\%$, while for the unstable light nuclei (A = 3 - 20) it is of the same order, as for ^{3}He. Hence the equation (3.2) can be written with sufficient accuracy only with the first term of the series:

$$\bar{I}_{np} = 1 - \frac{1}{2\ln 2}\left(\frac{\beta}{A}\right)^2 = 1 - \frac{1}{2\ln 2}\left(\frac{A-2z}{A}\right)^2 \text{ bits/nucleon} \qquad ..(3.3)$$

The equation (3.3) expresses in a simplified form the dependence between the information for proton-neutron composition of the atomic nuclei and the basic parameters of this composition - the mass number A and the number of protons z. It can be seen that in the case of $\beta > 0$ (which includes the great majority of the atomic nuclei), $\bar{I}_{np}$ increases with the increase of the mass number and the atomic number, and it diminishes with the increase of the isodifferent number and the number of neutrons. The depedence of the information on β, z and n reverses however for a limited number of nuclei having negative isodifferent number ($\beta<0$, $A<2z$, $A>2n$).

An evaluation can be made for the limits of values within which $\bar{I}_{np}$ is defined. For the 2β-stable nuclei:

$$0 \leqslant \frac{\beta}{A} \leqslant 0,212; \; 0,965 \leqslant \bar{I}_{np} \leqslant 1 \text{ bits/nucleon} \quad ......(3.4)$$

$^{3}_{2}He$ is the only exception from the latter inequalities ($\frac{\beta}{A}$ = 0,33, $\bar{I}_{np}$ = 0,918 bits). Most of the radioactive nuclei also obey to (3.4) although the low limit of this inequality is broadening at higher mass numbers to 0,95 bits and for some cases of light radioactive nuclei $\bar{I}_{np}$ is of the same order as for ^{3}He.

4. Equations for the Differential Information for Proton-Neutron Composition

Simplified equations can be also derived for the differential information ΔI_{np}, defined already by equation (2.4):

$$\Delta I_{np} = I^{A}_{np} - I^{A-1}_{np} = A(\bar{I}^{A}_{np} - \bar{I}^{A-1}_{np}) + \bar{I}^{A-1} \quad ......(4.1)$$

$$\Delta I_{np} = 1 - \frac{1}{2\ln 2} \cdot \frac{\beta}{A}^{2} \left[1 - \frac{(1 \pm \frac{1}{\beta})^{2}}{(1 - \frac{1}{A})} \right] \quad ....(4.2)$$

In the equation (4.2) the sign "+" is related to nucleus with mass number A, having one proton more in comparison with the nucleus with mass number A-1, while the sign "-" signifies the presence of one more neutron at an analogous ratio of the mass number. These two signs correspond to the presence of two almost symmetrical parts of the function ΔI_{np} (fig.2) whose values are higher and smaller respectively than 1 bit. Alternatively, it is not difficult to estimate that the first term in equation (4.1) will have a relatively low value (0,1 ÷ 0,3 bits), determining only a deviation from the value 1 bit, which is rather close to the value of the second term. All 2β -stable isotopes are within these limits (0,7 ÷ 1,3 bits) with exception of ${}^{2}_{1}H$. Also for the radioactive nuclei the differenti al information remains within the same limits. This interval broadens only for some of the lightest nuclei, where it is from 0,49 bits/nucleus for the pair ${}^{4}_{3}Li/{}^{3}_{2}He$ up to 1,61 bits/nucleus for the pair ${}^{5}_{3}Li/{}^{4}_{3}Li$.

5. "Defect" of Information

The total information for proton-neutron composition of one nucleus, expressed in bits according to equation (2.2), is a magnitude quite near to the mass number A. It seems convenient for the difference between these quantities to be introduced as an additional information characteristic of each atomic nucleus called "defect" of information by analogy with the defect of mass upon atomic nuclei formation. This difference decreases with the increase of the mass number. It decreases, also, for the restricted number of nuclei with $\beta < 0$, but in the principal case $\beta > 0$ it increases with the increase of the isodifferent number:

$$\triangle I^{*}_{np} = A - I_{np} = \frac{1}{2\ln 2}\cdot\frac{\beta^{2}}{A} = \frac{1}{2\ln 2}\cdot\frac{(A-2z)^{2}}{A} \text{ bits/nucleus..(5.1)}$$

The values of the "defect" of information are for all 2β -stable nuclei in the range 0 - 3,2%, and in most cases they remain smaller than 1%. The unique exception here is ${}^{3}_{2}He$ (8,1%). Also for the radioactive nuclei $\triangle I^{*}_{np} \leqslant 4\%$, and only for several of the lightest ones it raises to higher relative values (e.g. for ${}^{6}_{4}Be$ and ${}^{9}_{6}C$ it is about 8,2%).

When the number of protons and neutrons in a given atomic nucleus is equal, each of these particles carries exactly 1 bit information (equation 2,3). Then $\triangle I^{*}_{np}$ expressed the loss of information upon atomic nucleus formation, caused by deviation of the nucleus from the symmetrical state $n = z$. Also, since this deviation leads to a decrease of the binding energy, it may be supposed, that inversely proportional dependence exists between the "defect" of the information, and the "defect" of the mass. The final section of this work looks for a concrete expression of this dependence. Here we shall draw attention to the idea that the defect of information can be treated as a negative component of the binding energy. This idea is supported by the presence of the so called parameter of the relative symmetry of the nucleus, δ, with a negative sign, in Weizsäcker's equation [5] for the energy of atomic nucleus. This quantity coincides by virtue of a coefficient to defect of the information:

$$\delta = -\varepsilon \frac{(A - 2z)^2}{A} \quad [\text{energy}] \quad ....(5.2)$$

The remarkable coincidence between equations (5.1) and (5.2) can be considered as evidence for the usefulness of the information theory upon the study of the atomic nuclei. It gives an opportunity to derive an expression for the equivalence between information for proton-neutron composition and the binding energy of the atomic nuclei. Since the coefficient ε = 18,1 Mev, then the following equation will be in force:

$$E_b = - k.\Delta I^{*}_{np} \quad ....(5.3)$$

where k = 25,1 mev/bit.

6. Information Equations for the Four Main Groups of Nuclides

As a consequence of (3.3), (4.2) and (5.1), information equations can be drawn for the four groups of nuclides: isobars, isodifferents, isotopes, and isotones.

T A B L E 1

Information Equations for the Four Main Groups of Nuclides

Eq.No	Equations	Constants
	I. Isobars (A=const)	
6.1	$\bar{I}_{np}^{ib} = 1 - c\beta^2 = 1 - c(A-2z)^2$	$c = (2A^2 \ln 2)^{-1}$
6.2	$\Delta I_{np}^{ib} = a - bz$	$a = \frac{2(A+1)}{A \ln 2}$, $b = \frac{4}{A \ln 2}$
6.3	$\Delta I_{np}^{*} = cA(A - 2z)^2$	
	II. Isodifferents (β=n-z=const)	
6.4	$\bar{I}_{np}^{id} = 1 - \frac{d}{A^2}$	$d = \frac{\beta^2}{2 \ln 2}$
6.5	$\Delta I_{np}^{id} = 2\left[1 + \frac{d}{A(A - 2)}\right]$	
6.6	$\Delta I_{np}^{*} = \frac{d}{A}$	
	III. Isotopes (z=const)	
6.7	$\bar{I}_{np}^{ip} = 1 - \frac{1}{2 \ln 2}(1 - \frac{e}{A})^2$	$e = 2z$
6.8	$\Delta I_{np}^{ip} = f + \frac{g}{A(A-1)}$	$f = 0{,}2786$
6.9	$\Delta I_{np}^{*} = \frac{A}{2 \ln 2}(1 - \frac{e}{A})^2$	$g = \frac{4z^2}{2 \ln 2}$
	IV. Isotones (n=const)	
6.10	$\bar{I}_{np}^{in} = 1 - \frac{1}{2 \ln 2}(\frac{e'}{A} - 1)^2$	$e' = 2n$
6.11	$\Delta I_{np}^{in} = f + \frac{g'}{A(A-1)}$	$g' = \frac{4n^2}{2 \ln 2}$
6.12	$\Delta I_{np}^{*} = \frac{A}{2 \ln 2}(\frac{e'}{A} - 1)^2$	

7. Information Equations and the Systematics of Nuclides

The equations for the isotopes, isobars, isotones, and isodifferents introduced above, may be applied for the information interpretation of the nuclear periodicity and the systematics of the nuclei. These Equations connect the four kinds of nuclides (which are the foundation of their systematics expressed into the "Chart of Nuclides" [2]) in a common quantitative scheme. Each nuclide in this scheme is situated at the intersection of the four lines, the isobars, isotopes, isotones, and isodifferents lines, each of which is described by three information equations (6.1 - 6.12). In

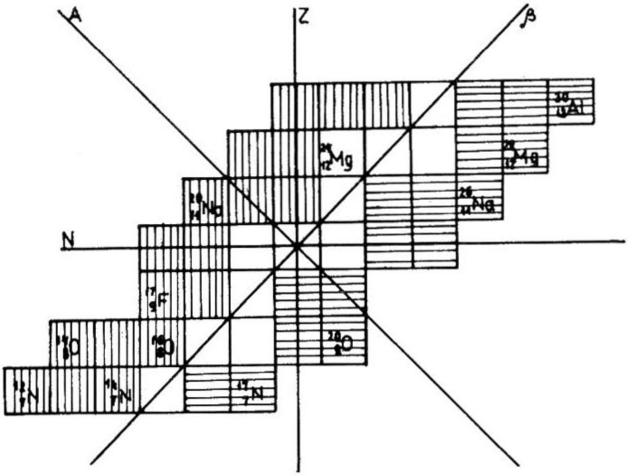

Fig.3 A fragment of the Chart of Nuclides with the lines of the four main groups of nuclides : isobars (A), isotopes (z), isotones (n), and isodifferents (β), which are described by information equations.

fig.3 a fragment is shown of the Chart of nuclides with the axes determining the place of a given nuclide.

Along with that of interest is the fact of similarity in form of equation (3.3) to the previously derived expression for the average spin information of one electron [6]. In the latter, the atomic number of the element z is an analogue to the mass number A and the number of the single electrons in the electron shell ($a = e_{-1/2} - e_{+1/2}$) takes the place of the number of the "single" neutrons which remain unpaired with protons ($\beta = n - z$):

$$\bar{I}_{sp} = 1 - \frac{1}{2\ln 2}\left(\frac{a}{z}\right)^2 \quad \text{bits/electron}\ldots\ldots\ldots(7.1)$$

This coincidence in the information equations of one electron and one nucleon allows an analogy to be found between structure and properties of the atomic nuclei and the electron shells.

T A B L E 2

Information Analogy between Nuclides and Chemical Elements

No.	NUCLIDES Parameters	NUCLIDES Groups	CHEMICAL ELEMENTS Parameters	CHEMICAL ELEMENTS Groups
1.	β=const, A≠const	Isodifferents	a=const, z≠const	Vertical groups
2.	A→A+1, $\beta \to \beta+1$ (n=const)	Isotones	z→z+1, a→a+1	$(s^1, p^1 \div p^3, d^1 \div d^5, f^1 \div f^7)$ Periods
3.	A→A+1, $\beta \to \beta-1$ (z=const)	Isotopes	z→z+1, a→a−1	$(s^2, p^4 \div p^6, d^6 \div d^{10}, f^8 \div f^{14})$
4.	A=const, β≠const $\Delta\beta=2$	Isobars	z=const, a≠const $\Delta a=2$	Electronic states of a chemical element with different spin multiplicity

As can be seen, the isodifferents and isobars have separate analogues, whilst the isotopes and isotones are common analogues to the elements in a given period of the Periodic System. Half of the elements in each period are analogues to the isotones. These elements have sequental filling of the atomic orbitals with single electrons ($a \rightarrow a+1$). The second half of the elements in the period, where each atomic orbital is populated by a second electron ($a \rightarrow a-1$), are analogues of the isotopes. It may be expected that the analogy between chemical elements and nuclides considered above, will turn to be useful not only for the systematics of the nuclides, but also when looking for a correlation between nuclear and electronic properties.

8. The Correlation between Binding Energy and Information for Proton-Neutron Composition of Atomic Nuclei

In the section concerned with the "defect" of information, it was already suggested by the Weizsäcker's equation [5], that a possible connection between binding energy and the information for proton-neutron composition of the atomic nuclei exists. It was shown that the defect of information increases in conjunction with the decrease of nuclear binding enrgy at greater deviations from the symmetrical proton-neutron distribution ($n=z$).

The dependence between either of defect of information, or the total information for proton-neutron composition, and the binding energy, is traced here at an equal deviation from the symmetrical case $n = z$, i.e. for groups of isodifferent nuclei ($\beta = n - z = \text{const}$). It was found that the binding energy in this case also grows with the decrease of the defect of information (or according to equation (6.6) - with the increase of the mass number A):

$$E_b = b + \frac{k}{\Delta I^{*}_{np}} \qquad \text{......(8.1)}$$

Along with that, direct linear correlation between binding energy and information for proton-neutron composition was derived for the isodifferent nuclei:

$$E_b = a.I^{id}_{np} + b \qquad \text{......(8.2)}$$

From equations (2.2) and (6.4) a parallel increase of the two quantities in (8.2) follows with the mass number increase.

Equation (8.1) and (8.2) may be considered as supporting Baker's ideas [7,8] that the binding energy is a function of the isodifferent number n-z only. These equations are given below for some series of isodifferents. The values of the coefficient k in them depend on the isodifferent number:

$$k = 8539\beta^{1.819} \qquad ....(8.3)$$

T A B L E 3

Constants in the Correlation Equations between the Binding Energy and the Information for Proton-Neutron Composition of Atomic Nuclei*

β	a	b	k	β	a	b	k
5	8.22	27.62	148.9	30	6.42	302.37	4266.9
10	7.95	65.38	579.0	40	5.85	439.21	6977.2
15	7.73	96.16	1275.1	50	5.42	550.61	10096.0
20	6.82	212.07	1999.8	55	5.42	562.32	12252.3

* a in meV/bit, b in meV, k in meV.bit

The equation (8.2) is more accurate at greater values of the isodifferent number. In this way, for $\beta \geqslant 15$, the relative error $\varepsilon < 1\%$; when $5 \leqslant \beta \leqslant 15$ for some of the nuclei $\varepsilon = 1 \div 5\%$, although for the most of them it remains smaller than 1%; for $\beta < 5$, along with the approximate equal number of nuclei with $\varepsilon < 1\%$ and $\varepsilon = 1 \div 5\%$, some series with $\varepsilon > 5\%$ appear.

The correlations between the information for proton-neutron composition of the atomic nuclei and such basic characteristics as their binding energy, introduced by equations (8.1) and (8.2), can be reviewed as a supplementary argument for the applicability and usefulness of Information Theory in studying atomic nuclei. The prove of this concept was the basic aim of the present work.

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